Theory of Geometrical Methods for Design of Laser Shaping Systems

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Beam Shaping Applications

In a holographic projection processing system featured on 10 January 1999 issue of *Applied Optics*, a two-lens beam shaping optic increased the quality of micro-optical arrays.
Concepts and Context

Expansion system

Laser

Spatial filter

Beam reshaping system

Optical system

Detector

Binary optics

San Diego, 2 August 2000

SPIE’s 45th Annual Meeting, Laser
Beam Shaping Conf., 4095-01
Historical Background

• Kreuzer, US patent 3,476,463, 1969:
  “Coherent light optical system yielding an output beam of desired intensity distribution at a desired equi-phase surface.”
Historical Background, cont.


Overview of Geometrical Methods

• Conservation of energy within a bundle of rays – geometrical optics intensity law.

• Ray trace equations.

• Constant optical path length condition.
Geometrical Optics Intensity Law

- Conservation of energy within geometrical optics – intensity law – follows from scalar wave equation:

\[
\left( \nabla^2 + n^2 k_0^2 \right) \varepsilon(r) = 0 \quad \text{Assume } \varepsilon(r) = \varepsilon_0(r) \exp[i k_0 S(r)]
\]

\[
\left( \nabla S \right)^2 = n^2
\]

\[
x_0 \nabla S \cdot \nabla x_0 + x_0^2 \nabla^2 S = 0
\]

\[
\nabla \cdot (f \mathbf{v}) = f \nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla f
\]

\[
\nabla \cdot \left( I \frac{\nabla S(r)}{n(r)} \right) = \nabla \cdot \left( I \mathbf{a}(r) \right) = 0 \quad I_1 dA_1 = I_2 dA_2
\]
Ray Trace Equations

- Ray Trace Equations:

\[
\frac{d}{ds} \left( n(r) \frac{dr}{ds} \right) = \nabla n(r)
\quad \quad \quad \quad \quad \quad \quad r(s) = as + b
\]

- Law Reflection & Refraction:

\[
A = a - 2\hat{n}(a \cdot \hat{n})
\]

\[
n'A = na + [n' \cos i' - n \cos i] \hat{n}
\]
Constant Optical Path Length Condition

- Impose the constant optical path length condition for all rays:

\[ OPL(C) = \int_C n(x, y, z) ds \]

\[ (OPL)_0 = (OPL)_r \]
Optical Design of Laser Beam Shaping Systems: Global Optimization vs. Solution of Differential Equations

- Global Optimization with discrete & continuous variables:
  - Beam shaping merit function
    \[ M = M_{\text{Diameter}} M_{\text{Collimation}} M_{\text{Uniformity}} \]

- Geometrical methods leads differential equations:
  \[ z(r) = \int f(r) \, dr + C \quad Z(r) = z(r) + g(r) \]
Merit Function Topography for Beam Shaper/Projector

\[ M = \frac{1}{\mu} \exp\left[ -0.01 \{50 - P_N\}^2 \right], \]

\[ \mu = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \{u(P_i) - \bar{u}\}^2} \]

\[ \bar{u} = \frac{1}{N} \sum_{i=1}^{N} u(P_i). \]

Zoomed Area (above)

Thin Lens Element

Shaping Element

Input Plane

Output Surface

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Beam Profile on Output Surface

\[ u(P_i) = \sigma(\rho_i) \left( \frac{\cos(i_i^{\text{in}}) \rho_i (\rho_i - \rho_{i-1}) \cos(x_i^{\text{out}})}{\cos(i_i^{\text{out}}) P_i (P_i - P_{i-1})} \right) . \]
Application of the Geometrical Methods - Differential Equations

- One mirror system used to uniformly illuminate a curved surface.
- Two lens system used to expand and transform an input laser beam into a top-hat collimated output beam.
- Two mirror system used to transform a 2-axis Gaussian input beam into a top-hat collimated output beam.
One-mirror Profile Shaping System

- Ray trace equation
  
  \[-(R - r)z'^2 + 2(Z - z)z' + (R - r) = 0\]

- Energy balance
  
  \[I_{in}(r)2\pi r dr = I_{out}(R)2\pi R \left[ dR^2 + dZ^2 \right]^{1/2} = I_{out}(R)2\pi R \sqrt{1 + \left( \frac{dZ}{dR} \right)^2} dR\]

- Differential equation of mirror
  
  \[\frac{z''}{z'} = \frac{1}{(R - r)} \left\{ \frac{I_{in}(r)}{I_{out}(R)} \left( \frac{r}{R} \right) \left[ \left( \frac{1 - z'^2}{1 + z'^2} + \frac{2z'}{(1 + z'^2)} \left( \frac{dZ}{dR} \right) \right] \right\} - 1 \]
Beam Shaping Designs of McDermitt, Ph.D Dissertation, University of Mississippi, 1972

Collimated Gaussian input beam uniformly illuminating Cylindrical receiver (McDermitt, p. 84).

Energy Balance:

\[ R = \sqrt{\frac{r_0^2}{2I_{out}} \left[ 1 - \exp \left( -2\frac{r^2}{r_0^2} \right) \right]} \]

\[ I_{out} = \frac{r_0^2}{2R_{max}^2} \left[ 1 - \exp \left( -2\frac{r_{max}^2}{r_0^2} \right) \right] \]

Ray Trace Equation:

\[ z' = \frac{-(R - r)(Z - z) \pm (n/n_0)(R - r)\sqrt{(Z - z)^2 + (R - r)^2}}{(1 - (n/n_0)^2)(Z - z)^2 - (n/n_0)^2(R - r)^2} \]

Constant OPL:

\[ (Z - z) = \frac{n(n - n_0)d + \left[ r_0^2(n - 1)^2d^2 + (n^2 - n_0^2)(R - r)^2 \right]^2}{n^2 - n_0^2} \]
Input and Output Beam Profiles
GA Solution to the Two-lens Shaper Problem
Two-Mirror System

Input Intensity with 1-to-3 beam waist in x-to-y directions:

Parameters:

\[ r_0 = 1; \quad x_0 = \frac{r_0}{2}; \quad x_{\text{max}} = r_0 \]

\[ y_0 = \frac{3r_0}{2}; \quad y_{\text{max}} = 3r_0 \]

\[ X_{\text{max}} = 2r_0 = Y_{\text{max}} \]
Input and Output Beam Coordinates for Elliptical Gaussian-to-circular Top-hat System

X vs. x

Y vs. y

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Sag of First Mirror

Parameters:

\[ r_0 = 1; \ x_0 = \frac{r_0}{2}; \ x_{\text{max}} = r_0 \]
\[ y_0 = \frac{3r_0}{2}; \ y_{\text{max}} = 3r_0 \]
\[ X_{\text{max}} = 2r_0 = Y_{\text{max}} \]

\[ L = 10; \ h = 5 \]
\[ l_0 = \sqrt{L^2 + h^2} \]

\[ z(x, y)(L + l_0) = \frac{1}{2} \left( x^2 + y^2 \right) + \frac{r_0^2}{\sqrt{2\pi} \ \text{erf}(2\sqrt{2})} \]
\[ \times \left\{ 4 - \exp \left[ -2 \left( \frac{2x}{r_0} \right)^2 \right] - 3 \exp \left[ -2 \left( \frac{2y}{3r_0} \right)^2 \right] - \frac{2\sqrt{2\pi}}{r_0} \left[ x \text{erf} \left( \frac{2\sqrt{2}x}{r_0} \right) + y \text{erf} \left( \frac{2\sqrt{2}y}{3r_0} \right) \right] \right\} \]
Summary and Conclusions

• Geometrical methods of laser beam shaping uses:
  – Conservation of energy within a bundle of rays,
  – Ray trace equations,
  – Constant optical path length condition.
• The resulting differential equations have been used to design 1 & 2 reflective/refractive systems.
• Enable construction of a laser beam shaping merit function used during global optimization with genetic algorithms.