Assignment 9, due Friday, 10th December

Theoretical:

1. Prove the Corollary (*given in lecture*): Consider the linear boundary-value problem

$$Ly(x) \equiv -y'' + p(x)y' + q(x)y = r(x), \quad a < x < b \tag{1}$$

with boundary conditions $y(a) = \alpha$ and $y(b) = \beta$. Let $p(x)$ and $q(x)$ satisfy the following conditions,

$$|p(x)| \leq P,$$
$$0 < \tilde{Q} \leq q(x) \leq \hat{Q},$$
and
$$h \leq \frac{2}{P}.$$

Then, the tridiagonal linear system resulting from the finite difference method, $Aw = r$, with coefficients

$$a_j = -\frac{1}{2}\left(1 + \frac{h}{2}p(x_j)\right),$$
$$b_j = \left(1 + \frac{h^2}{2}q(x_j)\right),$$
and
$$c_j = -\frac{1}{2}\left(1 - \frac{h}{2}p(x_j)\right).$$

for $1 \leq j \leq J$, has a unique solution w through LU decompositon. *Hint: Use the Theorem given in the lecture.*

2. Assume $p(x) \equiv 0$ in (1). Devise a fourth-order-accurate difference approximations to L.

4. Find explicitly the coefficients $\{a_{k,l}\}$ for $k, l = 1, 2, \ldots, m$, for the equation

$$-y'' + y = f,$$

assuming that the space $\mathcal{A}$ is spanned by chapeau functions on an equidistant grid.

Computational:

No computation!