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MA 110 Homework 4 Answers

This homework assignment is to be written out, showing all work, with problems numbered, code number on each page, and answers clearly indicated. The assignment is to be handed in by 8:00 AM, Tuesday, December 5. Late assignments will not be accepted after the key is posted on Dec. 7.

1. Four fair coins are tossed at once. The sequence of heads and tails that shows up is observed.
   a. What is the sample space for this experiment (list all outcomes)?

HHHH	HHHT	HHTH	HHTT
HTHH	HTHT	HTTH	HTTT
THHH	THHT	THTH	THTT
TTHH	TTHT	TTTH	TTTT

16 outcomes. All outcomes are equally likely.

   b. What is the probability that exactly three heads are showing?
   p(exactly 3 H) = event size/sample size = 4/16 = 1/4
   c. What is the probability that at least three heads are showing?
   p(at least 3 H) = p(exactly 3H) + p(4H) = 4/16 + 1/16 = 5/16
   d. What is the probability that the number of heads showing is not the same as the number of tails?
   p(#H differs from #T) = 1-p(2H and 2T) = 1-6/16 = 1-3/8 = 5/8
   e. Make up an event that might occur in this situation and compute its probability.
   Many answers are possible.

2. Two fair 6-sided dice are rolled at once. The pair of numbers showing up is observed.
   a. How many outcomes are in the sample space for this experiment?
   By multiplication principle it is 6 x 6 = 36. All outcomes are equally likely.
   b. What is the probability that the sum of the faces showing is 9?
   Event = {(3,6), (4,5), (5,4), (6,3)}
   p(sum is 9) = 4/36 = 1/9
   c. What is the probability that the difference (larger minus smaller) of the faces showing is 3?
   Event = {(1,4), (4,1), (2,5), (5,2), (3,6), (6,3)}
   p(difference is 3) = 6/36 = 1/6
   d. What is the probability that both dice show even numbers and the sum is 8?
   The two events conjoined are not independent, so the probability is not simply the product of p(both even) and p(sum 8). However, the compound event is easy to list, so we compute the probability from that.
   Event = {(2,6), (4,4), (6,2)}
   p(both even and sum 8) = 3/36 = 1/12
   e. What is the probability that the product of the faces showing is at least 15?
   Event = {(3,5), (5,3), (3,6), (6,3), (4,4), (4,5), (5,4), (4,6), (6,4), (5,5), (5,6), (6,5), (6,6)}
   p(product at least 15) = 13/36

3. A box contains 2 green marbles and 4 white marbles.
   a. A single marble is drawn at random from the box. What is the probability that it is green?
   Any of the six marbles has the same chance of being drawn at random, so we consider how many are green of the total in the box.
   p(G) = 2/6 = 1/3
   [For future reference, p(W) = 4/6 = 2/3.]

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b. Two marbles are drawn, but the first is replaced before drawing the second. What is the probability that both are green?
Because of "replacement," the two draws are independent.
p(GG) = p(G) x p(G) = 1/3 x 1/3 = 1/9

c. Two marbles are drawn, but the first is not replaced before drawing the second. What is the probability that both are green?
The two draws are not independent. (Recall G|G means "green, given green already drawn.")
p(G|G) = 1/5, since a green marble has been removed leaving 1 out of 5 green.
p(GG) = p(G) x p(G|G) = 1/3 x 1/5 = 1/15

d. Two marbles are drawn without replacement. What is the probability that they are the same color?
Since events GG and WW do not overlap,
p(both same) = p(GG) + p(WW)
p(GG) = 1/15 (from part c above)
p(WW) = p(W) x p(W|W) = 2/3 x 3/5 = 2/5 (similar to part c)
p(both same) = 1/15 + 2/5 = 1/15 + 6/15 = 7/15

e. Two marbles are drawn without replacement. What is the probability that they are not the same color?
p(not same) = 1- p(both same) = 1-7/15 = 8/15
Alternate derivation:
p(not same) = p(GW) + p(WG) = p(G) x p(W|G) + p(W) x p(G|W) =
= (1/3 x 4/5) + (2/3 x 2/5) = 4/15 + 4/15 = 8/15

4. Three unusual coins are tossed at once. One has both sides heads. One is loaded so that the probability of it coming up heads is 1/3. One is fair (so not really unusual).

a. What is the probability of rolling three heads?
The tosses are independent, so we can apply the probability multiplication principle.
p(HHH) = p(H 1st coin) x p(H 2nd coin) x p(H 3rd coin) =1 x 1/3 x ½ = 1/6

b. What is the probability of rolling three tails?
p(TTT) = p(T 1st coin) x p(T 2nd coin) x p(T 3rd coin) = 0 x 2/3 x ½ = 0

c. What is the probability of rolling exactly two heads?
Event = {HHT, HTH, THH}
p(exactly 2 H) = p(HHT) + p(HTH) + p(THH)
p(exactly 2 H) = (1 x 1/3 x ½) + (1 x 2/3 x ½) + (0 x 1/3 x ½)
p(exactly 2 H) = 1/6 + 1/3 + 0 = 1/2

d. What is the probability of rolling at least two heads?
Call above Event1. Need to include Event2 = {HHH}.
p(at least 2H) = p(exactly 2 H) + p(HHH) = ½ + 1/6 = 2/3

e. Make up a compound event that might occur in this situation and compute its probability. Many answers are possible.

5. Nine fair coins are tossed at once.
There are 2^9 = 512 outcomes, all equally likely.

a. What is the probability that exactly one is a head?
There are 9 outcomes with exactly 1 H.
p(exactly 1 H) = 9/512

b. What is the probability that exactly two are heads?
There are (9 x 8)/2 = 36 outcomes with exactly 2 H.
p(exactly 2 H) = 36/512 = 9/128

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c. What is the probability that at most two are heads?
p(at most 2 H) = p(0 H) + p(exactly 1 H) + p(exactly 2 H)
p(at most 2 H) = 1/512 + 9/512 + 36/512 = 46/512 = 23/256

d. What is the probability that at least two are heads?
At least 2 H is same event as not just 0H or 1 H.
p(at least 2 H) = 1-[p(0H) + p(1H)] = 1-[1/512 + 9/512] = 1- 5/256 = 251/256

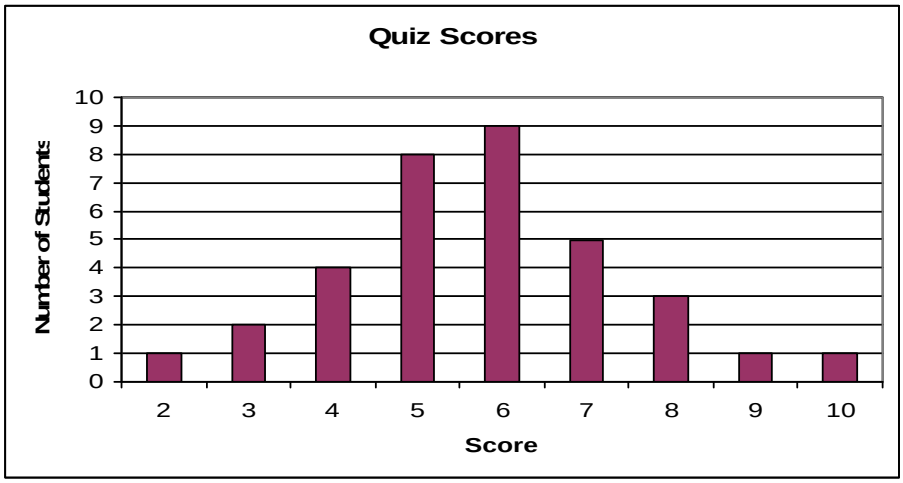
6. There are five tests in a Stat 101 class, each counting equally toward the test average. Suppose your scores on the first four tests are 81, 86, 90, and 96. What must you make on the fifth test to bring your average up to 90? (Assume no rounding.)
Since the average is 90 with no rounding, the 5 scores must add up to 5 x 90 = 450.
Hence, the needed score is 450-81-86-90-96 = 97.

Table 1.1

Table 1.1 is a frequency table of scores of a group of students on a 10-point mathematics quiz. Questions 7-15 refer to this table.

Quiz Score	2	3	4	5	6	7	8	9	10
Number of Students	1	2	4	8	9	5	3	1	1
Running total	1	3	7	15	24	29	32	33	34

7. Construct a bar graph displaying the data in Table 1.1. Be sure to label your axes appropriately.



8. The total number of students taking the quiz is ? 34

9. Construct a pie chart based on Table 1.1 with categories as follows: score 0-4, score 5-7, and score 8-10.

Category	Number	Percentage	Pie Angle
Score 0-4	7	20.5%	74^o
Score 5-7	22	64.7%	233^o
Score 8-10	5	14.7%	53^o
Total	34	100%	360^o

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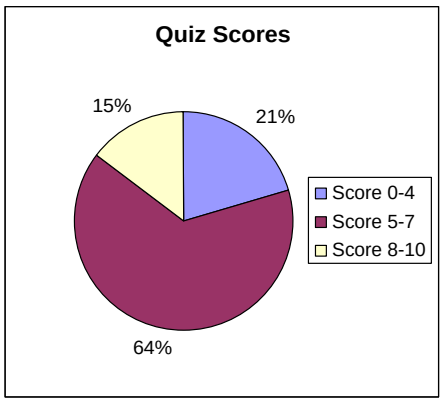


10. The average score on the quiz is ?
    Sum of scores = (1x2) + (2x3) + (4x4) + … + (1x10) = 196
    Average = Sum/Number = 196/34 = 5.76

11. The median score on the quiz is ?
    Position of Median: between 34/2 = 17th and 18th data point.
    Median = 6

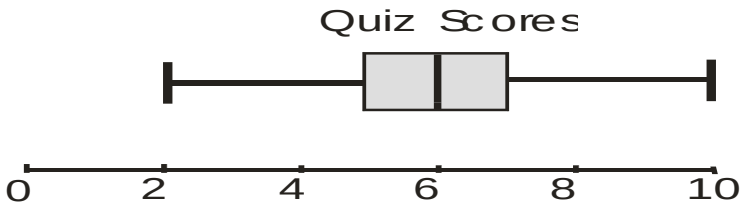
12. The first and third quartiles on the quiz are ?
    Position: 34/4 rounds up to 9th data point.
    Count 9 from bottom: $Q_1 = 5$
    Count 9 from top: $Q_3 = 7$

13. The minimum and maximum on the quiz are ? Min = 2 and Max = 10

14. The interquartile range on the quiz is ? $IQR = Q_3 - Q_1 = 7-5 = 2$

15. Determine the five number summary of the quiz scores and construct a box plot using the five number summary of the quiz scores.
    Five Number Summary of Quiz Scores

Min	Q_1	Median	Q_3	Max
2	5	6	7	10



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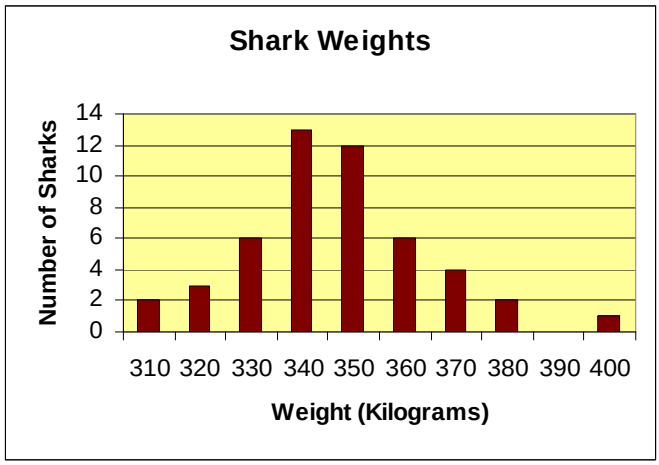
Questions 16-22 refer to the adjacent bar graph displaying the weight of great white sharks caught in the Bay of Biscay last June.



16. What is the total number of sharks caught?
    Add up the heights of the bars: 2 + 3 + 6 + … + 2 + 1 = 49

17. What is the average weight of the sharks caught?
    Add up the weights, taking into account the frequencies:
    (2 x 310) + (3 x 320) + … + (2 x 380) + (1 x 400) = 16,980.
    Average = sum/number = 16,980/49 = 346.5 kg

18. What is the median weight of the sharks caught?
    Position of Median: 49/2 rounds up to 25. Find 25th weight.
    Median = 350 kg

19. What are the first and third quartile of the shark weights?
    Position: 49/4 rounds up to 13.
    Count 13 from bottom: Q_1 = 340 kg.
    Count 13 from top: Q_3 = 360 kg.

20-22. Re-do questions 17-19 on the assumption that an additional shark weighing 390 kg was caught.

20. Sum = 16,980 + 390 = 17,370
    Average = sum/number = 17,370/50 = 347.4 kg

21. Position of Median: 50/2 → median between 25th and 26th weights.
    Median = 350 kg

22. Position: 50/4 rounds up to 13.
    Count 13 from bottom: Q_1 = 340 kg.
    Count 13 from top: Q_3 = 360 kg.