Probability 3

An ***event*** is any set of outcomes of a random experiment; that is, any ***subset*** of the sample space of the experiment.

- The ***probability*** of a given event is the ***sum*** of the probabilities of the outcomes in the event.
- <u>If all outcomes are equally likely</u>, then the probability of an event is the ***event size divided by the sample size***.

Examples of equally likely outcomes

- Tossing 3 fair coins
- Rolling 2 fair dice
- Drawing a colored marble from a set of equal numbers of colors.

Examples of not equally likely outcomes

- Tossing an unfair coin
- Drawing a colored marble from a set of unequal numbers of colors.

Example

Three fair coins are tossed simultaneously. We observe whether "heads" or "tails" lands face up on the coins. What is the probability of tossing at most two heads?

Sample Space:

Sample Size:

Event:

Event size:

Probability: p(at most 2 H) =

Example

A pair of fair 6-sided dice is rolled, and we observe the faces which show up.

Sample Space **Sample size N = 36**

(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

What is the probability that the sum of the faces showing adds up to 10?

Event: **Event size:**

Probability: p(sum is 10) =

What is the probability that the product of the faces showing is less than 6?

What is the probability that the difference of the faces showing is less than or equal to 2? (Assume difference = larger – smaller, or 0 if equal.)

Combining Probabilities of Events

When combining probabilities remember that the answer, if a probability, must lie between 0 and 1, inclusive!

Given two events E and F, the most common ways of combining then are

- E **and** F – the event E and the event F both occur.
- E **or** F – either event E occurs, or event F occurs, or both occur.
- **not** E – the event E does not occur.
- F **given** E – the event F occurs given that the event E has occurred. We symbolize "F given E" by F|E.

Two events E and F are ***independent*** if the occurrence of one has no influence on the probability of the other occurring.

For independent events, $p(F) = p(F|E)$ and $p(E) = p(E|F)$.

Probability Combination Principles 1

Multiplication principle for independent events

Suppose that E and F are independent events. Then

$$p(\text{E and F}) = p(\text{E}) \text{ x } p(\text{F})$$

Example

Suppose that you toss a coin two times and get two heads in a row.

(a) What was the probability of that happening?

(b) What is the probability of the next toss being a head?

Probability Combination Principles 2

Subtraction principle for complementary events

The set of outcomes in the sample space that are **not** in the event **E** are in the event **not E**. The probability of **not E** is

$$p(\text{not E}) = 1 - p(\text{E})$$

Example

Five fair coins are tossed. What is the probability of at least one head?

Too much to add up?

Note that "at least one head" is the same as "not all tails."

Example

A pair of fair 6-sided dice is rolled, and we observe the faces which show up.

Sample Space **Sample size N = 36**

(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

What is the probability that the sum of the dice is not 10?

Probability Combination Principles 3

Multiplication principle for dependent events

Suppose that E and F are not independent events. Then

$$p(\text{E and F}) = p(\text{E}) \times p(\text{F}|\text{E})$$

Recall that F|E means the event F occurs given that E has occurred.

In this case we try to "adjust" the sample space and consequent probability to reflect that E has occurred.

The following examples show the contrast between multiplication of probabilities for **independent** events and for **dependent** events.

Example (Independent Events)

A jar contains six marbles, 2 red and 4 white. We draw two marbles from the jar and observe the colors.

Drawing with replacement. Suppose that we **replace** the first marble before drawing the second. What is the probability of drawing a white marble then a red marble?

Example (Dependent Events)

A jar contains six marbles, 2 red and 4 white. We draw two marbles from the jar and observe the colors.

Drawing without replacement. Suppose that we do **not** replace the first marble before drawing the second. What is the probability of drawing a white marble then a red marble?

Probability Combination Principles 4

Mutually Exclusive Events

Two events are ***mutually exclusive*** if when one occurs the other cannot occur.

Addition

Suppose that E and F are **mutually exclusive** events. Then

$$p(\text{E or F}) = p(\text{E}) + p(\text{F})$$

Suppose that E and F are **not** mutually exclusive events. Then

$$p(\text{E or F}) = p(\text{E}) + p(\text{F}) - p(\text{E and F})$$

That is, we must subtract the probability of the overlap.

Example

A jar contains six marbles, 2 red and 4 white. We draw two marbles from the jar and observe the colors. Suppose we draw the two marbles from the jar **without replacement**. What is the probability of drawing marbles of different colors? (Be sure to consider RW and WR.)

Example

An unfair coin is tossed. The probability of heads (H) coming up on the coin is only 1/3.

(a) What is the probability of tails (T) coming up?

(b) Now we toss the coin twice. What is the probability of HH?

(c) Again tossing twice, what is the probability of HT?

(d) Again tossing twice, what is the probability of TH?

(e) Again tossing twice, what is the probability of exactly one H?

(f) Again tossing twice, what is the probability of at least one head?

Example

A deck of cards contains 52 cards, 13 of each suit: spades, hearts, diamonds, and clubs. Two cards are drawn from the deck without replacement.

(g) What is the probability of drawing two spades from the deck?

(h) What is the probability of drawing a spade and a heart?

(i) What is the probability of drawing two cards of different suits?