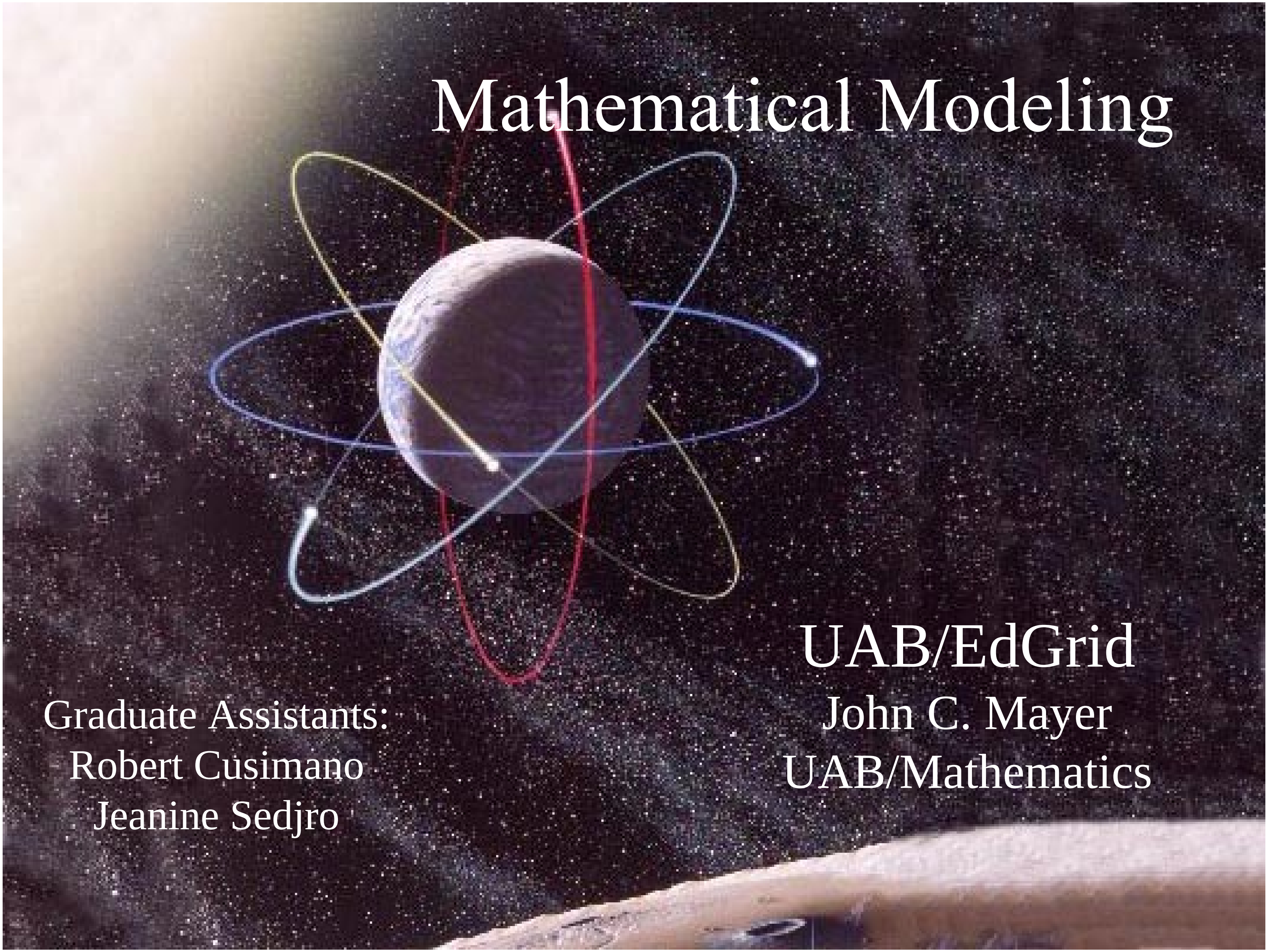


Mathematical Modeling

UAB/EdGrid

John C. Mayer
UAB/Mathematics

Graduate Assistants:
Robert Cusimano
Jeanine Sedjro

Mathematical Modeling

- Creates a mathematical representation of some phenomenon to better understand it.
- Matches observation with symbolic representation.
- Informs theory and explanation.

The success of a mathematical model depends on how easily it can be used, and how accurately it predicts and how well it explains the phenomenon being studied.

Mathematical Modeling

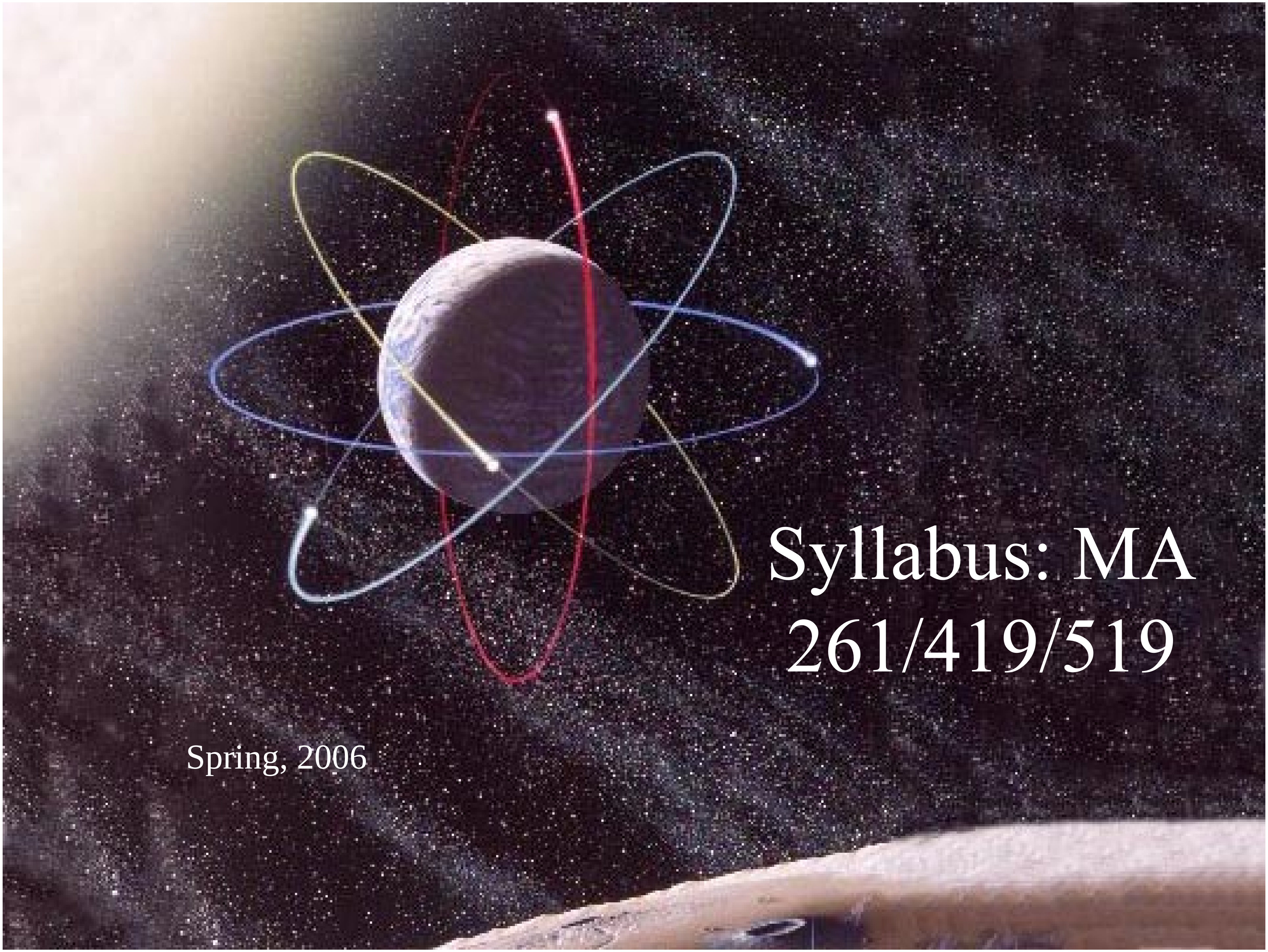
- A mathematical model is central to most computational scientific research.
- Other terms often used in connection with mathematical modeling are
  - Computer modeling
  - Computer simulation
  - Computational mathematics
  - Scientific Computation

Mathematical Modeling and the Scientific Method

- How do we incorporate mathematical modeling/computational science in the scientific method?

Mathematical Modeling Problem-Solving Steps

- **Identify problem area**
- **Conduct background research**
- **State project goal**
- **Define relationships**
- **Develop mathematical model**
- **Develop computational algorithm**
- **Perform test calculations**
- **Interpret results**
- **Communicate results**



Syllabus: MA 261/419/519

Spring, 2006

Syllabus: MA 261/419/519

Introduction to Mathematical Modeling

- Prerequisite: Calculus 1
- Goals
  - Learn to build models
  - Understand mathematics behind models
  - Improve understanding of mathematical concepts
  - Communicate mathematics
- Models may be mathematical equations, spreadsheets, or computer simulations.

Contact Information

- Instructor: Dr. John Mayer - CH 490A
  - mayer@math.uab.edu - 934-2154
  - TBA
- Assistant: Mr. Robert Cusimano - CH 478B
  - rob5236@uab.edu - 934-2154
  - MW 5:15 – 8:00 PM (computer lab CB 112)
- Assistant: Ms. Jeanine Sedjro – TBA
  - TBA
  - MW 4:00 – 6:45 PM (computer lab CB 112)

Class Meetings

- **Lectures**.
  - Monday and Wednesday
  - 5:30 – 6:45 PM
  - CB15 112.
- **Computer lab**.
  - Monday and Wednesday
  - 4:00 – 5:30 PM
  - 6:45 – 8:00 PM
  - CB15 112.

Software Tools

- Spreadsheet:
  - *Microsoft* EXCEL
- Compartmental Analysis and System Dynamics programming:
  - *Isee Systems* STELLA

Computer Lab

- The only computer labs that have STELLA installed are CB15 112 and EDUC 149A.
- **Always** bring a formatted 3.5" diskette to the lab.
- Label disk: "Math Modeling, Spring, 2006,
  Your Name,
  Math Phone Number: 934-2154"

Assignments

- One or two assignments every week.
- First few assignments have two due dates.
  – Returned next class.
  – Re-Graded for improved grade.
- Written work at most two authors.
  – You may work together with a partner on the computer.

Midterm Tests

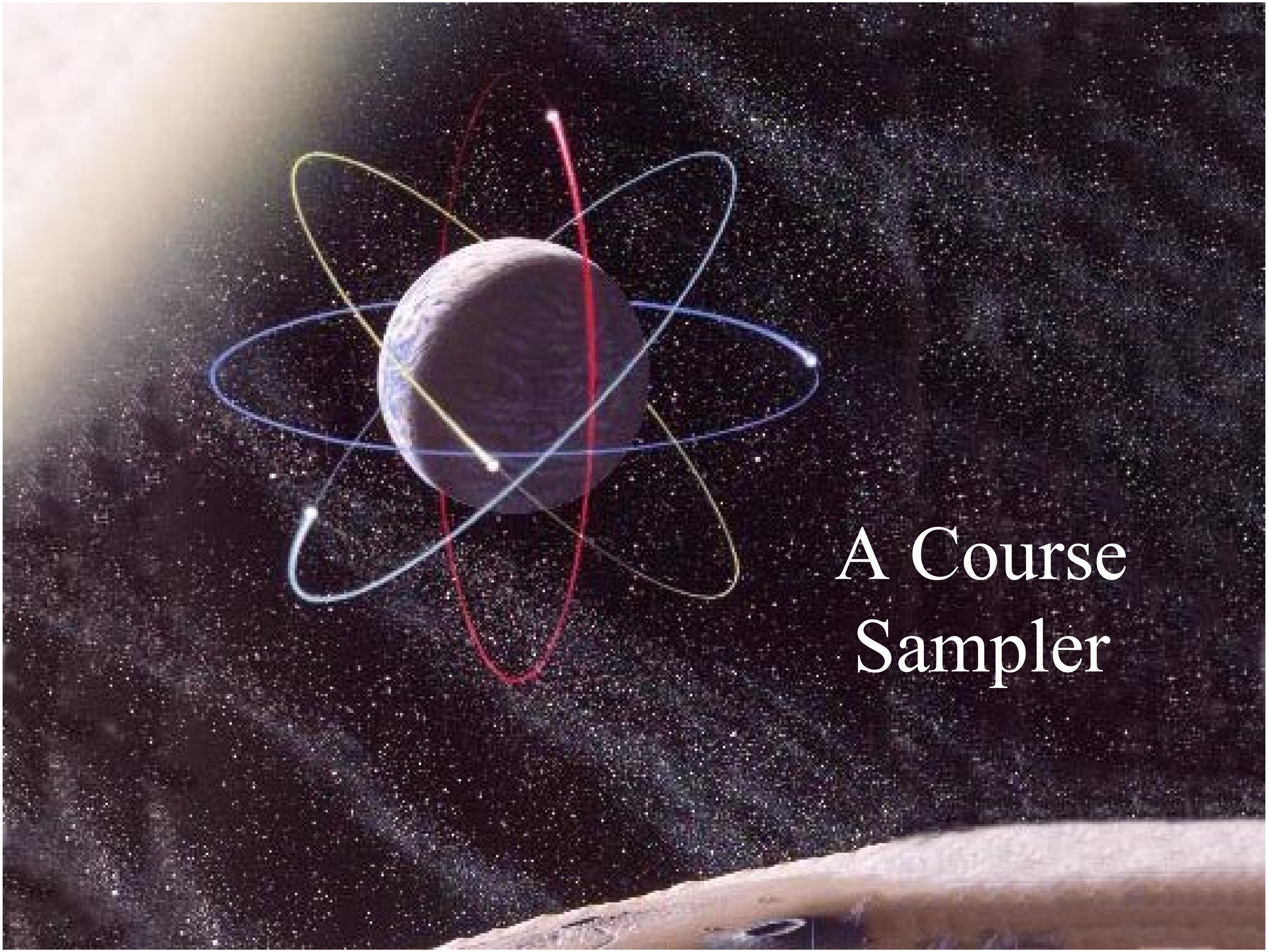
- Two tests, one about every 5 weeks.
- Given in the computer lab.
- Basic building blocks relevant to the kinds of models we are constructing.
- Mathematics and logic behind the computer models.

System Dynamics Stories and Projects

- **System Dynamics Stories.**
  - Scenarios describing realistic situations to be modeled.
  - **Entirely independent work – no partners.**
  - Construct model and write 5-10 page technical paper (template provided).
- **Project due date to be announced**.
  - Begin work about middle of term.
  - Preliminary evaluation three weeks prior to due date.

Grading

	MA 261	MA 419	MA 519
Assignments	40%	30%	30%
Tests (2)	30%	30%	25%
Model 1	10%	10%	10%
Model 2	na	na	10%
Paper	20%	20%	15%
Lesson Plan	na	10%	10%


A Course
Sampler

Topics for Tools

- **Spreadsheet Models**
  - Data analysis: curve fitting.
  - Recurrence Relations and difference equations.
  - Cellular Automata: nearest-neighbor averaging.
- **Compartmental Models**
  - Introduction to System Dynamics.
  - Applications of System Dynamics.
  - System Dynamics Stories (guided projects).

Spreadsheet Models: Excel

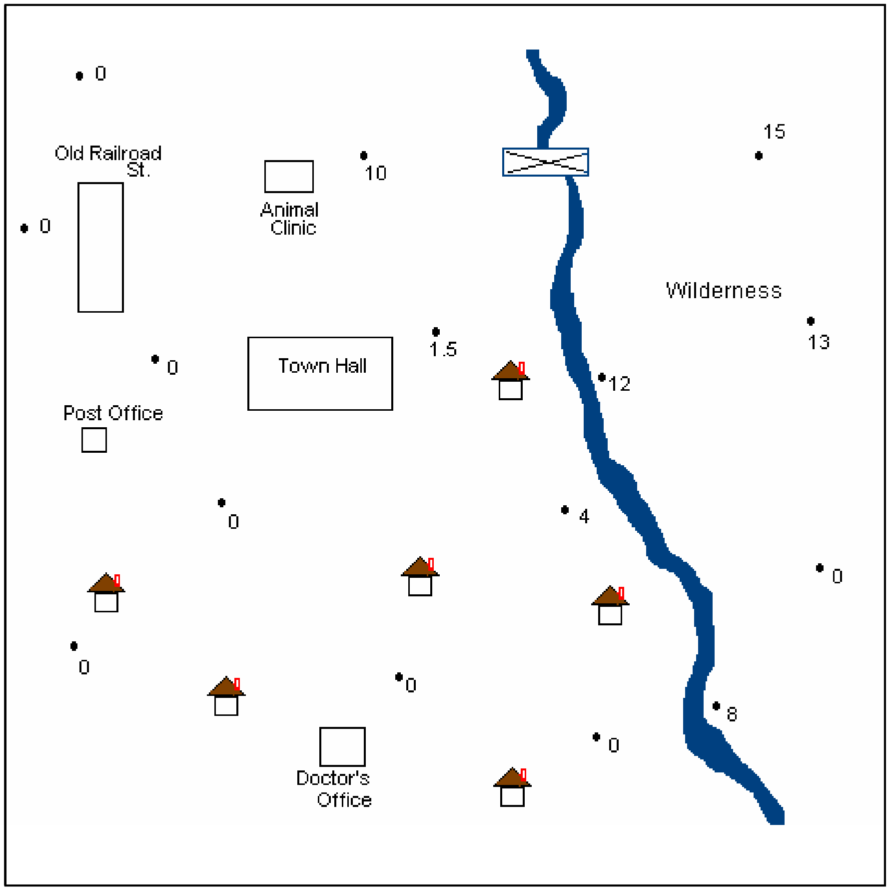
- Curve fitting – introduction to (linear) regression
- Difference Equations: modeling growth
- **Nearest-neighbor averaging**

Morteville

by Doug Childers

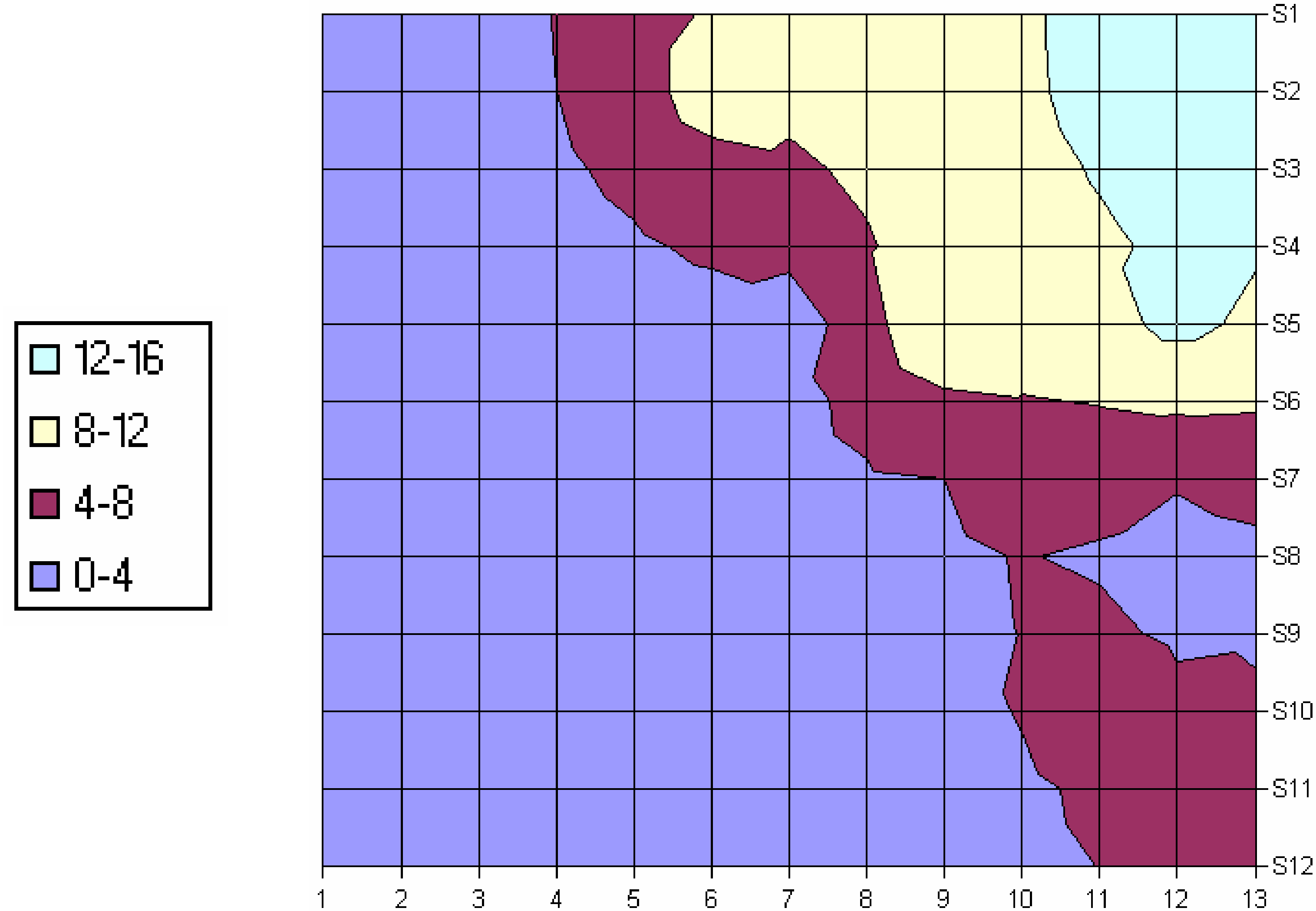
- Anthrax detected in Morteville
- Is terrorism the source?
- Infer geographic distribution from measures at several sample sites
- Build nearest neighbor averaging automaton in Excel
- Form hypothesis
- Get more data and compare
- Revise hypothesis

Morteville View 1



0.2	0	2.1	4.1	6.3	8.4	9	9.7	11	12	13	14	14
0.3	0.9	2.2	4	6.3	10	9	9.4	10	12	13	15	14
0	0.9	1.9	3.3	5	6.7	7.4	8.6	10	11	12	13	13
0.3	0.7	1.2	2.4	3.5	4.5	5.3	7.6	10	11	12	13	12
0.3	0.3	0	1.4	2.3	2.6	1.5	6.6	12	10	11	13	11
0.3	0.3	0.3	0.8	1.6	2.2	2.9	5.1	7.2	7.8	8.2	8.6	8.5
0.2	0.2	0.2	0	1.1	1.8	2.6	3.6	4	5.3	5.5	4.9	5.5
0.1	0.1	0.2	0.4	0.9	1.4	2.1	2.9	3.5	4.1	3.6	0	3
0.1	0	0.2	0.4	0.6	0.9	1.6	2.2	3	4.1	4.8	3.4	3.6
0.1	0.1	0.2	0.3	0.3	0	1	1.6	2.2	4.3	8	5.2	4.5
0.2	0.2	0.2	0.3	0.3	0.4	0.7	0.8	0	3	5	4.8	4.6
0.2	0.2	0.3	0.3	0.4	0.5	0.7	0.9	1.2	2.8	4.1	4.5	4.5

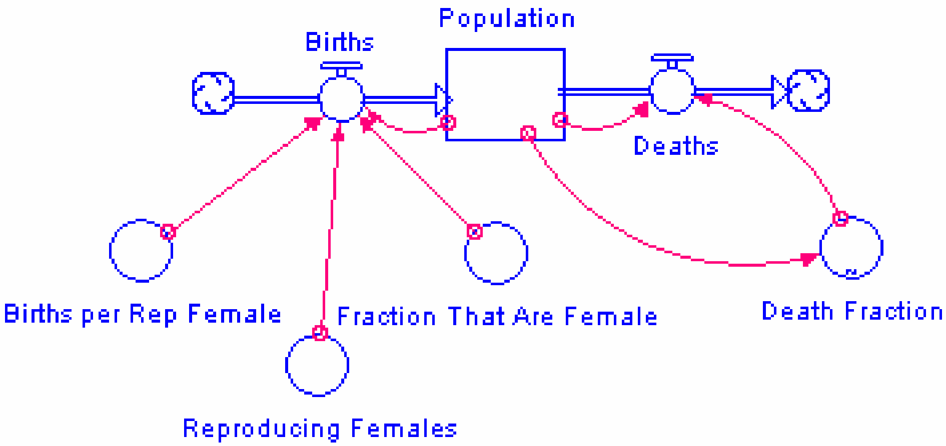
Anthrax Distribution 1

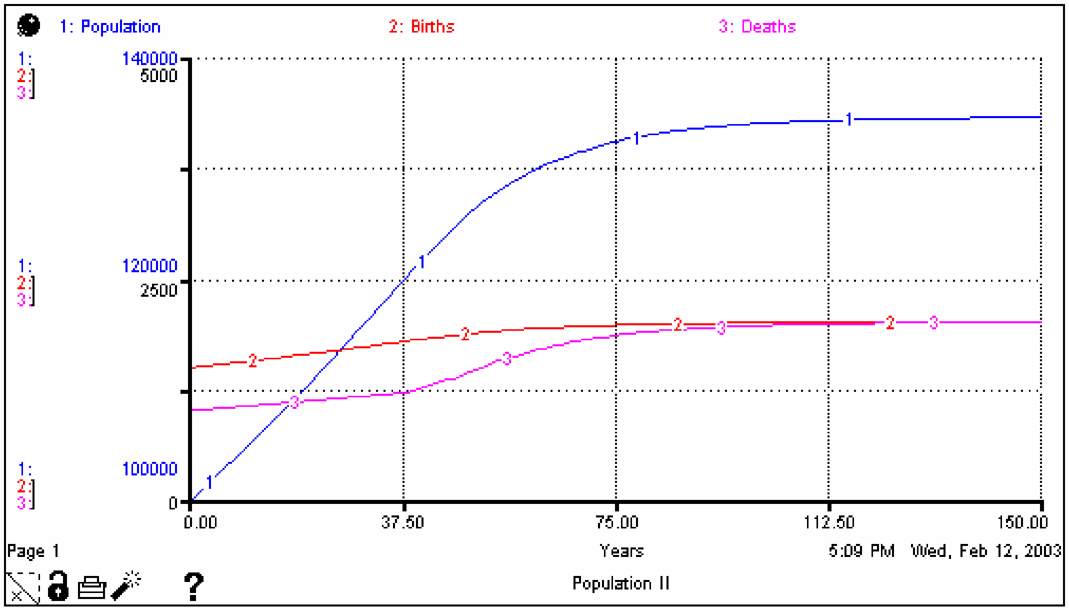


Compartmental Modeling

- How to Build a Stella Model
- **Simple Population Models**
- **Generic Processes**
- Advanced Population Models
- Drug Assimilation
- Epidemiology
- **System Stories**

Population Model





- Stella Model
- Equations
- Graphical Output

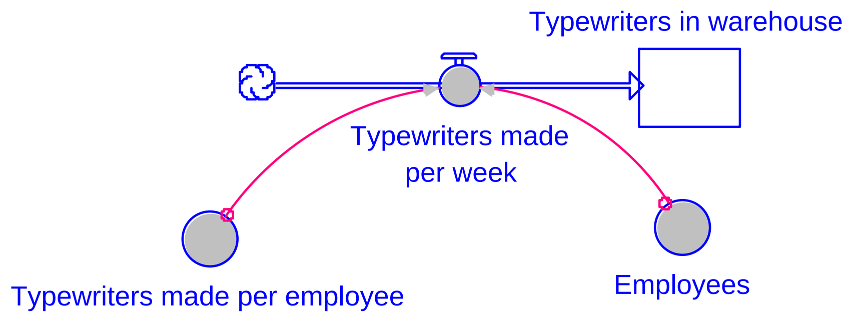
Population(t) = Population(t - dt) + (Births - Deaths) * dt
INIT Population = 100000 {people}

INFLOWS:
Births = Population*Fraction_That_Are_Female*Reproducing_Females*Births_per_Rep_Female {people/year}
OUTFLOWS:
Deaths = Population*Death_Fraction {people/year}
Births_per_Rep_Female = 66/1000 {people/1000 rep females/year}
Fraction_That_Are_Female = 0.5 {females/people}
Reproducing_Females = .45 {rep females/female}
Death_Fraction = GRAPH(Population)
(120000, 0.01), (125000, 0.011), (130000, 0.013), (135000, 0.015), (140000, 0.017), (145000, 0.019), (150000, 0.02)

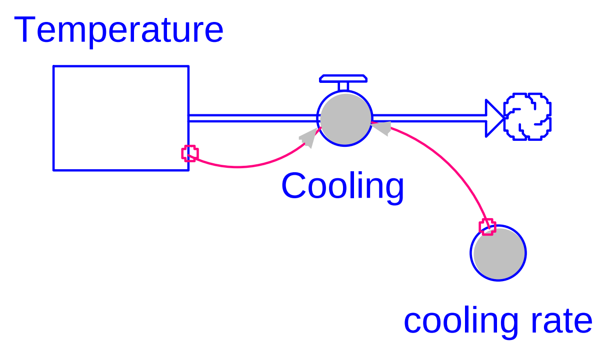
UAB

Generic Processes

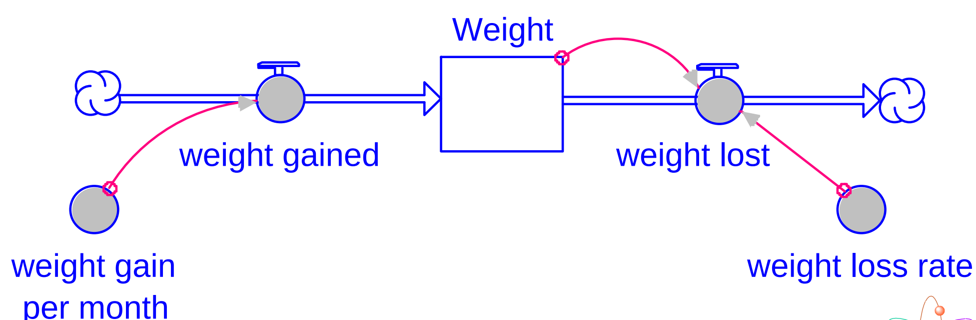
- Linear model with external resource



- Exponential growth or decay model

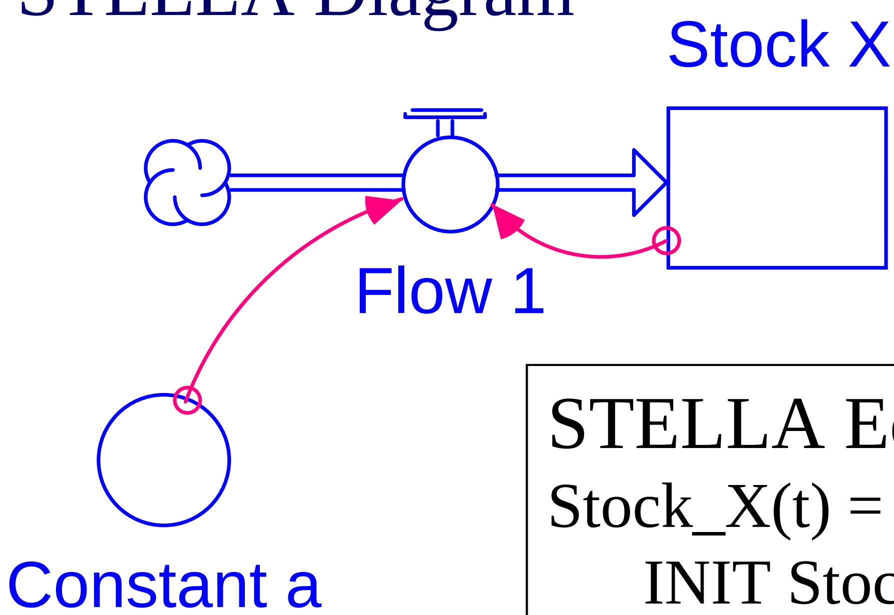


- Convergence model



Calculus Example

STELLA Diagram



Flow is proportional to X.

STELLA Equations:

Stock_X(t) = Stock_X(t - dt) + (Flow_1) * dt

INIT Stock_X = 100

Flow_1 = Constant_a*Stock_X

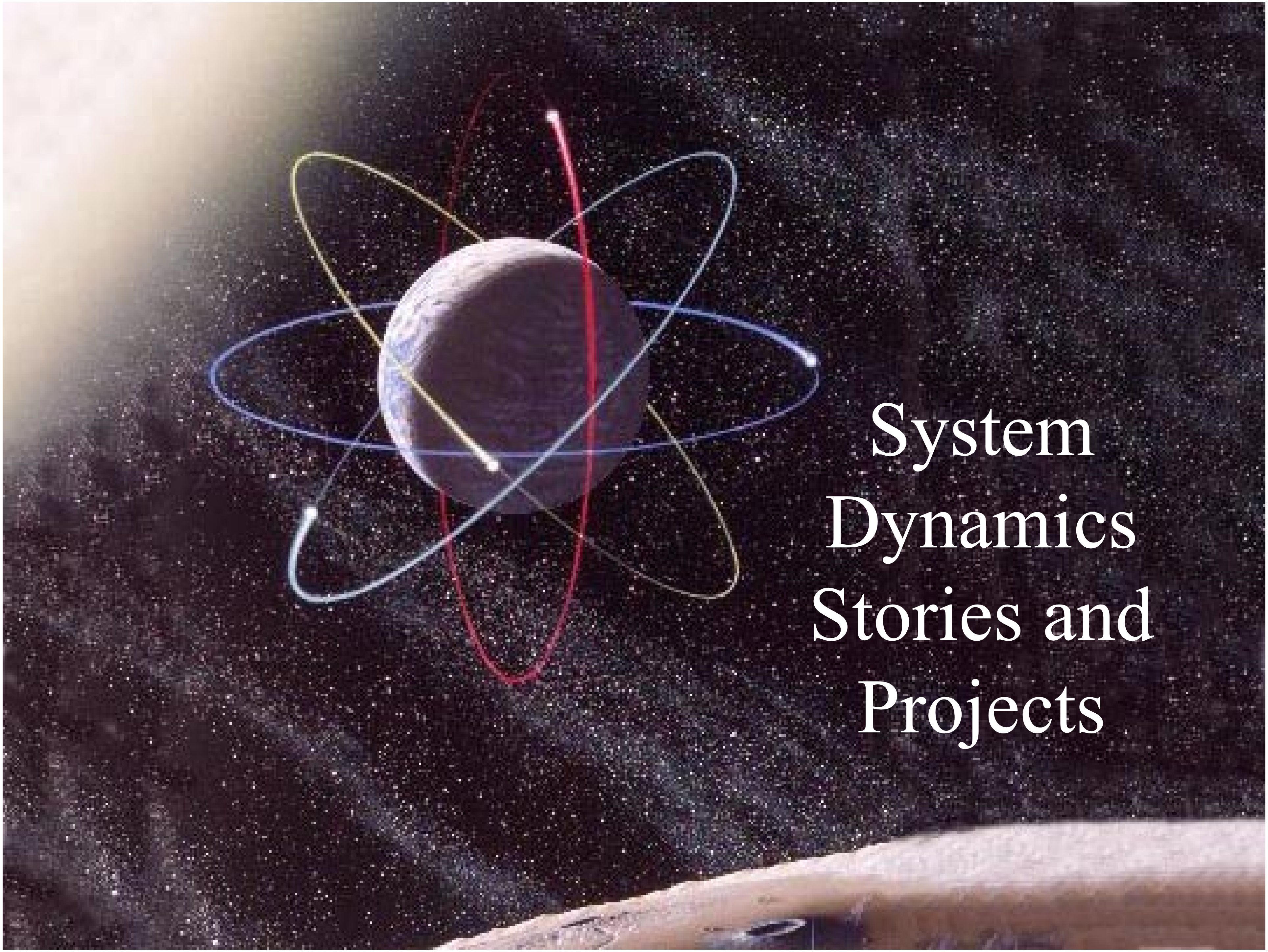
Constant_a = .1

Connecting the Discrete to the Continuous

- Stock_X(t) = Stock_X(t - dt) + (Flow_1) * dt
- (Stock_X(t) - Stock_X(t - dt))/dt = Flow_1
    - Flow_1 = Constant_a*Stock_X(t – dt)
- (X(t) - X(t - dt))/dt = a X(t - dt)
- Let dt go to 0
- dX/dt = a X(t)   (a Differential Equation)

Solution to DE

- $dX/dt = a\, X(t)$
- $dX/X(t) = a\, dt$
- Integrate
- $\log(X(t)) = at + C$
- $X(t) = \exp(C)\exp(at)$
- $X(t) = X(0)\exp(at)$


System
Dynamics
Stories and
Projects

System Dynamics Stories and Projects

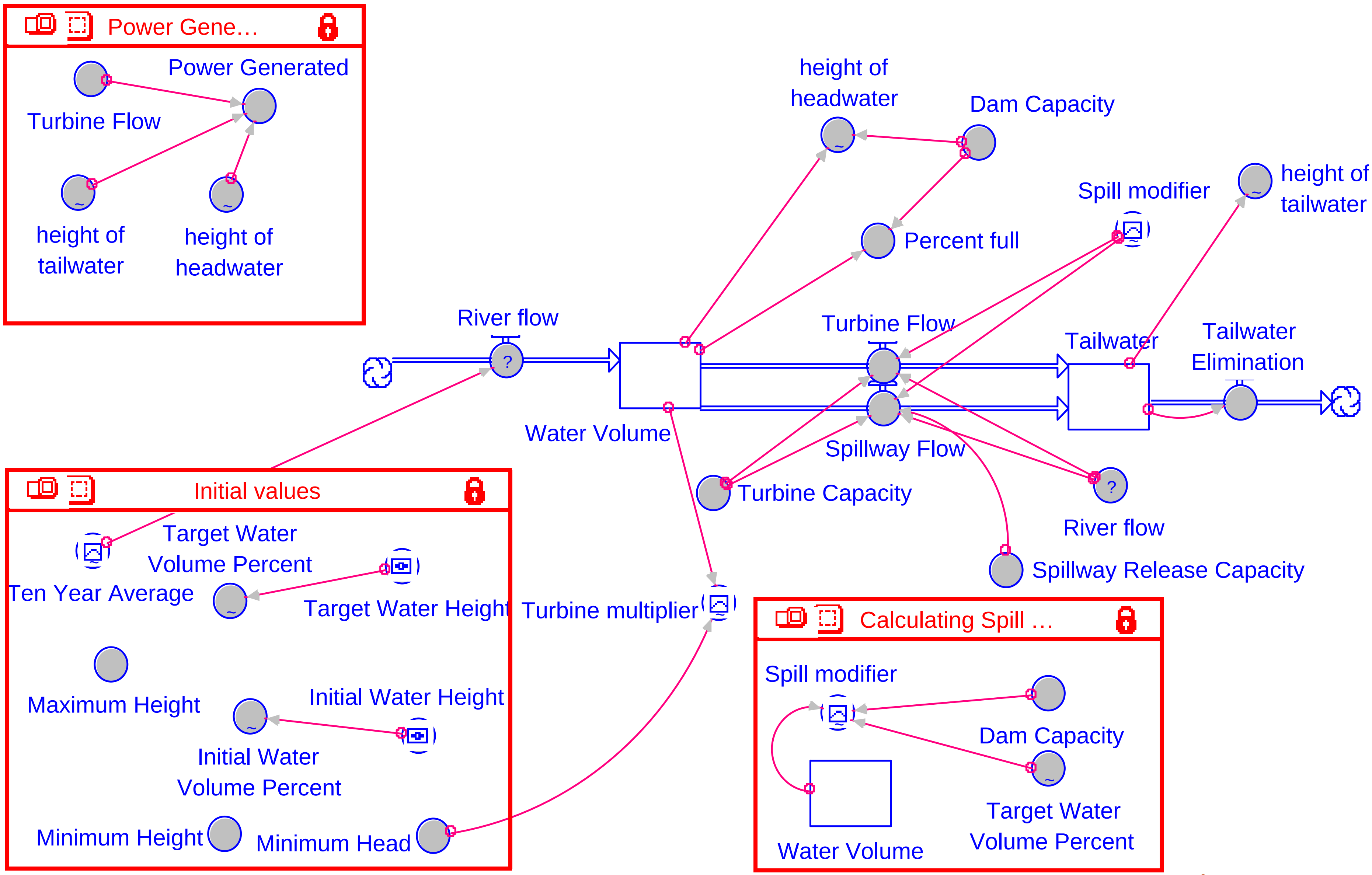
- Scenarios describing realistic situations to be modeled.
- Fully independent work.
- Construct model.
- Write 5-10 page technical paper (template provided).

Modeling a Dam 2

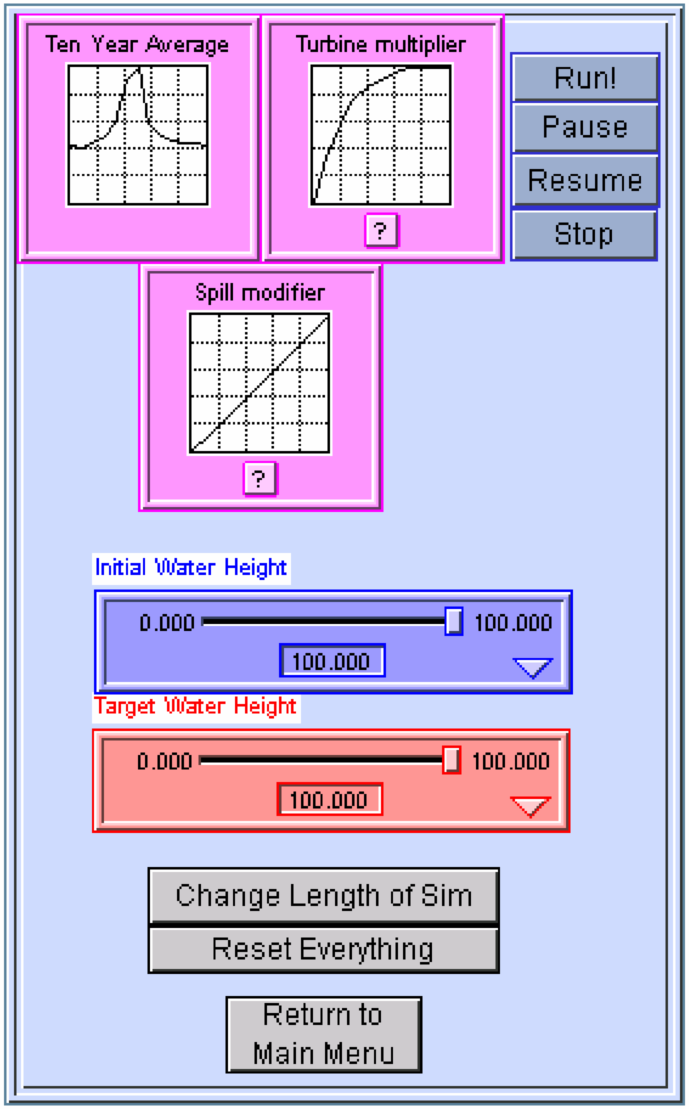
Boysen Dam

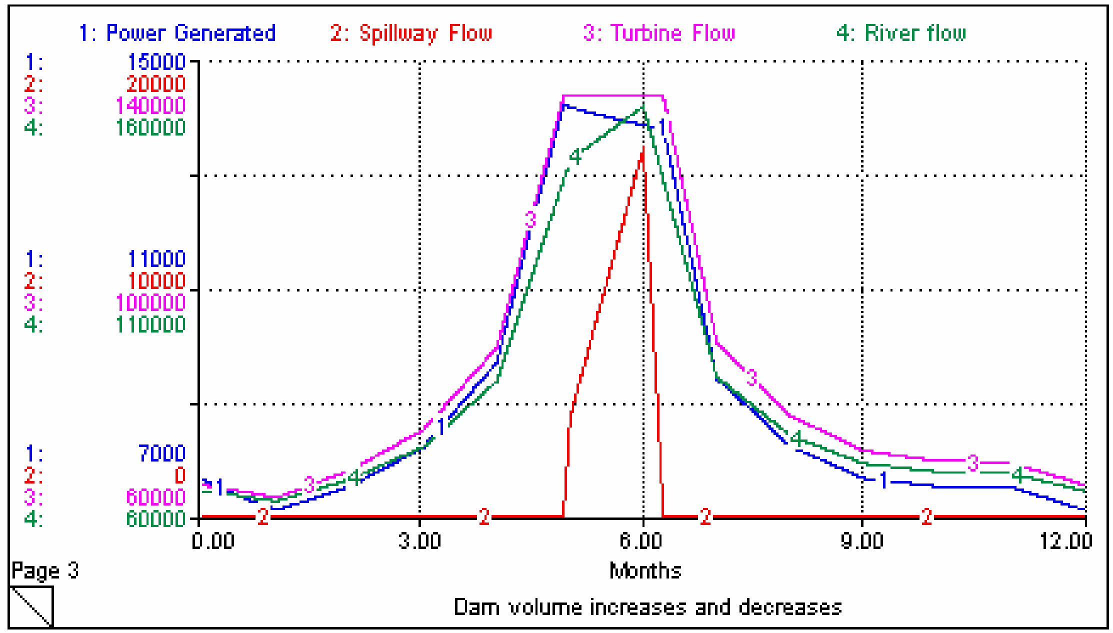
- Boysen Dam has several purposes: It "provides regulation of the streamflow for power generation, irrigation, flood control, sediment retention, fish propagation, and recreation development." The United States Bureau of Reclamation, the government agency that runs the dam, would like to have some way of predicting how much power will be generated by this dam under certain conditions.

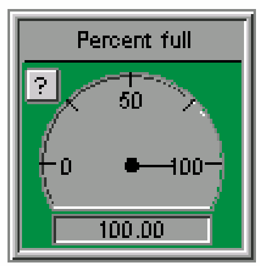
– Clinton Curry


Power Gene…
Power Generated
Turbine Flow
height of tailwater
height of headwater
Initial values
Target Water Volume Percent
Ten Year Average
Target Water Height
Maximum Height
Initial Water Height
Initial Water Volume Percent
Minimum Height
Minimum Head
River flow
Water Volume
height of headwater
Dam Capacity
Percent full
Spill modifier
height of tailwater
Turbine Flow
Tailwater
Tailwater Elimination
Spillway Flow
Turbine Capacity
River flow
Spillway Release Capacity
Turbine multiplier
Calculating Spill …
Spill modifier
Dam Capacity
Target Water Volume Percent
Water Volume

Boysen Dam







Simulation Time	12.0
Water Volume	892,226.0
Target Water Height	100.0
Power Generated	7,094.0

Modeling a Smallpox Epidemic

- One infected terrorist comes to town
- How does the system handle the epidemic under different assumptions?

– Alicia Wilson