MA 110 Homework 2

This homework assignment is to be written out, showing all work, with problems numbered and answers clearly indicated. The assignment is due to be handed in by Tuesday, October 10. Turn in all your work, but I suggest you keep a copy of the question sheets to study for the test coming up on October 17. Late assignments will not be accepted after the key is made available.

Table 2.1 refers to a situation where four players (A, B, C, and D) agree to divide a cake fairly by the Lone Divider method. The table shows how each player values each of the four slices that have been cut by the divider.

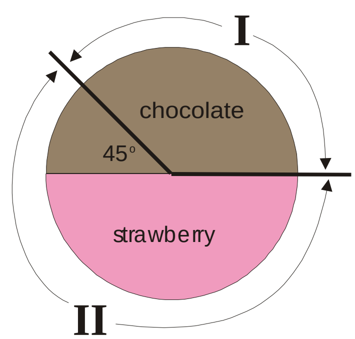
Table 2.1

Player \ Slice	s_1	s_2	s_3	s_4
A	32%	20%	24%	24%
B	25%	25%	25%	25%
C	25%	15%	30%	30%
D	26%	26%	26%	22%

1. Assuming all players play honestly and intelligently, which player was the divider?

2. Assuming they play honestly and intelligently, what should each chooser's declaration be?

3. Ted and Carol buy a half-chocolate, half-strawberry cake for $12.00. They want to divide it by the divider-chooser method. Ted likes chocolate twice as much as strawberry. Carol likes strawberry three times as much as chocolate.
   a. What is the value of the chocolate half of the cake to Ted? What is the value of the strawberry half of the cake to Ted?
   b. What is the value of the chocolate half of the cake to Carol? What is the value of the strawberry half of the cake to Carol?
   c. Suppose Ted is the divider and cuts the cake as shown into pieces I and II. Show that pieces I and II are of equal value to Ted.
   d. Which piece will Carol choose? Explain why.



Tables 2.2 and 2.3 refer to a situation in which five players (one divider and four choosers) are going to divide a cake fairly by the Lone Divider method. The divider has cut the cake into five slices $\{s_1,s_2,s_3,s_4,s_5\}$ and the choosers declarations are as follows

Table 2.2	Declaration
Chooser 1	$\{s_1, s_3\}$
Chooser 2	$\{s_2\}$
Chooser 3	$\{s_1, s_5\}$
Chooser 4	$\{s_3, s_4\}$

Table 2.3	Declaration
Chooser 1	$\{s_1, s_3\}$
Chooser 2	$\{s_3\}$
Chooser 3	$\{s_2, s_5\}$
Chooser 4	$\{s_1, s_3\}$

4. If the choosers' declarations are as in Table 2.2, describe a fair division of the cake.

5. Referring again to Table 2.2, describe a fair division of the cake different from your answer to #4, or explain why there is only one way to do it fairly.

6. If the choosers' declarations are as in Table 2.3, describe how to proceed to obtain a fair division of the cake? [Hint: be sure you correctly identify the "picky group."]

7. With reference to Table 2.3, explain what is possibly unfair about the following division of the cake: Divider gets s_4; Chooser 2 gets s_3; Chooser 3 gets s_2; Choosers 1 and 4 recombine pieces s_1 and s_5 into a new cake, and play lone divider again to divide the remaining cake fairly between them.

8. Three players, A, B, C, agree to divide some candy by the method of markers. Assume that you are player A and that you like Reese's Pieces (denoted R, below), but do not particularly like plain M&Ms (denoted M) or peanut M&Ms (denoted P). Place your markers below so that you will be guaranteed to receive a fair share.

P M M P R M P R P M P M M R P M M R R P M R M P

8. – 10. Four heirs, A, B, C, and D inherit four items from their uncle's estate: a sofa, a wine cellar, a car, and a rare book. They decide to divide the items among themselves by the method of sealed bids. Their bids are as in the bid table below. Complete the bid and allocation tables below and describe the final settlement, including what cash amounts each player has ultimately paid or gained.

Bids	**Players**				
Item	**A**	**B**	**C**	**D**	**High Biddder**
sofa	1,500	2,500	3,500	3,000	
wine	13,000	11,500	11,000	12,000	
car	4,500	4,000	5,500	6,000	
book	2,000	2,000	2,500	3,000	
Total Value					
Fair Share					

Allocation				
Player	**Item(s)**	**Value**	**Share**	**Put in (Take out)**
A				
B				
C				
D				
			Surplus	
			Share of Surplus	

Describe the final settlement for each player below (including cash balance).

11. Three players (A, B, and C) are fairly dividing some items by the Method of Markers. They have marked the linear array below as shown. Describe the allocation of items to each player and indicate what items, if any, are leftover.

```
1   2     3     4     5     6     7     8     9     10    11    12    13
               ↑      ↑     ↑                ↑       ↑      ↑
               B      A     C                C       B      A
```

12. Four players (A, B, C, and D) are fairly dividing some items by the Method of Markers. They have marked the linear array below as shown. Describe the allocation of items to each player and indicate what items, if any, are leftover.

```
1   2     3     4     5     6     7     8     9     10    11    12    13    14    15
               ↑      ↑     ↑        ↑↑       ↑   ↑↑      ↑↑          ↑      ↑
               D      C     A        AB       C   BD      CA          B      D
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13. Alice and Ted divorce amicably and have to divide the house they share. They decide to use the method of sealed bids. Alice bids $130,000 and Ted bids $160,000. What is the final result of applying the method of sealed bids? Be sure to address any exchange of cash.

14. You and two friends buy a rectangular cake, shown below. The cake is 1/2 chocolate, 1/4 strawberry, and 1/4 vanilla. You decide to divide it among the three of you by the lone divider method. You are the divider. You like strawberry just as much as vanilla, and you like vanilla twice as much as chocolate. Make cuts of the cake into the appropriate number of pieces so that you will receive a fair share. Label the pieces carefully.

Chocolate	
Strawberry	Vanilla

15. Four players (A, B, C, and D) agree to divide some candy by the method of markers. Assume that you are player A and that you like Reese's Pieces (denoted R, below) twice as much as plain M&Ms (denoted M) or peanut M&Ms (denoted P). Place your markers below so that you will be guaranteed to receive a fair share. [Hint: assign a numerical value to each piece.]

M P M P M R P M M P R P M P R M M R P M M R M P M R

16. Three players (A, B, and C) agree to divide some candy by the method of markers. Assume that you are player B and that you like plain M&Ms (denoted M, below) three times as much as Reese's Pieces (denoted R) or peanut M&Ms (denoted P). Place your markers below so that you will be guaranteed to receive a fair share.

M P P P R R P M P P R P M P R

Table 3.1 describes the small country Pardoxia with four states and populations as shown.

Table 3.1					
State	Ariza	Birma	Culpa	Doma	Total
Population (1,000's)	11,035	8025	845	95	

17. Determine a Hamilton's Method apportionment of the seats of the legislature of Pardoxia with 200 seats.

18. Suppose that the legislature is increased to 201 seats. What is now the Hamilton apportionment of the Pardoxian legislature? Did anything paradoxical happen? Explain

19. Suppose that 130,000 people migrate from Ariza to Birma because of a devastating hurricane. The populations of Culpa and Doma remain the same. How does this change the Hamilton apportionment? Did anything paradoxical happen? Explain.

20. In each apportionment above, determine which states received their upper quota of seats, and which states received their lower quota. Explain why Hamilton's Method always satisfies the Quota Rule.

21. A school district has four schools with student populations in the following Table 4.1. The district must buy busses to transport all students. Assume that a bus holds 45 students, and each bus can be used for just one trip. The district decides to apportion busses to schools by Hamilton's Method. How many busses does each school get? Explain what the standard divisor and standard quota represent in this example. Is this a good solution?

Table 4.1					
School	High	Middle 1	Middle 2	Elementary	Total
Students	999	310	300	776	