Counting 1

A ***counting problem*** is a problem in which we want to count the number of objects in a collection or the number of ways something occurs or can be done.

- At a local restaurant, for a fixed price one can buy a lunch consisting of 1 drink, 1 meat, and 3 different vegetables. If there are 5 drinks, 7 meats, and 13 vegetables available, how many different fixed price lunches are there?
- In a Global City election, there are five candidates. How many different ways can the candidates finish if there are no ties? In how many ways can the first three places be filled?
- A *Kaleidoscope* reporter comes to visit a 25 student class to interview 4 students. In how many ways can the 4 students be chosen?

You will need a calculator!

Counting by Systematic Listing

- Involves coming up with an actual list of all the possible objects in the collection or ways of doing something.
- Is practical only for short lists.
- Uses a **systematic** listing approach so as not to miss any objects, ways, or outcomes.

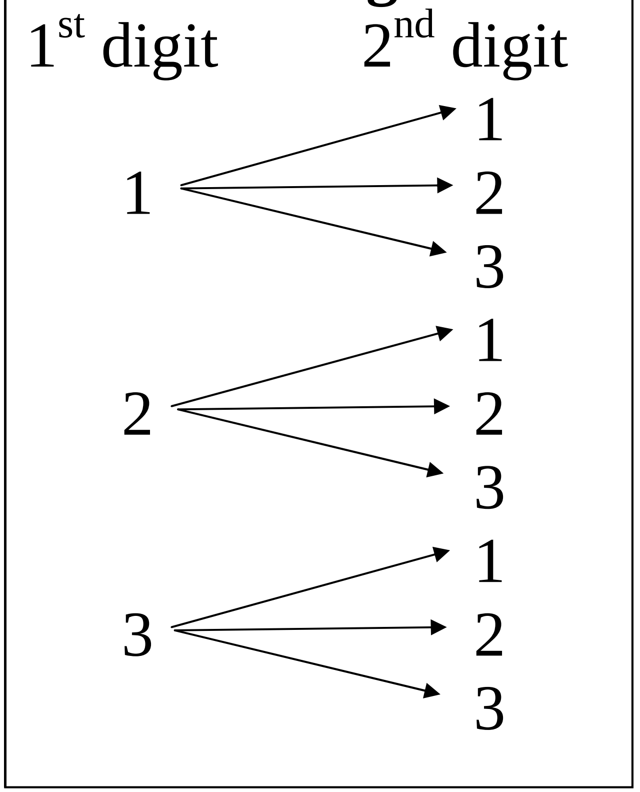
Example

Determine the number of two-digit numbers that can be written using only the digits from the set {1, 2, 3}.

Product Table

	1	2	3
1	11	12	13
2	21	22	23
3	31	32	33

Tree Diagram



Example

Toss a nickel, a dime, and a quarter. Observe whether a head (H) or a tail (T) comes up on each coin. How many outcomes are there?

[What systematic listing technique might we use?]

But sometimes the list is too long to actually "list."

Multiplication Rule

When something takes place in stages or steps, to find the number of ways it can occur, multiply the number of ways each individual stage can occur.

Example

Determine the number of two-digit numbers that can be written using only the digits from the set {1, 2, 3}.

________________ Number of ways to choose 1^{st} digit	X	________________ Number of ways to choose 2^{nd} digit	=	________________ Total number of 2-digit numbers

We call the above set of boxes a ***slot diagram***.

Example

Toss a nickel, a dime, and a quarter. Observe whether a head (H) or a tail (T) comes up on each coin. How many outcomes are there?

________________ Number of ways nickel can come up.	X	______ Dime	X	______ Quarter	=	__________ Total outcomes

Example

A male sales representative has 5 ties, 7 shirts, 4 pants, and 3 jackets. If an outfit consists of one tie, one shirt, one pants, and one jacket, how many outfits does this fellow have?

Example

At a local restaurant, for a fixed price one can buy a lunch consisting of 1 drink, 1 meat, and 3 different vegetables. If there are 5 drinks, 7 meats, and 13 vegetables available, how many different fixed price lunches are there?

Example

A Mississippi license plate consists of three letters of the alphabet, followed by a magnolia blossom or a mockingbird, followed by three digits from the set {0, 1,…, 9}. How many such plates can be made?

Sum Rule

If a collection of ***s*** objects can be divided into two **non-overlapping** (**disjoint**) pieces of sizes ***m*** and ***n***, then the whole collection is of size $s = m + n$ objects.

Note that the Sum Rule generalizes to more than two pieces, provided that each pair of pieces is disjoint.

Example

Each week Sally and Dan go out to dinner, to a movie, or to a play. If there are 24 restaurants in town, 7 movie theaters, and 5 playhouses, how many different places can they go to before they have to repeat a place?

___________	+ ____________	+ ________________	= ______________
restaurants	movies	playhouses	total places

Example

A local restaurant offers at lunchtime a drink and salad lunch or a drink, soup, and salad lunch. If there are 3 salads, 4 soups, and 5 drinks, how many different lunches are there?

______ drink	x	______ salad	+	______ drink	x	______ soup	x	______ salad	=
drink and salad lunch				drink, soup, and salad lunch					

= ______ drink & salad lunch + ______ drink, soup, & salad lunch = ______ total number of lunches

[Can you think of an alternate way to solve this problem?]

Example

Radio stations in the United States are assigned a sequence of four call letters (for example, WBHM). East of the Mississippi River, they all start with W. West of the Mississippi, they all start with K (for example, KORN). Letters can be repeated. How many such call letter sequences are there?

Example

How many three-letter sequences selected from the regular alphabet do not consist of the same three letters repeated?

Construct a corresponding slot diagram. We have been asked to find the first slot below. It is easiest to find the second slot and the total slot and work from there.

__________ Sequences with not all 3 letters the same	+	__________ Sequences with all 3 letters the same	=	__________ All 3-letter sequences

Rule of Complements

If a collection contains ***s*** objects, and the number of objects with a certain property is ***m***, then the number of objects that do not have that property is $\boldsymbol{n = s - m}$.

The example above of 3-letter sequences not consisting of the same three letters was of this type. Here is another.

Example

Out of a litter of 4 puppies, how many ways are there to have at least one male?

Sometimes the break-up of a list into pieces does not create disjoint pieces. There may be some overlap.

Example

Each week Sally and Dan go out to dinner, to a movie, or to a play. If there are 24 restaurants in town, 7 movie theaters, and 5 playhouses, how many different places can they go to before they have to repeat a place, assuming 3 of the playhouses are dinner theaters, and 2 of the restaurants also show movies?

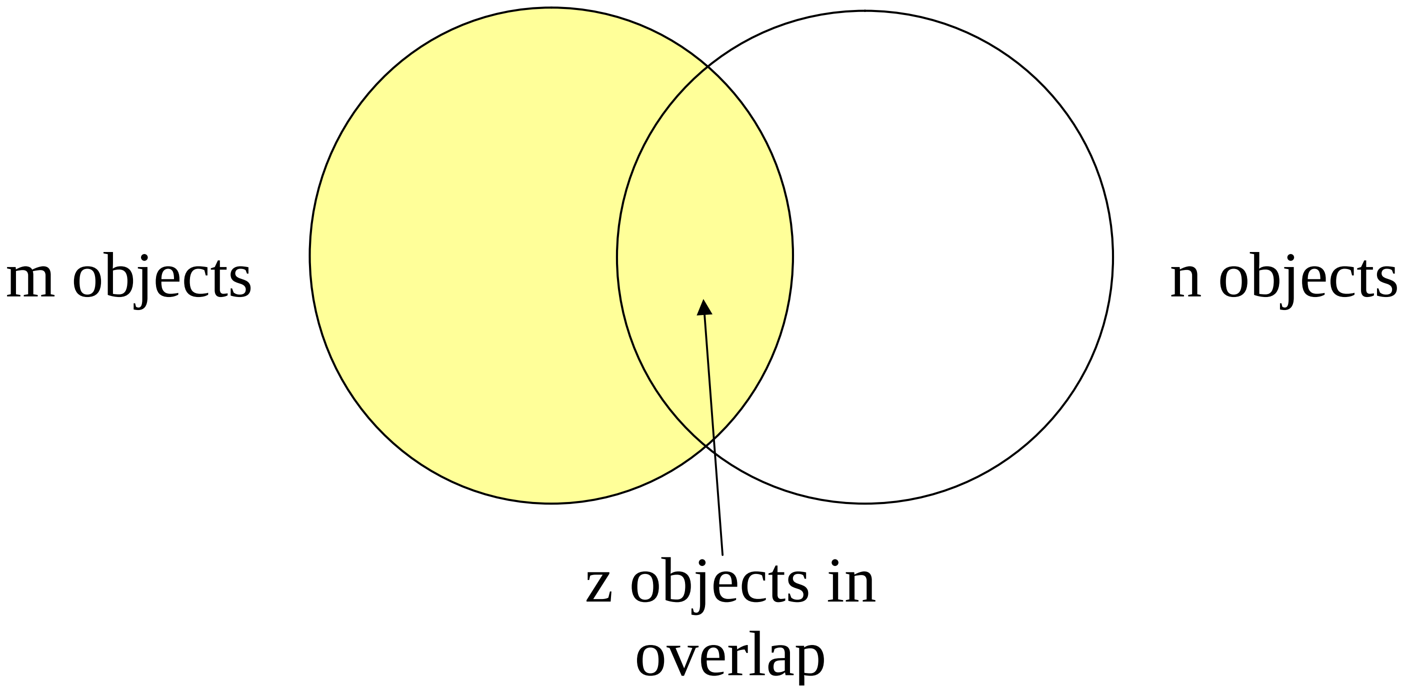
In this case we need a more general version of the Sum Rule.

Sum Rule with Overlap

If the collection of objects to be counted can be divided into two pieces of sizes m and n, and the pieces have z objects in common, then the total s of objects in the collection is

$$s = m + n - z.$$

We can picture this situation with a ***Venn diagram***:



Example

If we form 2-digit numbers using digits from the set {1, 2, 3, 4, 5, 6}. How many outcomes are there that contain exactly one 1 or exactly one 4?

Example

If we form 3-digit numbers using digits from the set {1, 2, 3, 4, 5, 6}. How many outcomes are there that contain exactly one 1 or exactly one 4?

Example

A Mississippi license plate consists of three letters of the alphabet, followed by a magnolia blossom or a mockingbird, followed by three digits from the set {0, 1,…, 9}. If no letter or digit can be repeated, how many such plates can be made?

Example

In a Global City election, there are five candidates. (a) How many different ways can the candidates finish if there are no ties? (b) In how many ways can the first three places be filled?

Example

A Mississippi license plate consists of three letters of the alphabet, followed by a magnolia blossom or a mockingbird, followed by three digits from the set {0, 1,…, 9}. If the number part cannot begin with a string of 0's, how many such plates can be made?

Example

At a twin convention 10 pairs of twins, otherwise unrelated, meet for dinner. Identical and fraternal twins are present. During the dinner conversation, 11 different people are heard to say, “this is my brother,” and 9 different people are heard to say “this is my sister.” (a) What is the maximum, and what is the minimum, number of males at the dinner table? (b) Of females? (c) What is the maximum, and what is the minimum, number of pairs of identical twins?