Counting 2

Ordered and Unordered Arrangements

Sometimes when we count, we are interested in counting the objects in a particular order, and at other times, order does not matter.

- In a Global City election, there are five candidates. If there are no ties, in how many ways can the first three places be filled?
  - In answering this question, it is important who is 1^{st}, 2^{nd}, 3^{rd}.
- A *Kaleidoscope* reporter comes to visit a 25 student class to interview 4 students. In how many ways can the 4 students be chosen?
  - In this case, it doesn't matter who is interviewed first.

Ordered arrangements are called ***permutations***. Unordered arrangements are called ***combinations***.

Example

How many ordered arrangements (permutations) are there of the letters **UAB**?

By the multiplication principle, we have

________ x ________ x _________ = __________

permutations.

Counting Ordered Arrangements

The number of ordered arrangements, or ***permutations***, of n objects taking all n at a time, is

$$n(n-1)(n-2)\ldots(2)(1) = {}_{n}P_{n} = n!$$

Note: n! is read “n factorial.”

Example

In a Global City election, there are five candidates. If there are no ties, in how many ways can the first three places be filled?

By the multiplication principle, we have

________ x ________ x _________ = __________

permutations.

The number of ordered arrangements, or ***permutations***, of n objects taking r at a time, is

$$n(n-1)(n-2)\ldots(n-r+1) = {}_nP_r = n!/(n-r)!$$

Example

We want to use 8 symbols to make ID codes for 300 people, with each code to consist of 3 different symbols. Is this possible?

Counting Unordered Arrangements

An arrangement of a set of objects selected without regard to their order is called a ***combination*** of the objects.

The combination of n objects, taken r at a time, is denoted ${}_nC_r$.

Example

How many double scoops of ice cream of different flavors are possible at a 31-flavors ice cream store?

If order were important, we would have _____ x _____ = _____ possibilities. But we would have counted both “vanilla-chocolate” and “chocolate-vanilla.” Each combination has a “sister” or duplicate in the opposite order.

The number of combinations without order is

________ x ________ / _________ = ________

duplicates

division

Counting Unordered Arrangements

The number of unordered arrangements, or ***combinations***, of n objects taking r a time, is

$${}_n\mathbf{C}_r = {}_nP_n/r! = n(n-1)(n-2)\ldots(n-r+1)/r! = (n!/(n-r)!)/r!$$

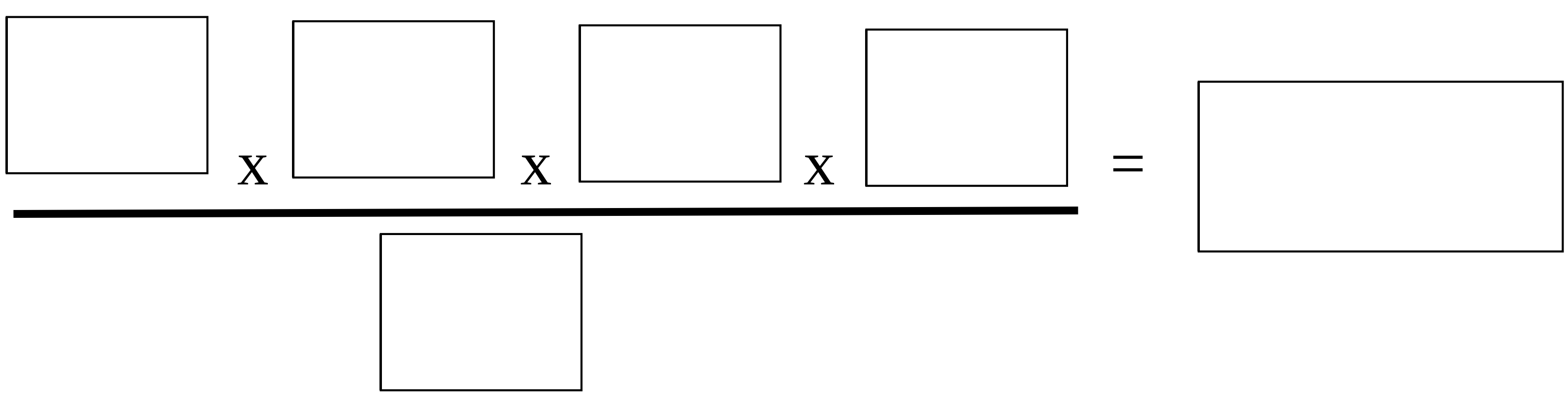
Hence, $\mathbf{{}_nC_r = n!/((n-r)!r!)}$

Note: the division by r! removes the duplicates.

Example

A Kaleidoscope reporter comes to visit a 25 person class to interview 4 students. In how many ways can the 4 students be selected from the class of 25?

Slot diagram approach:



Example

A Kaleidoscope reporter comes to visit a 25 person class to interview 4 students. In how many ways can the 4 students be selected from the class of 25?

Combination approach:

n = 25 and r = 4

$${}_{25}C_4 = 25!/(4!(21!)) = \_\_\_\_\_\_\_\_\_\_\_$$

Example

A 3-topping pizza at the Olde Tyme Pizza Shoppe has 3 different toppings selected from 12 possibilities. How many 3-topping pizzas are there?

Example

Double-toppings are permitted at the Olde Tyme Pizza Shoppe, but not triple. How many 3-topping pizzas are possible if repeating one topping is permitted?

Example

A jar contains 5 yellow jellybeans and 4 red jellybeans. In how many ways can 3 jellybeans be selected

(a) If all are yellow?

(b) If all are red?

(c) If at least two are yellow?

(d) If at most two are red?