PH100-5T Mid-Term

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Instructions. Total exam with worth 100 points. The number of points for each question are given. There is a 20 point extra credit problem at the end of exam. Partial credit will be given based on the work you show on your paper. If you need extra space, write on back of exam pages and give directions to where your work is located for each question. Circle your answer.

1. Consider a particle whose position as a function of time is given by

$$x(t) = (1 + 2t + 3t^2)m. \qquad \#$$

a. (5 pts) What is the average velocity (speed) between the times $t = 0$ and $t = 2$

$x_{av} = \frac{x(2s)-x(0)}{2s} = 8\frac{m}{s}$

b. (5 pts) What is the instantaneous velocity (speed) at $t = 1s$?

$v(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t)-x(t)}{\Delta t} = (2 + 6t)\frac{m}{s}$

$v(1s) = 8\frac{m}{s}$

c. (5 pts) What is the average acceleration between the times $t = 0$ and $t = 2s$?

$a_{av} = \frac{v(2s)-v(0)}{2s} = 6\frac{m}{s^2}$

d. (5 pts) What is the instantaneous acceleration at $t = 1s$?

$a(t) = \lim_{\Delta t \to 0} \frac{v(t+\Delta t)-v(t)}{\Delta t} = 6\frac{m}{s^2}$

2. (10 pts) The brakes on your automobile are capable of creating a deceleration of $17\frac{ft}{s^2}$. If you are going 85 mph and suddenly see a state tropper, what is the minimum time in which you can get your car under the 55 mph speed limit? (Hint: $1mph = 1.467\frac{ft}{s}$.)

$v = v_0 + at$

$a = -17\frac{ft}{s^2};\ v_0 = 85mph;\ v = 55mph$

$t = \frac{v-v_0}{a} = \frac{(55-85)mph * 1.467\frac{ft/s}{mph}}{-17\frac{ft}{s^2}} = 2.59s$

3. A ball is thrown vertically upwards from the ground with a speed of $25.2\frac{m}{s}$.

a. (5pts) How long does it take to reach its highest point?

$v = v_0 - gt = 0$ at the highest point

$t = \frac{v_0}{g} = \frac{25.2\frac{m}{s}}{9.8\frac{m}{s^2}} = 2.57s$

b. (5 pts) How high does it rise?

$y = v_0 t - \frac{g}{2}t^2$

$y(2.57s) = 25.2\frac{m}{s} * 2.57s - \frac{9.8\frac{m}{s^2}}{2}(2.57s)^2 = 32.4m$

c. (10 pts) At what times will it be $27.0m$ above the ground and what are the speeds of the object at these times?

Assume $y(t) = 27.0m = v_0 t - \frac{g}{2}t^2$

Rearranging equation gives

$\frac{g}{2}t^2 - v_0 t + 27m = 0$

Find t by solving the above quadratic equation for t

$t = \frac{v_0 \pm \sqrt{v_0^2 - 27m*2*g}}{g} = \frac{25.2\frac{m}{s} \pm 10.2879\frac{m}{s}}{9.8\frac{m}{s^s}} = 3.612$ and $1.5216s$

$v(1.5216s) = v_0 - g(1.5216s) = +10.28\frac{m}{s}$
$v(3.612s) = v_0 - g(3.612s) = -10.28\frac{m}{s}$

4. (15 points) Suppose a particle starts at the origin with velocity $\mathbf{v}_0 = -3.5\mathbf{i} + 4.7\mathbf{j}$, in m/s. Its acceleration is $2.1\mathbf{i} + 1.1\mathbf{j}$, in m/s^2. What are the coordinates and velocity $5.0s$ later? What is the speed of the particle at $t = 5.0s$?

For motion with constant acceleration, the displacement of object as a funtion of time is given by

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \mathbf{a}\frac{t^2}{2}$

Evaluating the displacement $t = 5.0s$ gives

$\mathbf{r}(5s) = 5s[-3.5\mathbf{i} + 4.7\mathbf{j}]\frac{m}{s} + \frac{(5s)^2}{2}[2.1\mathbf{i} + 1.1\mathbf{j}]\frac{m}{s^2}$

$\mathbf{r}(5s) = [+8.75\mathbf{i} + 37.25\mathbf{j}]m$

For motion with constant acceleration, the velocity of a particle as a function of time is given by

$\mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a}\ t = [-3.5i + 4.7j]\frac{m}{s} + [2.1i + 1.1j]\frac{m}{s^2}\ t$

Evaluating the velocity of particle at $t = 5.0s$ gives

$\mathbf{v}(5s) = [7\mathbf{i} + 10.2\mathbf{j}]\frac{m}{s}$

The speed is the magnitude of the velocity which is for this case

$speed = v = \sqrt{v_x^2 + v_y^2} = \sqrt{49 + 104.04}\ \frac{m}{s} = 12.3\frac{m}{s}$

5. (15 points) 2. A projectile is fired with an initial velocity of $55m/s, 25^\circ$ above the horizontal. How long does it take to get to the highest point on its trajectory? How far above the launch point is the highest point? How far down range is the highest point?

The x and y components of initial velocity are given by

$v_{y0} = v_0 \sin 25^\circ = 23.244\frac{m}{s}$ and $v_{x0} = v_0 \cos 25^\circ = 49.845\frac{m}{s}$

The time for projectile to reach the top of its path is given when the vertical component of its velocity is equal to zero

$v_y = 0 = v_{yo} - gt$

Solving for t gives

$t_{top} = \frac{v_0 \sin 25^\circ}{g} = \frac{23.244}{9.8}s = 2.37s$

The highest point is found be evaluating $y(t_{top})$

$y(t_{top}) = v_{y0}t_{top} - \frac{g}{2}t_{top}^2 = 27.57m$

The distance down range of highest point is found by evaluating $x(t_{top})$

$x(t_{top}) = v_{x0}t_{top} = 118.1m$

6. (10 points) 3. An outfielder can throw a baseball at a speed of $35m/s$. At what angle above the horizontal should he throw it if he wants it to be caught by an infielder $95m$ away? Assume the throwing and catching heights are thesame.

We are given $v_0 = 35\frac{m}{s}$ What is the initial projection angle θ_0. We know from the range formula of a projectile

$R = \frac{v_0^2 \sin(2\theta_0)}{g} = 95m$

Solving for θ_0

$\theta_0 = \frac{1}{2}\sin^{-1}\left[\frac{gR}{v_0^2}\right] = .5\sin^{-1}\left[\frac{9.8*95}{35^2}\right] = 24.7^\circ$

7. (10pts)2. A rock is thrown horizontally from a $115m$ high cliff and strikes the ground $92.5m$ from the base of the cliff. At what speed was the rock initially thrown?

$x = v_{x0}t = 92.5m$

$y = \frac{g}{2}t^2 = 115m$

Now solve for the fall time from the expression for $y(t)$

$t_{Fall} = \sqrt{\frac{2*115m}{g}} = 4.84s$

Solving the x-position equation for v_{x0} and evaluating at $t = t_{Fall}$ gives

$v_{x0} = \frac{92.5m}{t_{Fall}} = 19.1\frac{m}{s}$

EXTRA CREDIT PROBLEM (20pts)

Two stones are simultaneously thrown at the same speed from the same height above the ground. One is thrown vertically upward, and the other vertically downward. The one thrown downward hits the ground in one second, while the other stone spent $2.5s$ in the air before hitting the ground. Determine the initial velocity of each stone; determine the height above the ground from which the stones were thown; and determine the impact velocit of each stone with the ground.

Stone #1

$y = H = v_0 t + g\frac{t^2}{2}$ where $t = 1s$ for stone #1. Rewriting this equation

$H = v_0(1s) + \frac{g}{2}(1s)^2 \ldots\ldots\ldots\ldots\ldots Eq.A$

Stone #2

$H = -v_0 t + \frac{g}{2}t^2$ where $t = 2.5s$ for stone #2.

Rewriting this equation

$H = -v_0(2.5s) + \frac{g}{2}(2.5s)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.B$

Multiple Eq A by (2.5) and add to Eq B

$3.5H = \frac{g}{2}[2.5 + 6.25]s = 42.875m$

$H = \frac{42.875m}{3.5} = 12.25m$

Both stones will hit the ground with the same speed which is given by (using stone #1 equations of motion)

$v = v_0 + gt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots Eq.C$

Solving for v_0 from Eq A

$v_0 = \frac{H - .5g(1s)^2}{1s} = (12.25 - 4.9)\frac{m}{s} = 7.35\frac{m}{s}$

Therefore, using Eq. C gives the speed stones hit ground

$v = (7.35 + 9.8)\frac{m}{s} = 17.15\frac{m}{s}$