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MA 110 Test 2 ANSWERS

Questions on this test count the number of points indicated [n], for a total of 95 points. (You get 5 additional points if you print your code correctly and legibly.)

1. [5] Consider a fair division problem involving 5 players. The phrase "a player receives a fair share" describes the fact that (circle the letter of the best answer)
   a. The player receives a share that is more valuable than that of any other player.
   b. The player receives a share that is at least as valuable as that of any other player.
   c. The player receives a share that, in every player's opinion, has a value that is equal to 20% or more of the total value.
   d. The player receives a share that, in that player's own opinion, has a value that is equal to exactly 20% of the total value.
   e. The player receives a share that, in that player's own opinion, has a value that is equal to 20% or more of the total value.
   f. No player thinks another player's share is worth more than his.

2. [5] Three players (one divider and two choosers) are going to divide a cake by the Lone Divider Method. The divider cuts the cake into three slices {s_1, s_2, s_3}. Chooser 1 values the slices at 41%, 30%, and 29% of the whole, respectively. Chooser 2 values the slices at 25%, 35%, and 40%, respectively. What should the choosers' declarations be?
   Chooser 1: s_1 Chooser 2: s_2, s_3

3. [15] Tables 2.1 and 2.2 refer to a situation in which five players (one divider and four choosers) are going to divide a cake fairly by the Lone Divider method. The divider has cut the cake into five slices {s_1, s_2, s_3, s_4, s_5} and the choosers declarations are as follows

Table 2.1	Declaration
Chooser 1	{s_1, s_4}
Chooser 2	{s_1, s_2}
Chooser 3	{s_2, s_3}
Chooser 4	{s_3, s_4}

Table 2.2	Declaration
Chooser 1	{s_2, s_4}
Chooser 2	{s_2}
Chooser 3	{s_1, s_5}
Chooser 4	{s_4}

   a. If the choosers' declarations are as in Table 2.1, describe a fair division of the cake. [5] See b below.

   b. If the choosers' declarations are as in Table 2.1, describe a fair division of the cake different from that in your answer to a, above. [5]
   The following are the only possible answers to a and b:

D: s_5	C1: s_1	C2: s_2	C3: s_3	C4: s_4
D: s_5	C1: s_4	C2: s_1	C3: s_2	C4: s_3

   c. If the choosers' declarations are as in Table 2.2, describe how to proceed to obtain a fair division of the cake? [5]
   Choosers 1, 2, and 4 are a picky group. Any correct answer must include them playing again with a recombined cake with at least s_2 and s_4 in it, and as many total pieces as players left. One correct answer is: Divider gets s_3; Chooser 3 gets s_1; Choosers 1, 2, and 4 recombine $s_2 + s_4 + s_5$ into a new cake and play again. A different possibility is that Divider gets s_3 and all choosers play again with a new cake made up of $s_1 + s_2 + s_4 + s_5$. (The latter is fair, but less efficient.)

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4. [10] Four players (A, B, C, and D) are fairly dividing some items by the Method of Markers. They have marked the linear array below as shown. Describe the allocation of items to each player and indicate what items, if any, are leftover.

D (1–3) A (5–8) B (10–11) C (14–16)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Markers (↑): D (between 3 and 4), A (between 4 and 5), B (between 5 and 6), D (between 6 and 7), C (between 7 and 8), A (between 8 and 9), B C (between 9 and 10), D (between 11 and 12), B (between 12 and 13), A C (between 13 and 14)

Leftover: 4, 9, 12, 13

5. [15] Three heirs, A, B, and C inherit two items from their uncle's estate: a piano and a painting. They decide to divide the items among themselves by the method of sealed bids. Their bids are in the bid table below. Complete the tables below and describe the final settlement, including what cash amounts each player has ultimately paid or gained.

Bids	**Players**			
Item	**A**	**B**	**C**	**High Biddder**
piano	$9,750	$8,500	$7,500	A
painting	$6,000	$8,000	$6,750	B
Total Value	$15,750	$16,500	$14,250	
Fair Share	$5,250	$5,500	$4,750	

Allocation				
Player	**Item(s)**	**Value**	**Share**	**Put in (Take out)**
A	piano	$9,750	$5,250	Put in $4,500
B	painting	$8,000	$5,500	Put in $2,500
C	none	$0	$4,750	Take out $4,750
			Surplus	$2,250
			Share of Surplus	$750

Describe the final settlement for each player below (including cash balance).

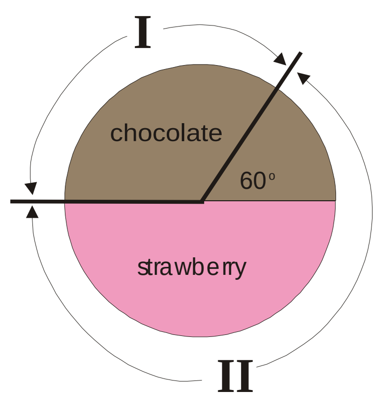
A: Gets piano and pays $4,500 – $750 = $3,750.
B: Gets painting and pays $2,500 - $750 = $1,750.
C: Gets no item, but get cash $4,750 + $750 = $5,500.
Note that the cash paid and the cash received balance to $0.

6. [8] Three players (A, B, and C) agree to divide some candy by the method of markers. Assume that you are player B and that you like Reese's Pieces (denoted R, below) twice as much as peanut M&Ms (denoted P), and you like plain M&Ms (denoted M) just as much as Reese's Pieces. Place your markers below so that you will be guaranteed to receive a fair share.

2	1	1	2	2	2	1	2	2	2	2	1	2	2	2	2	2	(30)
M	P	P	M	R	M	P	M	R	M	M	P	M	M	M	R	M	
					B↑						B↑						

7. [12] Ted and Alice buy a half-chocolate, half-strawberry cake for $24.00. They want to divide it by the divider-chooser method. Ted likes chocolate twice as much as strawberry. Alice likes chocolate three times as much as strawberry.
Parts a and b can be done by inspection, but parts c and d should show some work and/or explicit reasoning for full credit.

a. [3] What is the value of the chocolate half of the cake to Ted?
Let x = Value of strawberry to Ted. Then
2x = Value of chocolate. So 2x+x=24.
Then 3x=24 or x=8. Ted values chocolate $16.



What is the value of the strawberry half of the cake to Ted?
As shown above, Ted values strawberry $8.

b. [3] What is the value of the chocolate half of the cake to Alice?
Let x = Value of strawberry to Alice. Then
3x = value of chocolate to Alice. So 3x+x=24.
Then 4x=24 or x=6. Alice values chocolate $18.

What is the value of the strawberry half of the cake to Alice?
As shown above, Alice values strawberry $6.

c. Suppose Alice is the divider and cuts the cake as shown into pieces I and II. Show that pieces I and II are of equal value to Alice. [3]
In Alice's value system:
Value(II) = Value(strawberry) + $60^{o}/180^{o}$ × Value(chocolate)
Value(II) = 6 + 1/3 × 18 = 6 + 6 = 12
As $12 for piece II is half the value of the cake, she must value piece I at $12 also.

d. Which piece will Ted choose? Explain why. [3]
In Ted's value system:
Value(II) = Value(strawberry) + 1/3 × Value(chocolate)
Value(II) = 8 + 1/3 × 16 = 8 + 5.33= 13.33.
Since $13.33 is more than half the value of the cake, Ted will choose piece II.

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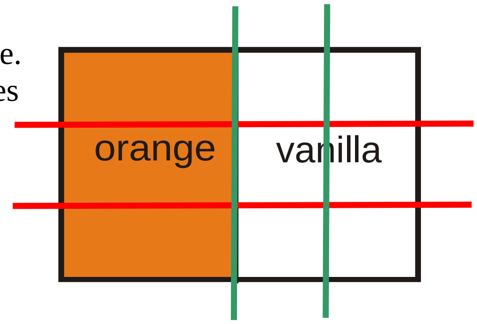
8. [5] Assume that you like vanilla twice as much as orange. Divide the orange/vanilla cake illustrated into three pieces of equal value to you.
There are many correct answers. Two are shown, one in red (horizontal) cuts and one in green cuts.



9. [15] The legislature of a country has 140 seats to be apportioned among four states A, B, C, and D, in proportion to their populations. The table below shows the populations of the states. Complete the table by filling in **all** unshaded blanks in order to produce a Hamilton apportionment.

State	A	B	C	D	Total	Points
Population	91,542	33,454	147,456	7,548	280,000	2
Standard Divisor	2,000					3
Standard Quotas	45.771	16.727	73.728	3.774	140.000	3
Round Down	45	16	73	3	137	2
Surplus Seats	1	0	1	1	3	2
Hamilton App't	46	16	74	4	140	3

10. [5] In the legislative apportionment for Paradoxia, the states A, B, and C receive the apportionments A – 9 seats, B – 5 seats, and C – 11 seats, using Hamilton's method with a house size of 150. The house size is increased to 151 but no populations change. When the apportionment is recalculated by the same method, the results are A – 10 seats, B – 4 seats and C – 12 seats. Why is this paradoxical and what paradox does this illustrate?
The paradox is that state B lost a seat even though the populations remained the same and a seat was added to the total. One would expect that B would retain its 5 seats and one lucky state would get the new seat as surplus seats were handed out. This is an example of the Alabama paradox.