

# MA 110 Homework 1

This homework assignment is to be written out, showing all work, with problems numbered and answers clearly indicated. Put your code number on each page. The assignment is due to be handed in by 8:00 AM, Tuesday, September 12. Late assignments will be penalized, and will not be accepted after the key is posted.

**Table 1.1**

Table 1.1 describes an election with four candidates (A, B, C, and D) and the preference schedule:

Number of Votes	8	7	6	4	3
1 <sup>st</sup> Choice	C	A	A	C	B
2 <sup>nd</sup> Choice	D	B	D	A	C
3 <sup>rd</sup> Choice	A	C	C	B	D
4 <sup>th</sup> Choice	B	D	B	D	A

- Using the plurality method, which candidate wins the election in Table 1.1? If there is a tie for winner, say who are tied.
- Using the Borda count method, which candidate wins the election in Table 1.1? If there is a tie for winner, say who are tied.
- Using the plurality-with-elimination method, which candidate wins the election in Table 1.1? If there is a tie for winner, say who are tied.
- Using the method of pairwise comparisons, which candidate wins the election in Table 1.1? If there is a tie for winner, say who are tied.
- Find the Condorcet candidate in the election in Table 1.1, or explain why there is not one.
- If in any answers to questions 1-4 you did find a tie for winner, use bottom-up comparison to break the tie.
- If in any answers to questions 1-4 you did find a tie for winner, use top-down comparison to break the tie.
- Rank the candidates in the election in Table 1.1 using the extended Borda count method. (Do not break ties, but state ranks with ties.)
- Rank the candidates in the election in Table 1.1 using the extended plurality-with-elimination method. (Do not break ties, but state ranks with ties.)
- An election is to be decided using the plurality method. There are five candidates and 212 votes. What is the smallest number of votes that a winning candidate can have if there are to be no ties for winner?
- An election is held among four candidates A, B, C, and D using the Borda count method. There are 24 voters. If candidate A received 55 points, candidate B received 71 points, and candidate C received 46 points, how many points did candidate D receive? Who won the election?
- An election with five candidates A, B, C, D, and E is held using the method of pairwise comparisons. Partial results of the pairwise comparisons are as follows: E wins 2 pairwise comparisons; C wins 2 and ties 1; A and B each win 1 and tie 1. How many pairwise comparisons did D win. Who wins the election?
- Write out a statement of each of the four fairness criteria.

14. Give an example of an election (not from the book) decided by Borda count which violates the majority criterion. [Hint: it can be done with a preference schedule with three candidates and two columns. Easy.]
15. Give an example of an election (not from the book) decided by Borda count which violates the Condorcet criterion, but which does not violate the majority criterion. [Hint: it can be done with a preference schedule with four candidates and three columns. Harder.]

**Table 1.2**

Table 1.2 describes an election with candidates (A, B, C, D, and E) and the preference schedule:

Number of Votes	9	6	4
1 <sup>st</sup> Choice	B	D	C
2 <sup>nd</sup> Choice	E	C	B
3 <sup>rd</sup> Choice	A	A	E
4 <sup>th</sup> Choice	C	B	D
5 <sup>th</sup> choice	D	E	A

16. Determine the winner of the election in Table 1.2 by each of the methods: Borda count, plurality-with-elimination, and pairwise comparisons.
17. The election in Table 1.2 is declared void. Disgusted with the initial outcome of the election, candidate D drops out. The voters vote again, with exactly the same preferences as in Table 1.2, but without candidate D in the race. Determine the winner of the election by each of the same three voting methods again.
18. Based solely upon the information in the answers to questions 16 and 17 which fairness criteria (if any) are violated in this election by each election method used?
19. Refer to the weighted voting system [q: 8, 3, 2, 1, 1]. For each value of q that is reasonable, list which player is a dictator (if any), which players (if any) have veto power (a dictator automatically has veto power), and which players are dummies (if any).
20. Refer to the weighted voting system [8: m, 4, 2, 1]. For each value of m that is reasonable, up to the point where player 1 is a dictator, list which players (if any) have veto power (but are not dictator), and which players are dummies (if any).
21. Give an example of a weighted voting system with five players, no dictator, exactly two players with veto power, and no dummies.
22. Give an example of a weighted voting system with four players, in which players one and two each have twice as many votes as each of the other players, there are no dummies, and for which 2/3 of the votes is required to pass a motion.
23. Find the Banzhaf power distribution of the weighted voting system [7: 3, 2, 2, 2].
24. Find the Banzhaf power distribution of the weighted voting system [10: 4, 4, 1, 1, 1].
25. Find the Banzhaf power distribution of the weighted voting system [5: 4, 3, 1, 1].
26. Find the Banzhaf power distribution of the weighted voting system [9: 9, 2, 2, 1, 1, 1, 1]. [Hint: you should have to do very little work.]
27. Find the Banzhaf power distribution of the weighted voting system [11: 3, 3, 2, 1, 1, 1]. [Hint: you should have to do very little work.]