Probability 2

An ***event*** is any set of outcomes of a random experiment; that is, any ***subset*** of the sample space of the experiment.

- The ***probability*** of a given event is the ***sum*** of the probabilities of the outcomes in the event.
- If all outcomes are equally likely, then the probability of an event is the ***event size divided by the sample size***.

Example

Two fair coins are tossed simultaneously. We observe whether "heads" or "tails" lands face up on the two coins. What is the probability of tossing at least one head?

Sample space: {HH, HT, TH, TT} Sample Size: N = 4

Event: {HH, HT, TH} Event Size: E = 3

Probability of event: p(at least one H) =

Example

Three fair coins are tossed simultaneously. We observe whether "heads" or "tails" lands face up on the coins. What is the probability of tossing at least two heads?

Example

A pair of fair 6-sided dice (one white, one black) is rolled, and we observe the faces which show up. We use (2,3) to indicate there is a 2 on the white die and a 3 on the black die.

Sample Space **Sample size N = 36**

(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

What is the probability that the sum of the faces showing adds up to 4?

Event: Event size:

Probability: $p(\text{sum is } 4) =$

What is the probability that the sum of the faces showing is 7?

What is the probability that the faces showing add up to a multiple of 5?

What is the probability that the product of the faces showing is less than or equal to 6?

Combining Probabilities

When combining probabilities remember that the answer, if a probability, must lie between 0 and 1, inclusive!

Given two events E and F, the most common ways of combining then are

- E **and** F – the event E and the event F both occur.
- E **or** F – either event E occurs, or event F occurs, or both occur.
- **not** E – the event E does not occur.
- F **given** E – the event E occurs given that the event F has occurred.
    - We symbolize “F given E” by F|E.

Two events E and F are ***independent*** if the occurrence of one has no influence on the probability of the other occurring.

Probability Combination Principles 1

Multiplication

Suppose that E and F are independent events. Then

$$p(\text{E and F}) = p(\text{E}) \text{ x } p(\text{F})$$

Suppose that E and F are not independent events. Then

$$p(\text{E and F}) = p(\text{E}) \text{ x } p(\text{F}|\text{E})$$

Recall that F|E means the event F occurs given that E has occurred.

Subtraction

The set of outcomes in the sample space that are not in the event E are in the event not E. The probability of not E is

$$p(\text{not E}) = 1 – p(\text{E})$$

Example

A pair of fair 6-sided dice (one white, one black) is rolled, and we observe the faces which show up.

Sample Space **Sample size N = 36**

(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

What is the probability that the sum of the dice is not 7?

Example

A jar contains five marbles, 2 green and 3 white. We draw two marbles from the jar and observe the colors.

Drawing with replacement. Suppose that we **replace** the first marble before drawing the second. What is the probability of drawing two green marbles?

Example

A jar contains five marbles, 2 green and 3 white. We draw two marbles from the jar and observe the colors.

Drawing without replacement. Suppose that we do **not** replace the first marble before drawing the second. What is the probability of drawing two green marbles?

Example

A jar contains five marbles, 2 green and 3 white. We draw two marbles from the jar and observe the colors. Suppose we the draw two marbles from the jar without replacement. What is the probability of drawing first a green marble and then a white marble?

Probability Combination Principles 2

Mutually Exclusive Events

Two events are ***mutually exclusive*** if when one occurs the other cannot occur.

Addition

Suppose that E and F are **mutually exclusive** events. Then

$$p(\text{E or F}) = p(\text{E}) + p(\text{F})$$

Suppose that E and F are **not** mutually exclusive events. Then

$$p(\text{E or F}) = p(\text{E}) + p(\text{F}) - p(\text{E and F})$$

Example

A pair of fair 6-sided dice (one white, one black) is rolled, and we observe the faces which show up.

Sample Space **Sample size N = 36**

(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

What is the probability that the sum of the dice is 7 or 11?

Example

A jar contains five marbles, 2 green and 3 white. We draw two marbles from the jar and observe the colors. Suppose we draw the two marbles from the jar without replacement. What is the probability of drawing a green marble and a white marble (in any order)?