Integration Methods

Euler

Runge-Kutta 2

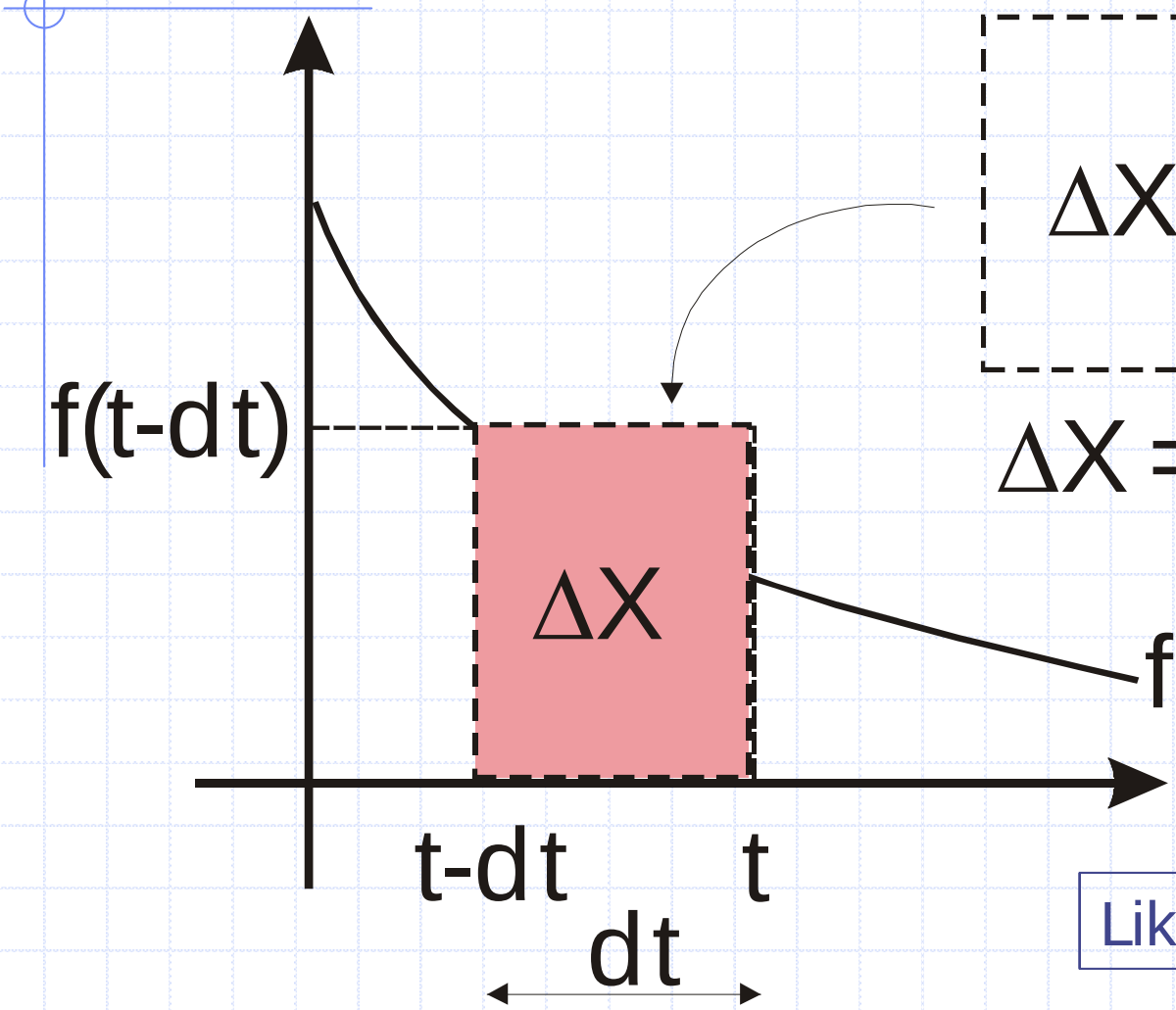
Runge-Kutta 4

Euler's Method

- Let Stock = X
- Let flow = f(t, X)
  [function of time, Stock]
- Compute X(t) from X(t-dt) and time.
  - ΔX = dt * f (t-dt, X(t-dt))
  - X(t) = X(t-dt) + ΔX

Euler's Method

Assume flow = f(t).



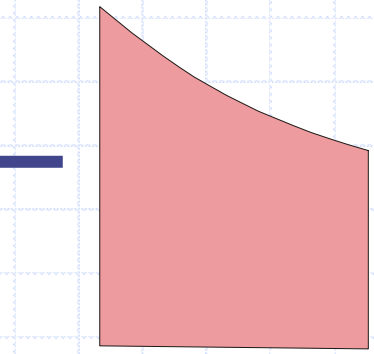
$\Delta X = dt * f(t-dt)$

flow = f(t)

Like Riemann sum.

Euler Integration Error

- Error = ΔX - area under flow curve

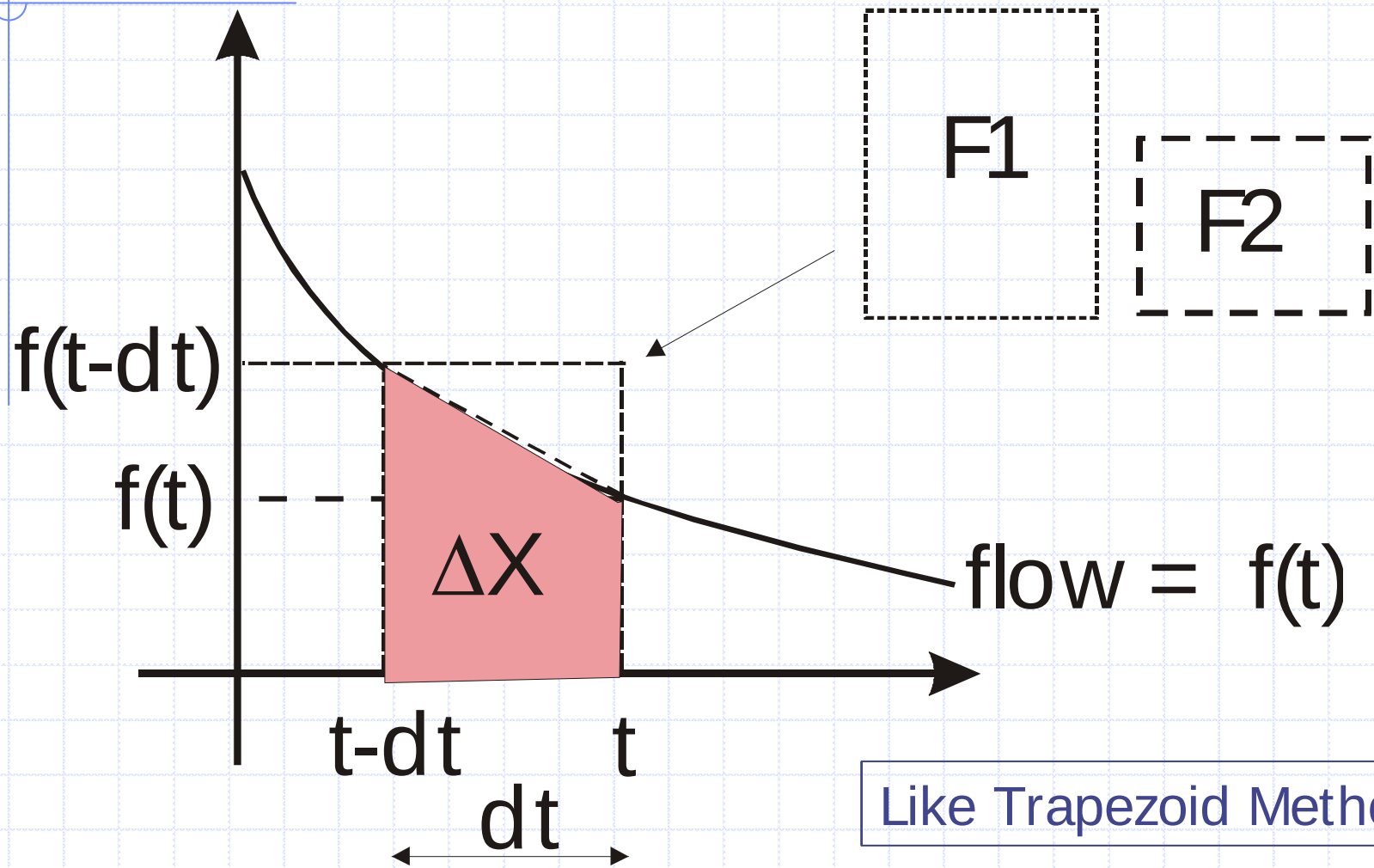
Error = ΔX 

Runge-Kutta 2

- Let Stock = X, flow = f(t,X)
- Estimates for stock updates:
  - F1 = dt * f(t-dt, X(t-dt))
  - F2 = dt * f(t, X(t-dt) + F1)
- $\Delta X = \frac{1}{2} * (F1 + F2)$
- $X(t) = X(t-dt) + \Delta X$

Runge-Kutta 2

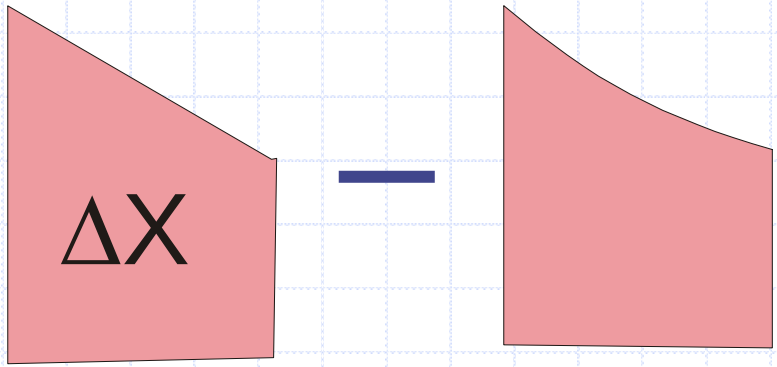
Assume flow = f(t).



Like Trapezoid Method.

RK2 Integration Error

- Error = ΔX - area under flow curve

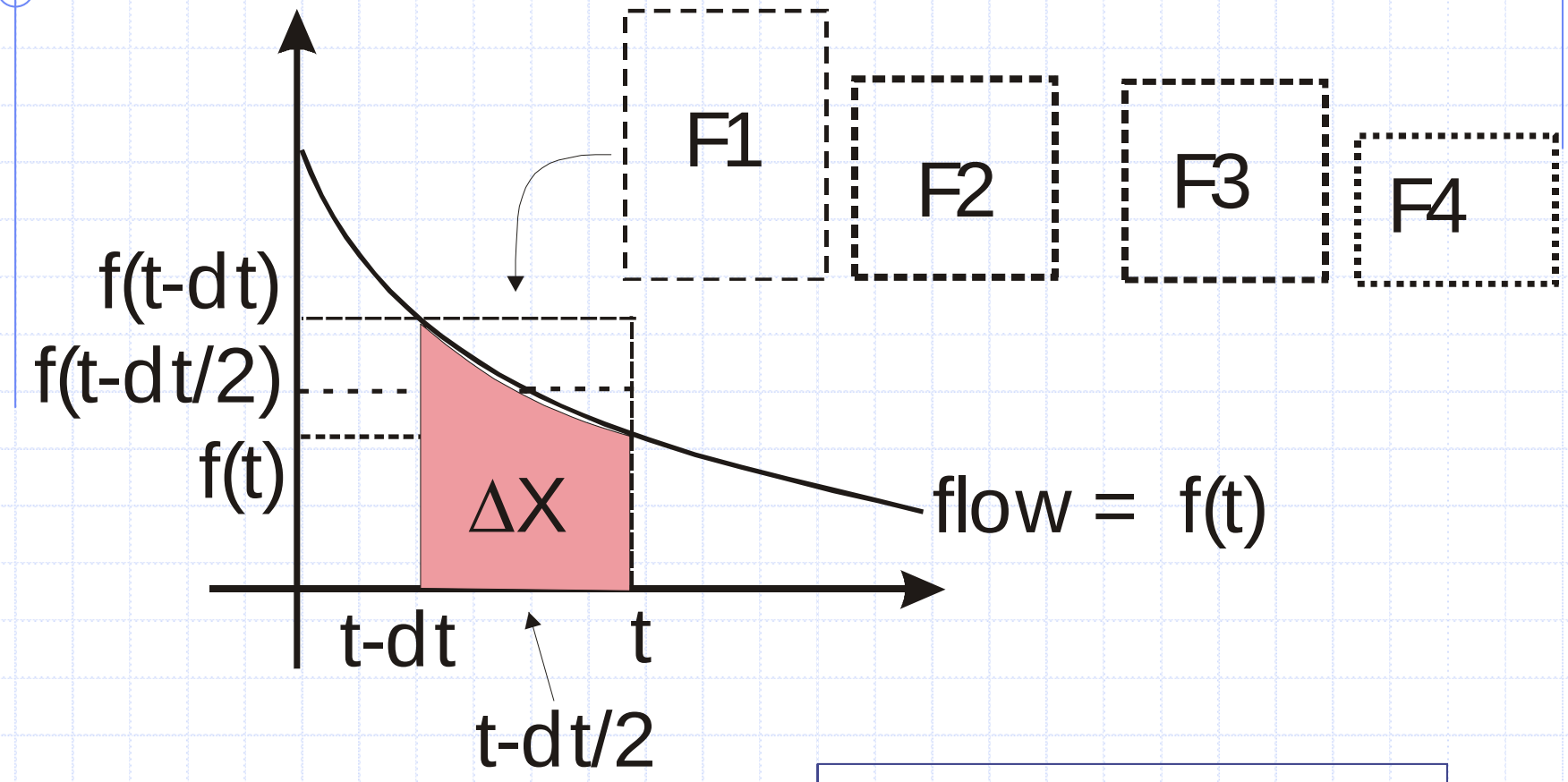
Error = 

Runge-Kutta 4

- Let Stock = X, flow = f(t,X)
- Estimates for stock updates:
  - F1 = dt * f(t-dt, X(t-dt))
  - F2 = dt * f(t-½ dt, X(t-dt) + ½ * F1)
  - F3 = dt * f(t-½ dt, X(t-dt) + ½ * F2)
  - F4 = dt * f(t, X(t-dt) + F3)
- $\Delta X = 1/6 * (F1 + 2*F2 + 2*F3 + F4)$
- $X(t) = X(t-dt) + \Delta X$

Runge-Kutta 4

Assume flow = f(t).



Like Simpson's Method.

What Method to Use?

- RK2 and RK4 are more accurate for same dt than Euler
- RK2 and RK4 work well for continuous systems
- Euler works poorly for oscillatory systems
- RK2 and RK4 work poorly with logic values and integers
- RK2 and RK4 work poorly with discrete systems
- RK2 and RK4 work poorly with conveyors