MA 110 Homework 3 ANSWERS

This homework assignment is to be written out, showing all work, with problems numbered and answers clearly indicated. If you feel that you must make assumption(s) not stated in the problem in order to solve it, then state your assumption(s). The assignment is due to be handed in by 8:00 AM, Thursday, Nov. 9. Late assignments will not be accepted after the key is posted.

Table 3.1

Table 3.1 describes the small country Mallosistra with four states and populations as shown.

State	Ariza	Birma	Culpa	Doma	Total
Population	20,072	7,568	2,840	1,520	

1. Determine a Jefferson's Method apportionment of the seats in the legislature with 160 seats.
2. Determine an Adams' method apportionment of the seats in the legislature with 160 seats.
3. Determine a Webster's Method apportionment of the seats of the legislature with 160 seats.

House size M =	160				
State	A	B	C	D	Total
Population	20,072	7,568	2,840	1,520	32,000
Standard Divisor			200.000		
Standard Quota	100.360	37.840	14.200	7.600	
Modified Divisor			198		
Modified Quota	101.374	38.222	14.343	7.677	
Jefferson App't	101	38	14	7	160
Modified Divisor			202.8		
Modified Quota	98.974	37.318	14.004	7.495	
Adams' App't	99	38	15	8	160
Modified Divisor			200		
Modified Quota	100.360	37.840	14.200	7.600	
Webster App't	100	38	14	8	160

4. Redo problems 1-3 with 164 seats in the legislature.

House size M =	164				
State	A	B	C	D	Total
Population	20,072	7,568	2,840	1,520	32,000
Standard Divisor			195.122		
Standard Quota	102.869	38.786	14.555	7.790	
Modified Divisor			192.900		
Modified Quota	104.054	39.233	14.723	7.880	
Jefferson App't	104	39	14	7	164
Modified Divisor			197.000		
Modified Quota	101.888	38.416	14.416	7.716	
Adams' App't	102	39	15	8	164
Modified Divisor			195.850		
Modified Quota	102.487	38.642	14.501	7.761	
Webster App't	102	39	15	8	164

5. Determine a Jefferson's Method apportionment of the seats in the legislature with 10 seats, but each state must get at least one seat.

6. Determine an Adam's Method apportionment of the seats in the legislature with 10 seats, but each state must get at least one seat.

House size M =	10				
State	A	B	C	D	Total
Population	20,072	7,568	2,840	1,520	32,000
Standard Divisor			3,200.000		
Standard Quota	6.273	2.365	0.888	0.475	
Modified Divisor			3200		
Modified Quota	6.273	2.365	0.888	0.475	
Jefferson App't	6	2	1	1	10
Modified Divisor			4000		
Modified Quota	5.018	1.892	0.710	0.380	
Adams' App't	6	2	1	1	10
Modified Divisor			3200		
Modified Quota	6.273	2.365	0.888	0.475	
Webster App't	6	2	1	1	10

7. In the apportionments of problems 1-6, state whether or not a violation of the quota occurred, which state(s), and if it was a violation of the upper quota or of the lower quota.
In Problem 2, Adams' method violated the lower quota for State A (Ariza).
In Problem 4, Jefferson's method violated the upper quota for State A (Ariza).

8. How many ways are there for a litter of four kittens to consist of at least two males? List all the possibilities.
{MMFF, MFMF, MFFM, FMMF, FMFM, FFMM, MMMF, MMFM, MFMM, FMMM, MMMM} There are 11 ways.

9. A license plate consists of three letters (AB…YZ) and four digits (01..89).
   a. How many license plates of this sort are there?
   Order matters on a license plate, and nothing was said about not repeating letters or digits, so repeats are OK.
   26 x 26 x 26 x 10 x 10 x 10 x 10 = 175,760,000
   b. How many license plates are there if no letter can be repeated?
   Only letters cannot be repeated; digits can be repeated.
   26 x 25 x 24 x 10 x 10 x 10 x 10 = 156,000,000
   c. How many license plates are there that start with A, B, or C?
   At this point some might assume the hypothesis of (a) and some of (b).
   Assuming repeat letters OK: 3 x 26 x 26 x 10 x 10 x 10 x 10 = 20,280,000
   Assuming no repeat letters: 3 x 25 x 24 x 10 x 10 x 10 x 10 = 18,000,000
   d. How many license plates are there with no Z in them?
   Assuming repeat letters OK: 25 x 25 x 25 x 10 x 10 x 10 x 10 = 156,250,000
   Assuming no repeat letters: 25 x 24 x 23 x 10 x 10 x 10 x 10 = 138,000,000
   e. How many license plates are there in which the number part does not begin with a string of zeros?
   Assuming repeat letters OK: 26 x 26 x 26 x 9 x 10 x 10 x 10 = 158,184,000
   Assuming no repeat letters: 26 x 25 x 24 x 9 x 10 x 10 x 10 = 140,400,000
   f. How many license plates are there in which a digit is repeated three times?
   Assuming repeat letters OK: 26 x 26 x 26 x 10 x 9 x 4 = 6,327,360
   Assuming no repeat letters: 26 x 25 x 24 x 10 x 9 x 4 = 5,616,000

g. How many license plates are there in which no digit is repeated three times?
Assuming repeat letters OK: 175,760,000 - 6,327,360 = 169,432,640
Assuming no repeat letters: 156,000,000 - 5,616,000 = 150,384,000

10. A restaurant offers a fixed price meal. You may choose either a soup or a salad, a main course, and three different vegetables.

 a. How many different meals are possible if there are 4 soups, 3 salads, 5 main courses, and 7 vegetables to choose from?
 (4 + 3) x 5 x 7 x 6 x 5 = 7,350

 b. How many different meals are possible if, in addition to the above meals, you may have soup and substitute a salad for one of your vegetables?
 New possibilities not counted in (a): 4 x 3 x 5 x 7 x 6 = 2,520
 Grand total: 7,350 + 2,520 = 9,870

11. Seven friends ABCDEFG line up to buy tickets for a movie. How many different orders are there in which they may line up?
 7 x 6 x 5 x 4 x 3 x 2 x 1 = 7! = 5,040
 How many different orders are there if A and F are always in line together?
 (5 x 4 x 3 x 2 x 1) x (2 x 1) x (6) = 1,440
 (order of other 5) x (AF or FA) x (placement of AF-pair)

12. Seven friends ABCDEFG sit at a round table for dinner. How many different (circular) orders are there in which they may sit together?
 5,040/7 = 720

13. A survey of 1000 subscribers to the *Los Angeles Times* revealed that 900 people subscribe to the daily Morning edition and 500 subscribe to both the daily and the Sunday editions. How many subscribe to the Sunday edition?
 daily + Sunday – daily and Sunday = total. Let x = number subscribing to Sunday. Then
 900 + x – 500 = 1,000
 x = 600
 How many subscribe to the Sunday edition only?
 600 – 500 = 100

14. Alice and Dan go to dinner and a play each Saturday night. If there are 7 restaurants in town and 4 playhouses, how many different dates can they have before they repeat?
 7x 4 = 28
 How would the answer be different if 2 of the restaurants were dinner theaters (that is, also playhouses)?
 If two restaurants are dinner theaters, then those are two dates. The remaining 5 restaurants and 2 theaters combine to give another 10 different dates. Thus:
 (5 x 2) + 2 = 12

15. A middle school Spanish club has 18 members, 8 boys and 10 girls. Answer the following:

 a. How many ways can a committee of four be chosen? (Assume order doesn't matter.)
 (18 x 17 x 16 x 15)/(4 x 3 x 2 x 1) = 3,060

 b. Now suppose the first person picked will be chair of the committee?
 Order only of first chosen matters.
 (18) x [(17 x 16 x 15)/(3 x 2 x 1)] = 12,240

c. Suppose instead the committee must be evenly divided between boys and girls?
No reason to suppose order of boys or girls matters.
[(8 x 7)/2] x [(10 x 9)/2] = 1,260

d. Suppose instead that the committee cannot consist of all boys nor all girls?
All boy committees: (8 x 7 x 6 x 5)/(4 x 3 x 2 x 1) = 70
All girl committees: (10 x 9 x 8 x 7)/(4 x 3 x 2 x 1) = 210
Remaining committees: 3,060 – 70 – 210 = <u>2,780</u>

16. Six names ABCDEF are written on separate slips of paper and put in a jar.

a. How many ways are there to select three different names randomly from the jar if order matters?
6 x 5 x 4 = 120

b. How many ways are there to select three different names randomly from the jar if order doesn't matter?
120/(3 x 2 x 1) = 20

c. In how many ways are there to select three slips from the jar, if the slips are replaced after choosing each of them? (Assume order matters.)
6 x 6 x 6 = 216

d. In how many ways are there to select three slips from the jar, if the slips are replaced after choosing each of them? (Assume order doesn't matter.)

[6]	+	[6 x 5]	+	[(6 x 5 x 4)/(3 x 2 x 1)] = 56
three same		two different		all three different

Clearly, there are only 6 ways to pick the same name thrice, and the thuird term in the sum, all 3 names different, without order, is obvious too. The "two different" part of the sum above may be confusing. There are 6 ways to pick the repeated name and 5 ways to pick the non-repeated name, for 30 ways together.

17. Three fair coins are tossed at one time.
The sample space is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}, and each outcome is equally likely.

a. What is the probability that they all come up heads? 1/8

b. What is the probability that exactly two come up heads? 3/8

c. What is the probability that at most two come up heads? 7/8
Note that this is the sam as "not all three heads:" 1-1/8 = 7/8

d. What is the probability that at least two come up heads? 1/2

18. A fair six-sided die is rolled.
The sample space is {1,2,3,4,5,6} and each outcome is equally likely.

a. What is the probability that a 6 shows up? 1/6

b. What is the probability that an even number shows up? 1/2

c. What is the probability that either a 1 or a 2 shows up? 1/3

19. A jar contains two red marbles and four white marbles. A marble is drawn from the jar at random.
The sample space is {R,W} but the outcomes are not equally likely.

a. What is the probability that the marble drawn is red? 1/3

b. What is the probability that the marble drawn is white? 2/3

c. What is the probability that the marble drawn is neither red nor white? 0

d. What is the probability that the marble drawn is either red or white? 1