PH523 Assignment No. 3

Due date: October 13, 2003

1. In the presence of gravity the energy-momentum tensor for an electromagnetic field is

$$T^{(EM)}_{\mu\nu} = \frac{1}{4\pi}\,(F_{\mu\lambda}\,F_{\nu}{}^{\lambda} - \frac{1}{4}g_{\mu\nu}\,F_{\lambda\sigma}\,F^{\lambda\sigma}).$$

Use Maxwell's equations in their general relativistic form to show that

$$T^{(EM)\mu\nu}_{;\nu} = -F^{\mu\nu}\,J_{\nu}.$$

Use the fact that $F^{\mu\nu}\,J_{\nu}$ is just the Lorentz 4-force per unit volume with which the electromagnetic field acts on charged matter to show that

$$(T^{(EM)\mu\nu} + T^{(Matter)\mu\nu})_{;\nu} = 0.$$

2. A conformally flat space is defined as one with a metric tensor of the form $g_{\mu\nu} = f(x^{\lambda})\,\eta_{\mu\nu}$, where f is an arbitrary positive function and $\eta_{\mu\nu}$ is the flat-space metric. Show that for such a metric the Weyl tensor, defined as

$$C^{\mu}_{\nu\rho\sigma} = R^{\mu}_{\nu\rho\sigma} - \frac{1}{2}(g^{\mu}_{\rho}\,R_{\nu\sigma} - R_{\nu\rho}\,g^{\mu}_{\sigma} - g_{\nu\rho}\,R^{\mu}_{\sigma} + R^{\mu}_{\rho}\,g_{\nu\sigma}) + \frac{1}{6}(g_{\nu\sigma}\,g^{\mu}_{\rho} - g_{\nu\rho}\,g^{\mu}_{\sigma})\,R,$$

is zero.