

MA 110 Quiz 7 **ANSWERS**

Show your work. State assumptions if you think they are needed or are unsure of mine.

1. How many different two-letter sequences can be formed using different letters from the word **QUIZ**?

$$\text{choices of first letter} \times \text{choices of second letter} = 4 \times 3 = 12$$

2. A New Jersey license plate consists of three letters (AB...XYZ) followed by three digits (012...89). How many such license plates are there?

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000$$

3. An English class has 20 students in it. In how many ways can a committee of four be chosen to bring complaints to the instructor? (No member of the committee has special distinction.)

$$\text{choices as if order mattered} / \text{duplicates} = (20 \times 19 \times 18 \times 17) / (4 \times 3 \times 2 \times 1) = 4,845$$

4. In how many ways can four books be put into two piles if each pile has exactly two books in it?

Possible assumptions: (1) the order of books in each pile matters, and (2) the order of the piles matters.

Possible answers: 3, 6, 12, 24, depending upon assumptions.

Default (natural) answer is 6. See below for alternatives under various assumptions. If appropriate explicit assumptions are stated and correctly implemented count 2 full credit + 1 extra credit = 3 points. If assumptions are stated, but flawed in execution, mark wrong but count 1+1 = 2 points. If no assumptions are stated, but one of the possible correct answers is given, mark incomplete and count 1 point.

(a) Assume not-(1) and (2): Ways to choose books for the first pile = $(4 \times 3) / (2 \times 1) = 6$. The books in second pile are now determined.

(b) Assume (1) and (2):

$$\begin{aligned} & (\text{books for first pile in order}) \times (\text{books for second pile in order}) = \\ & = (4 \times 3) \times (2 \times 1) = 24. \end{aligned}$$

(c) Assume not-(1) and not-(2):

$$\begin{aligned} & (\text{unordered ways to choose books in first pile}) / (\text{orders of piles}) = \\ & = [(4 \times 3) / (2 \times 1)] / (2) = 3. \text{ [Note that AB, AC, AD says it all.]} \end{aligned}$$

This is the answer as the other pile and its books are now determined.

(d) Assume (1) and not-(2):

$$\begin{aligned} & (\text{books for first pile in order}) \times (\text{books for second pile in order}) / (\text{order of piles}) = \\ & = (4 \times 3) \times (2 \times 1) / (2) = 12. \end{aligned}$$

The (natural) default correct answer is (a). If appropriate explicit assumptions are made, (b), (c), or (d) should be counted for full credit. Otherwise, only half credit.