

MA 110 Test 3 **Answers**

Questions on this test count the number of points indicated [n], for a total of 95 points. Show work for partial credit. (You get 5 points for printing your code number.)

- [15] The legislature of Appalchie has 100 seats to be apportioned among its four states Alab, Geog, Tenn, and Weva, in proportion to their populations. The table below shows the populations of the states. Complete the table by filling in all **unshaded** blanks to produce an apportionment by each of Jefferson's, Adams' and Webster's methods. (There is a scratch sheet on the last page.)

State	Alab	Geog	Tenn	Weva	Total	Points
Population	54,150	152,580	21,120	12,150	240,000	1
Standard Divisor	2,400					
Standard Quotas	8.800	63.575	22.563	5.063	100.00	2
Modified Divisor	2,350	2348	2353			
Modified Quota	8.987	64.928	23.043	5.170		
Jefferson App't	8	64	23	5	100	4
Modified Divisor	2,450	2430	2460			
Modified Quota	8.620	62.278	22.102	4.959		
Adams App't	9	63	23	5	100	4
Modified Divisor	2,405	2403	2406			
Modified Quota	8.7817	63.4428	22.5156	5.0520		
Webster App't	9	63	23	5	100	4

Note range of workable modified divisors above. See companion spreadsheet for testing divisors.

- [5] Did a violation of the quota occur in any of the apportionments in question 1? If so, state which apportionment(s), which state(s), and in each case if it was a violation of the upper quota or the lower quota.

No.

- [5] On a certain day in June, 350 people bought a ticket on the fast train from Birmingham to Atlanta and 500 people bought a ticket from Atlanta to Charleston. Suppose 730 people altogether bought tickets. How many people bought two tickets: one Birmingham to Atlanta and another Atlanta to Charleston?

$$BA + AC - \text{overlap} = \text{total}$$

$$350 + 500 - x = 730$$

$$x = 120$$

120 people bought both tickets.



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4. [5] Eight fair coins are tossed at one time and the sequence of heads (H) and tails (T) that comes up is observed. What does a typical element of the sample space look like? What is the size of the sample space?

A typical element is a sequence of 8 H or T, for example HTHHTTHH.

There are two ways each coin can come up. Hence, there are $2^8 = 128$ outcomes in the sample space.

5. [15] An Egyptian license plate consists of two digits, followed by two letters, followed by two more digits.

- a. How many such license plates are possible if the first digit cannot be 0?

$$9 \times 10 \times 26 \times 26 \times 10 \times 10 = 6,084,000$$

- b. If, in addition, no letter or digit can be repeated, how many such license plates are possible?

$$9 \times 9 \times 26 \times 25 \times 8 \times 7 = 2,948,400$$

- c. How many of the plates in part a have exactly three digits the same?

first 2 same + second 2 same = total (no overlap)

$$(9 \times 1 \times 26 \times 26 \times 1 \times 9 \times 2) + (9 \times 9 \times 2 \times 26 \times 26 \times 1 \times 1) = \\ = (109,512) + (109,512) = 219,024$$

6. [15] A committee of four members is to be formed from a school Spanish club containing 20 members, of whom 12 are boys and 8 are girls.

- a. If the order in which members are picked is not important, how many such committees are possible?

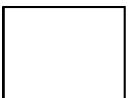
$$(20 \times 19 \times 18 \times 17) / (4 \times 3 \times 2 \times 1) = 4,845$$

- b. If the first chosen is chair, but the other three can be picked in any order, then how many such committees are possible?

$$20 \times [(19 \times 16 \times 17) / (3 \times 2 \times 1)] = 19,380$$

- c. If the committee must have equal numbers of boys and girls, but the order in which they are picked is not important, then how many such committees are possible?

$$[(12 \times 11) / (2 \times 1)] \times [(8 \times 7) / (2 \times 1)] = 1,848$$



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7. [15] Henry owns four books titled A, B, C, and D.
- In how many different ways can he arrange the four books in a row on his bookshelf?
 $4 \times 3 \times 2 \times 1 = 24$
 - In how many different many ways can he put them on a circular rotating tray?
 $(4 \times 3 \times 2 \times 1)/4 = 6$
 - In how many different many ways can he choose two books to keep and two to give away?
Order is not important.
Keep: $(4 \times 3)/2 = 6$. Give away is then determined. So, answer is 6.
8. [10] Three fair coins are tossed at one time. The sequence of heads (H) and tails (T) that comes up is observed.
The sample space is
{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
with sample size = 8. All outcomes are equally likely.
- What is the probability that at most two tails are observed?
Event at most 2 tails = {HHH, HHT, HTH, THH, HTT, THT, TTH}
with size = 7
 $\text{prob}(\text{at most 2 T}) = \text{event size}/\text{sample size} = 7/8$
 - What is the probability that at least two tails are observed?
Event at least 2 T = {HTT, THT, TTH, TTT}
with size = 4
 $\text{prob}(\text{at least 2 T}) = 4/8 = 1/2$
[Accept 4/8.]
9. [10] A jar contains one blue marble, two red marbles, and five white marbles. A marble is drawn from the jar at random.
Sample space is {B, R, W}. Outcomes are not equally likely.
- What is the probability that the marble drawn is white?
Since 5 out of 8 marbles are white, $\text{prob}(W) = 5/8$
 - What is the probability that the marble drawn is red?
Since 2 out of 8 marbles are red, $\text{prob}(R) = 2/8 = 1/4$
[Accept 2/8.]

