Code:____ANSWERS____ (Do not use name or student number)

MA 110 Test 1

Table 1.1 is the preference schedule for an election with four candidates (A, B, C, and D).

Table 1.1

Number of Votes	8	3	1	5	8
1st Choice 4	D 32	C 12	A 4	B 20	A 32
2nd Choice 3	B 24	A 9	B 3	C 15	C 24
3rd Choice 2	A 16	D 6	C 2	D 10	B 16
4th Choice 1	C 8	B 3	D 1	A 5	D 8

Complete Answer. Show your full work on each of the following problems for full credit. Points count as indicated [n].

1. [10] Using the Borda Count method, which candidate wins the election in Table 1.1? If there is a tie, say so and do not break it.
   A: 16 + 9 + 4 + 5 + 32 = 66
   B: 24 + 3 + 3 + 20 + 16 = 66 A and B are tied for winner.
   C: 8 + 12 + 2 + 15 + 24 = 61
   D: 32 + 6 + 1 + 10 + 8 = 57

2. [10] Using the Plurality-with-Elimination method, **rank** the candidates in the election in Table 1.1? If there is a tie, say so and do not break it.

Round	A	B	C	D	
1	9	5	3	8	1st: D
2	12	5		8	2nd: A
3	12			13	3rd: B
					4th: C

3. [5] There is a Condorcet candidate in the election in Table 1.1. Who is it?
   B vs A: 13-12 B
   B vs C 14-11 B
   B vs D 14-11 B
   B wins all three of his 1-1 comparisons, so is a Condorcet candidate.

4. [5] What do your answers to questions 1, 2, and 3 tell you about which fairness criteria are violated in this election and by which method(s)? (Answer fully, giving reasons.)
(1) B is a Condorcet candidate, so by the Condorcet Criterion, should win the election. Question 2 has D winning by Plurality-with-Elimination. So Plurality-with-Elimination violates the Condorcet Criterion.
(2) Question 1 has a tie between A and B for winner by Borda count. The issue is "Does tying for winner violate a criterion?" If so, then question 1 shows that Borda Count violates the Condorcet Criterion. If tying is as good as winning, then this particular election tells us nothing.

5. [5] In the election in Table 5.1, the plurality method produces a tie between A and D. Who wins if you break the tie by bottom-up comparison?

Table 5.1

Number of Votes	**8**	**8**	**7**	**4**
1st Choice	D	A	C	B
2nd Choice	B	C	B	D
3rd Choice	C	D	A	A
4th Choice	A	B	D	C

A has 8 last place votes to D's 7. So the tie is broken in favor of D.

6. [5] An election is to be decided using the plurality method. There are five candidates and 99 voters. What is the smallest number of votes that a winning candidate can have, if there can be no ties for the winner?
21 + 20 + 20 + 19 + 19 = 99 The smallest is 21. Any attempt to make it smaller will result in a tie for winner.

7. [5] Give an example of an election decided by Borda Count which violates the Majority Criterion.

Number of Votes	5	4
1st Place 3	A 15	B 12
2nd Place 2	B 10	C 8
3rd Place 1	C 5	A 4

A has a majority of first place votes. However, A has only 19 Borda points to B's 22 (and C's 13). So B wins by Borda Count. Hence, this election violates the Majority Criterion.

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Short Answer. Place your answer to each question in the space provided. Show work in the space below the question if you want to be considered for possible partial credit. Points count as indicated [n].

8. _3__ [5] An election is held among five candidates (A, B, C, D, and E) using the Method of Pairwise Comparisons. A and C get 2 points each , B and E each get 1 ½ points. How many points does D get?
There are 10 possible comparisons, so 10 comparison points.
A + C + B + E = 2 + 2 + 1.5 + 1.5 = 7.
Thus D must have 3 comparison points.

9. __C__ [5] An election is held among three candidates (A, B, and C) using the Borda count method. There are 18 voters. If candidate A received 36 points and candidate B received 31 points, who won the election?
18 ballots x 6 Borda points per ballot = 108 Borda points
Since A and B received a total of 36 + 31 = 67 points, C must have received 108 – 67 = 41 points. Hence, C is the winner.

10. [5] An election is held among four candidates (A, B, C, and D). Using a voting method we will call "method X," the winner of the election is A. Because of an irregularity, a recount is required. Before the recount, B drops out of the race. In the recount, using method X, D now wins. Based on this information, we can say that voting method X violates the <u>Independence of Irrelevant Alternatives</u> Criterion. [Accept: IIA.]

11._13__ [5] Refer to the weighted voting system [q: 9, 5, 5, 4, 1]. What is the <u>smallest</u> value the quota q can take?
The sum of the weights is 24. The quota must be more than half the sum of the weights in order to avoid deadlock. Hence, the smallest quota is 13.

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12.__11_ [5] Refer to the weighted voting system [q: 6, 4, 3, 1]. Find a value of the quota q for which there are exactly two players with veto power.
Let q = 11. Then without Player 1, the weight is only 8 and without Player 2, the weight is only 10. So they both have veto power. Without Player 3, the weight is 11, which meets the quota. So Player 3 and (the lighter) Player 4 do not have veto power.

13.__8__ [5] Refer to the weighted voting system [10: w, 4, 2, 1]. Find a value of the weight w of Player 1 for which there is exactly one dummy.
Let w = 8. Then Player 1 with Player 2 or Player 3 meets the quota. Player 4 cannot help meet the quota with any second player, but with any third he is superfluous. So Player 4 is the lone dummy. [Also correct: 4.]

14. [5] Find the Banzhaf power distribution of the weighted voting system [12: 4, 4, 4, 1, 1, 1]. [Hint: very little computation should be required.] (Answer in space below.)
The first three players together meet the quota. No other three players do. Hence, the last three players are dummies. The first three players have equal weight, so are interchangeable. Thus, the power distribution is
1/3, 1/3, 1/3, 0, 0, 0.

15. [15] Consider the weighted voting system [8: 5, 3, 2, 1]. Find the winning coalitions, critical players, and Banzhaf power distribution for this system.

Winning Coalitions [5]	**Critical Players** [5]
$\{P_1, P_2\}$	P_1, P_2
$\{P_1, P_2, P_3\}$	P_1, P_2
$\{P_1, P_2, P_4\}$	P_1, P_2
$\{P_1, P_3, P_4\}$	P_1, P_3, P_4
$\{P_1, P_2, P_3, P_4\}$	P_1

Banzhaf Power Distribution: [5]

P_1: 1/2 P_2: 3/10 P_3: 1/10 P_4: 1/10