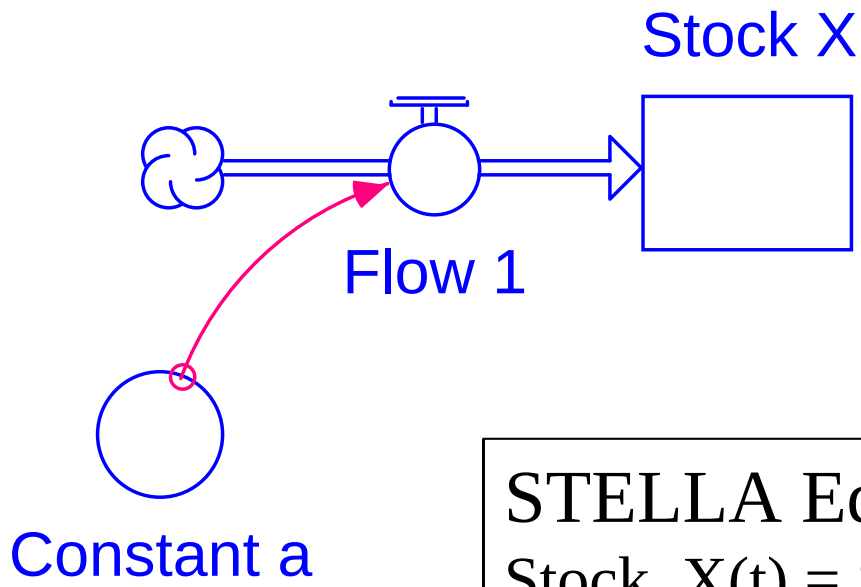
STELLA and Calculus

STELLA numerically simulates the solutions to systems of differential equations.

STELLA INTEGRATES!

Calculus Example 1

STELLA Diagram



Flow is constant.

STELLA Equations:

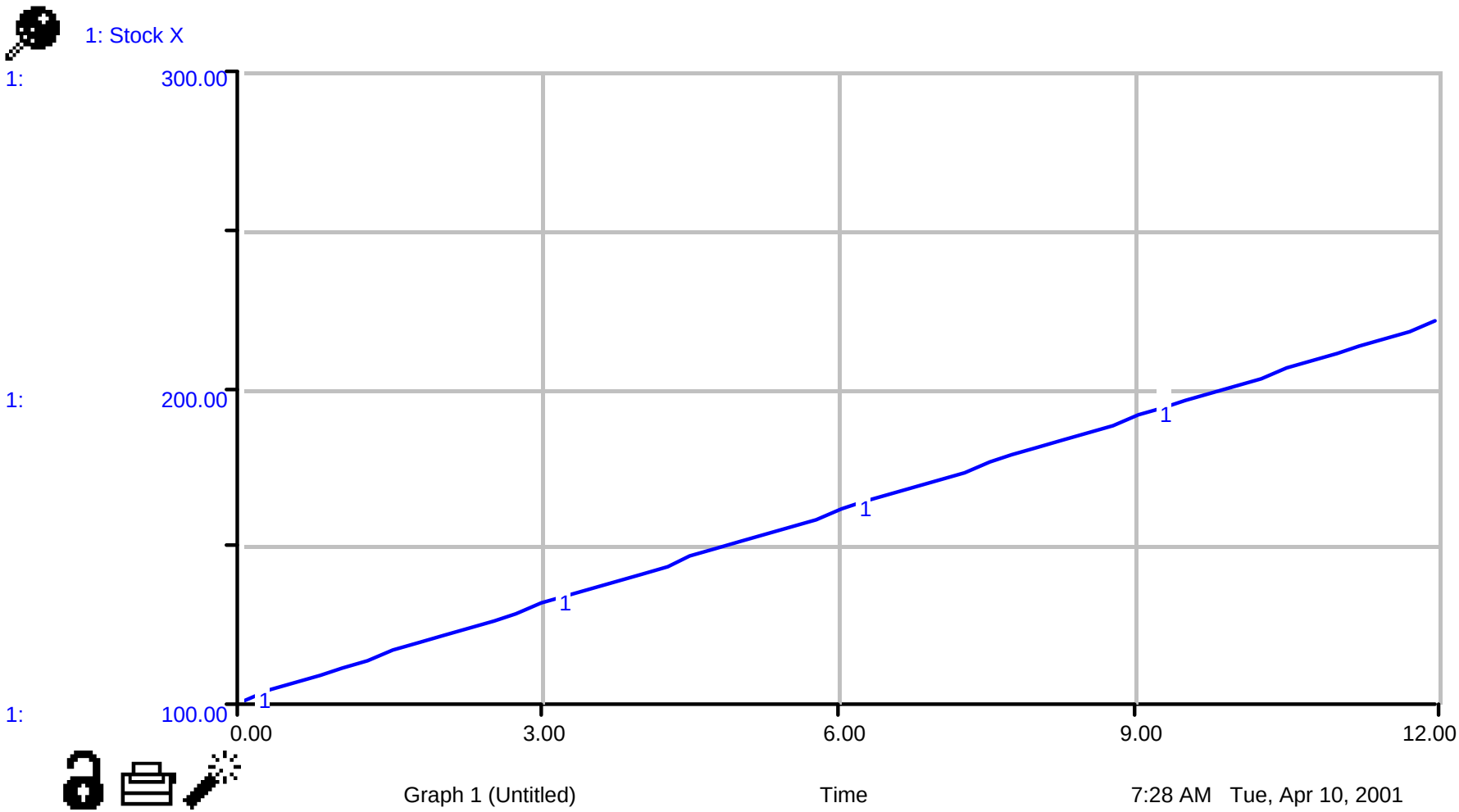
Stock_X(t) = Stock_X(t - dt) + (Flow_1) * dt

INIT Stock_X = 100

Flow_1 = Constant_a

Constant_a = 10

Example 1 Graph



Question?

- Does the graph have an equation?
- "Obviously"

$$X = 10t + 100$$

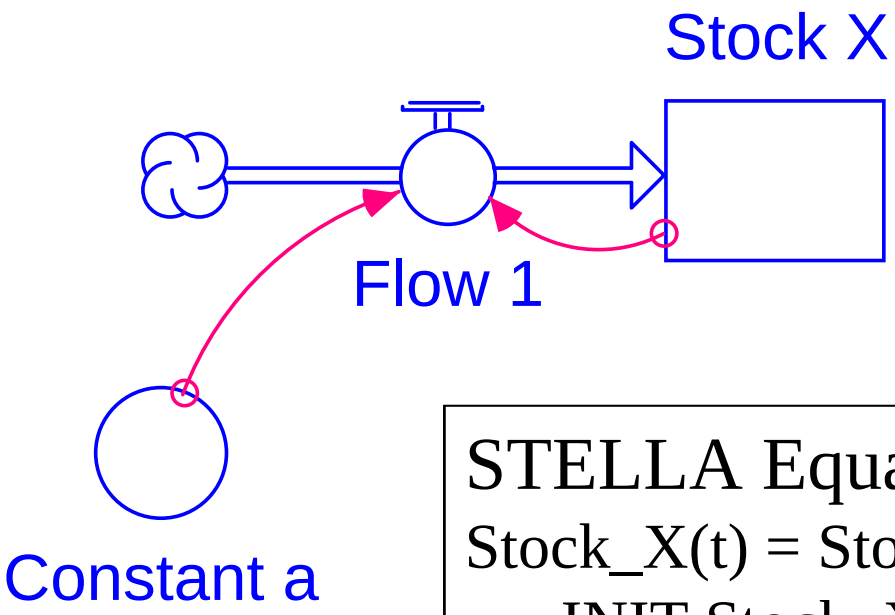
Differential Equation 1

- Stock_X(t) = Stock_X(t - dt) + (Flow_1) * dt
- (Stock_X(t) - Stock_X(t - dt))/dt = Flow_1
    - Flow_1 = Constant_a
- (X(t) - X(t - dt))/dt = a
    - let dt $\longrightarrow$ 0
- dX/dt = a (a differential equation)
- Solution to DE:

$$X = at + X(0)$$

Calculus Example 2

STELLA Diagram



Flow is proportional to X.

STELLA Equations:

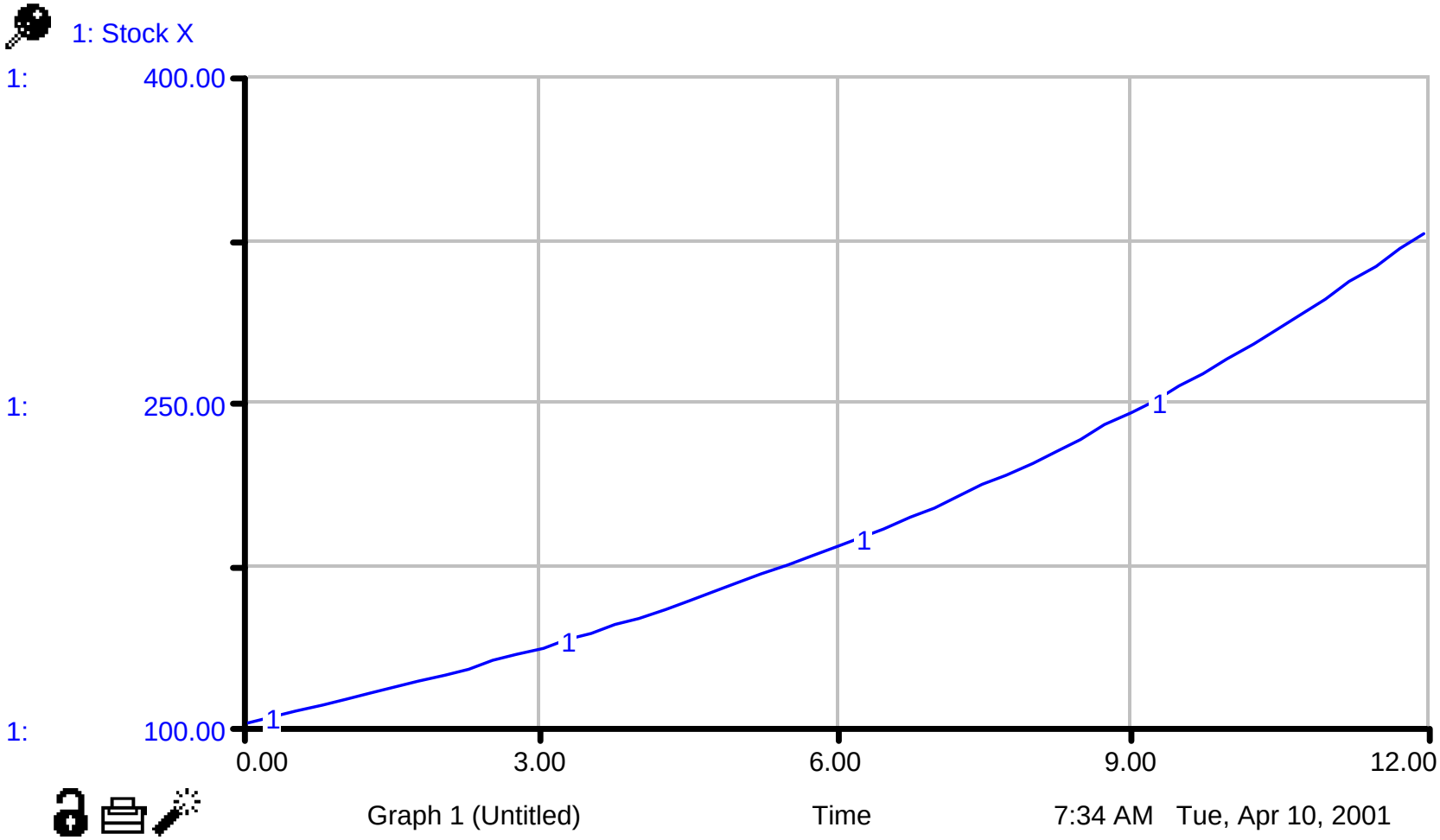
Stock_X(t) = Stock_X(t - dt) + (Flow_1) * dt

INIT Stock_X = 100

Flow_1 = Constant_a*Stock_X

Constant_a = .1

Example 2 Graph



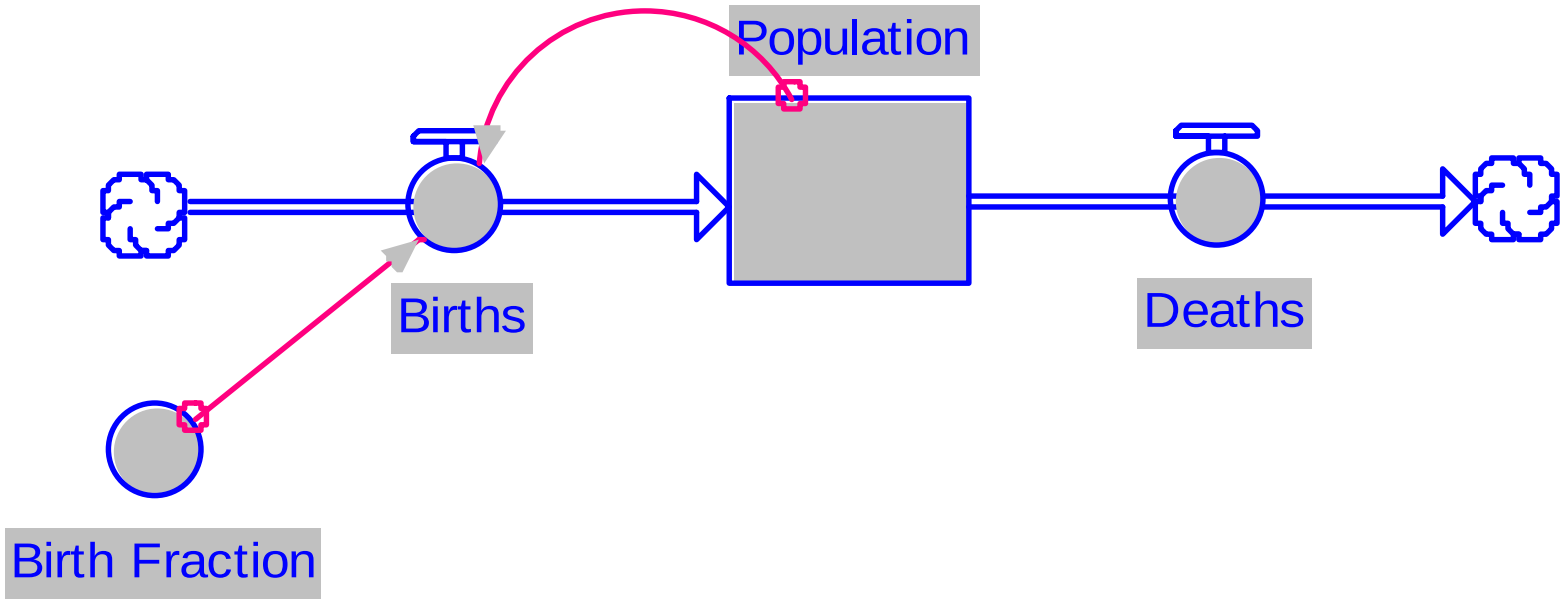
Differential Equation 2

- Stock_X(t) = Stock_X(t - dt) + (Flow_1) * dt
- (Stock_X(t) - Stock_X(t - dt))/dt = Flow_1
    - Flow_1 = Constant_a*Stock_X
- (X(t) - X(t - dt))/dt = aX(t-dt)
- dX/dt = aX (differential equation)
- Solution to DE:

$$X = X(0)\exp(at)$$

Calculus Example 3

STELLA Diagram



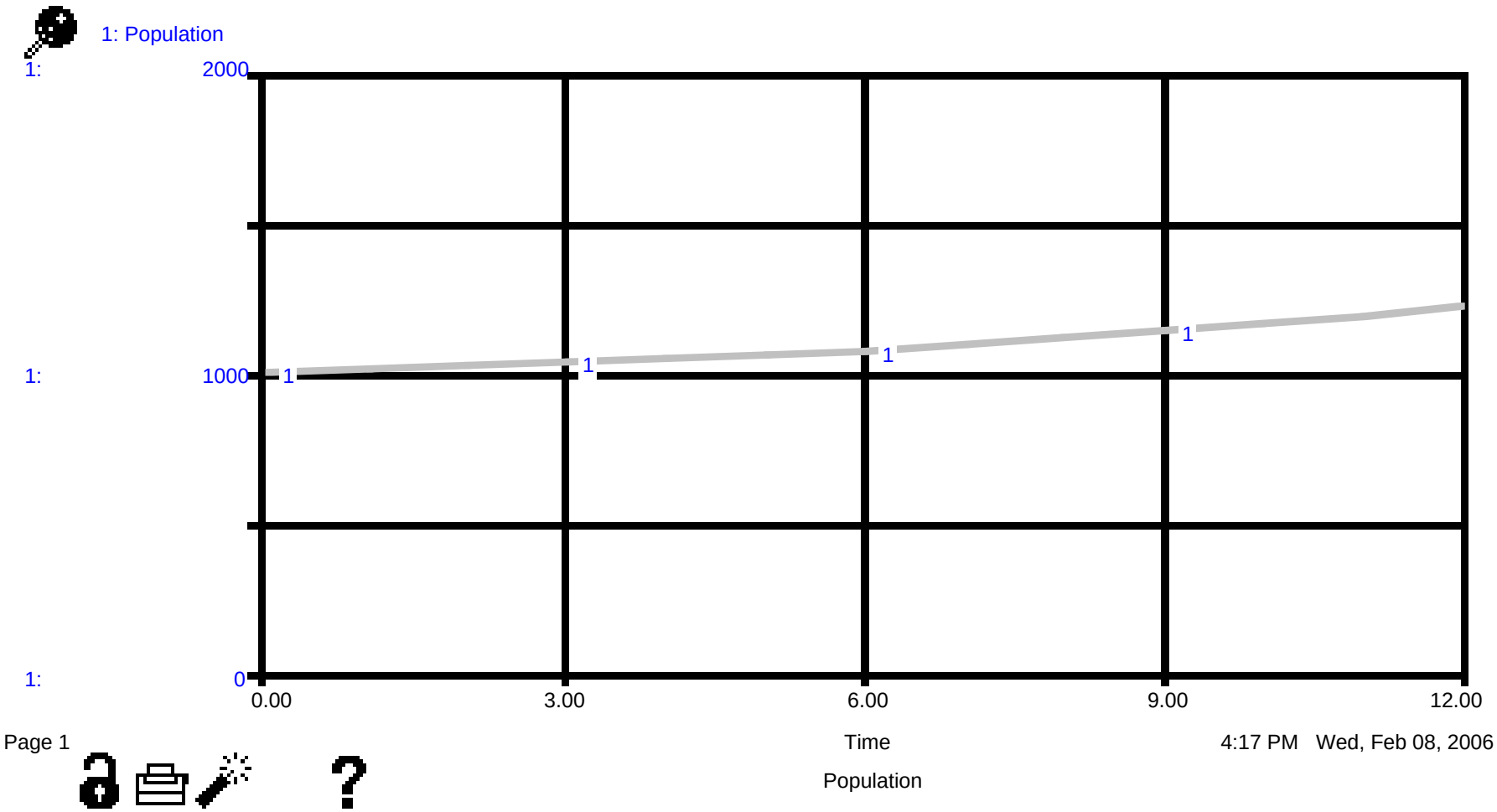
Two flows, an inflow and an outflow.

STELLA Equations 3

- Stock_X(t) = Stock_X(t - dt) + (Flow_1 - Flow_2) * dt
  - INIT Stock_X = 1000
- Flow_1 = Constant_a*Stock_X(t-dt)
- Flow_2 = Constant_b

  - Constant_a = .11
  - Constant_b = 100

Example 3 Graph



Differential Equation 3

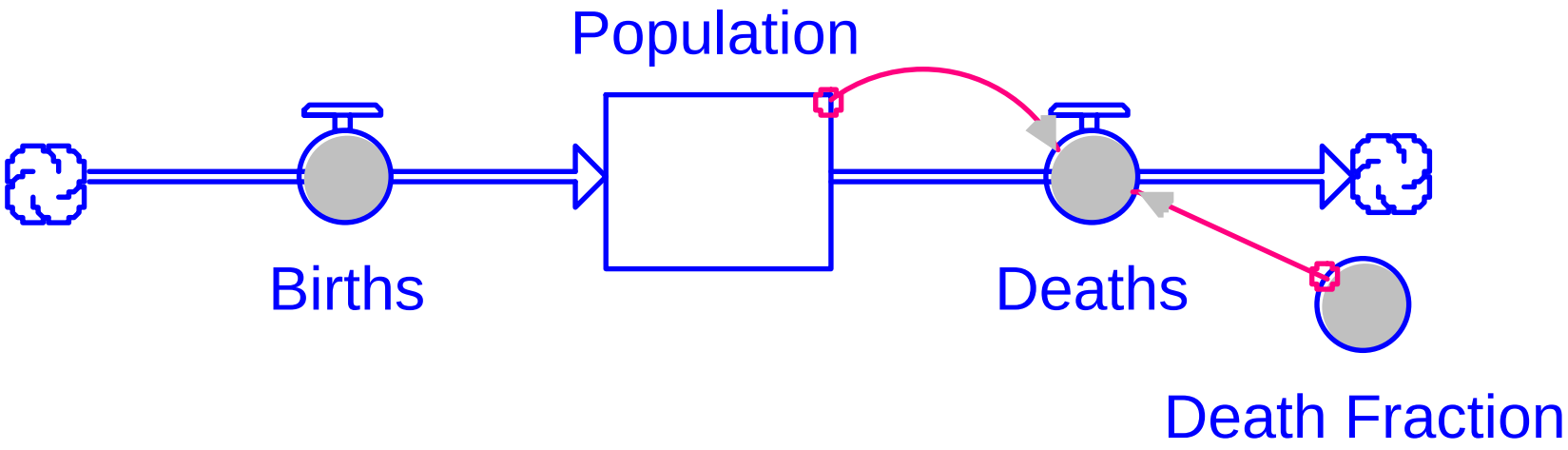
- Stock_X(t) = Stock_X(t - dt) + (Flow_1 - Flow_2) * dt
- (Stock_X(t) - Stock_X(t - dt))/dt = Flow_1 - Flow_2
  - Flow_1 = Constant_a*Stock_X(t-dt)
  - Flow_2 = Constant_b
- dX/dt = aX - b

Solution to DE:

?

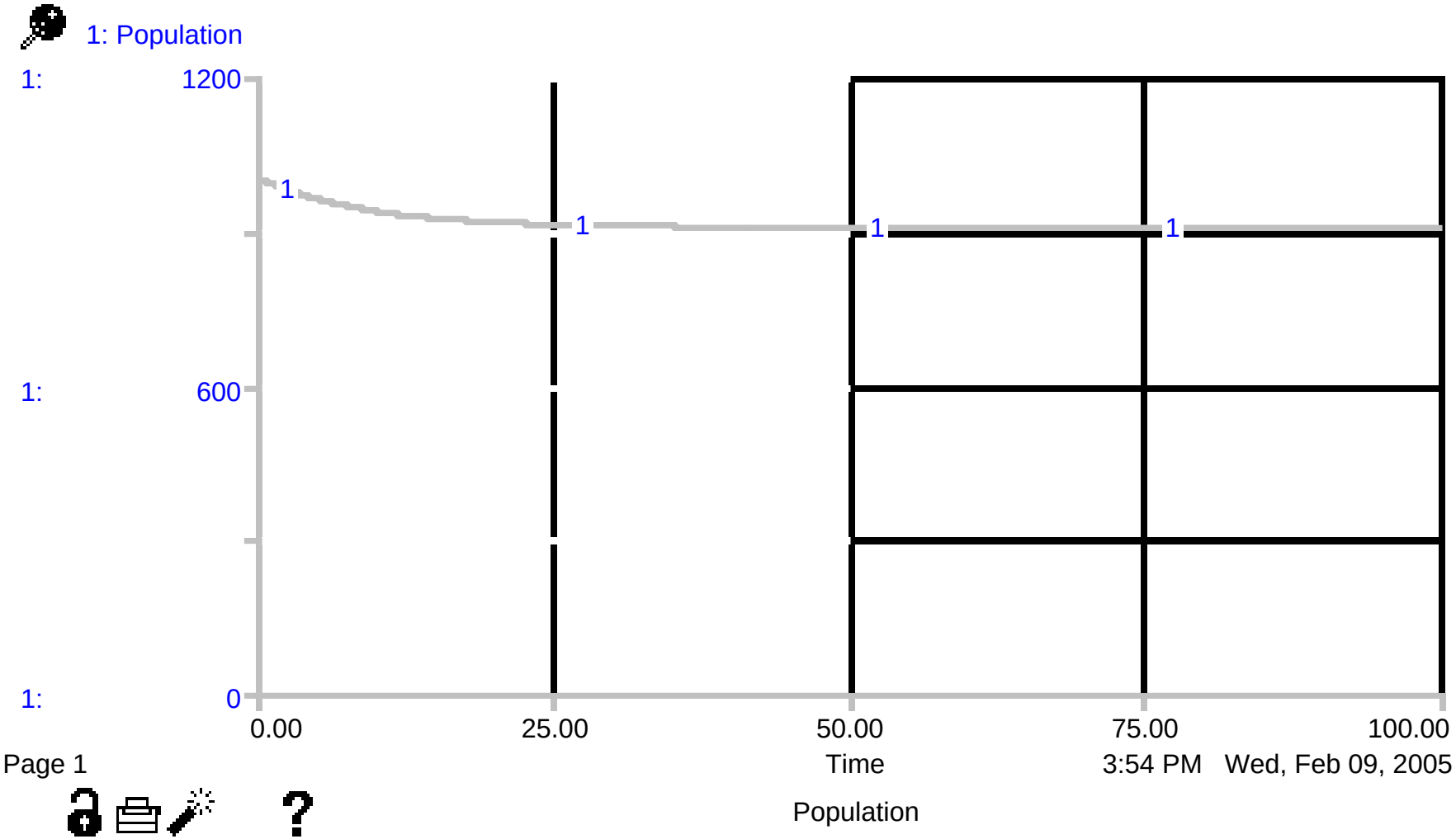
Calculus Example 4

STELLA Diagram



Two flows, an inflow and an outflow.

Example 4 Graph



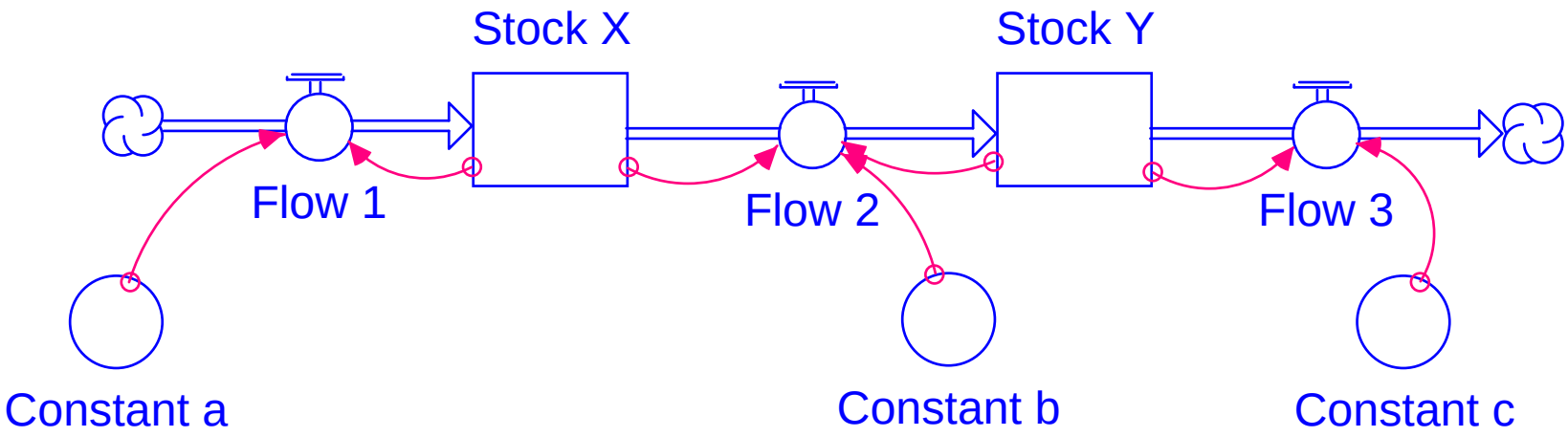
Differential Equation 4

- Stock_X(t) = Stock_X(t - dt) + (Flow_1 - Flow_2) * dt
- (Stock_X(t) - Stock_X(t - dt))/dt = Flow_1 - Flow_2
    - Flow_1 = Constant_a
    - Flow_2 = Constant_b*
- dX/dt = a - bX

Solution to DE:

?

Calculus Example 5
STELLA Diagram



Two stocks, X and Y, and three flows.

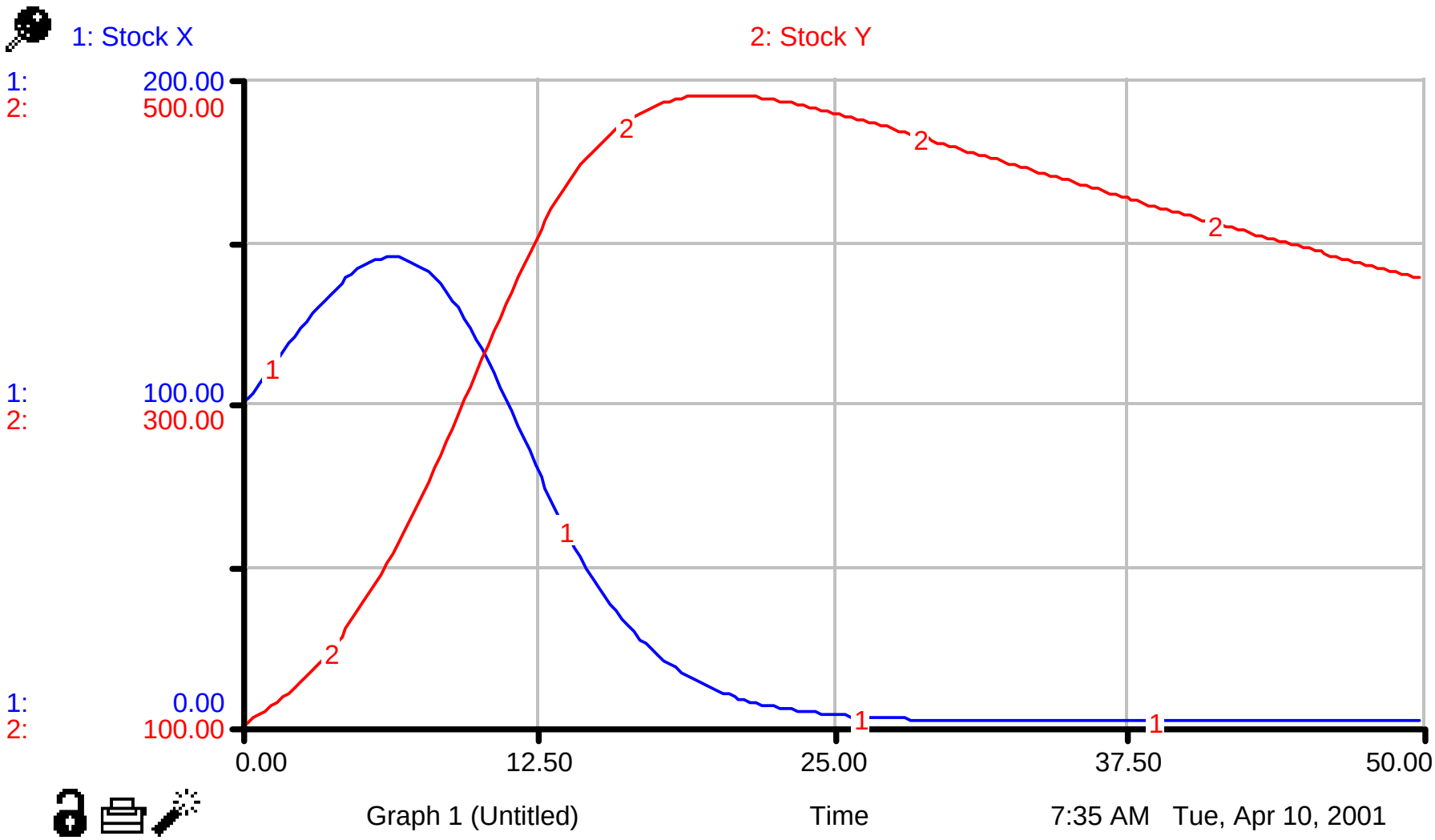
The outflow from Stock X is the inflow to Stock Y.

STELLA Equations 5

- Stock_X(t) = Stock_X(t - dt) + (Flow_1 - Flow_2) * dt
  - INIT Stock_X = 100
  - Flow_1 = Constant_a*Stock_X
  - Flow_2 = Constant_b*Stock_X*Stock_Y
- Stock_Y(t) = Stock_Y(t - dt) + (Flow_2 - Flow_3) * dt
  - INIT Stock_Y = 100
  - Flow_2 = Constant_b*Stock_X*Stock_Y
  - Flow_3 = Constant_c*Stock_Y

  - Constant_a = .2
  - Constant_b = .001
  - Constant_c = .01

Example 5 Graph



Differential Equations 5

How many differential equations do we need?

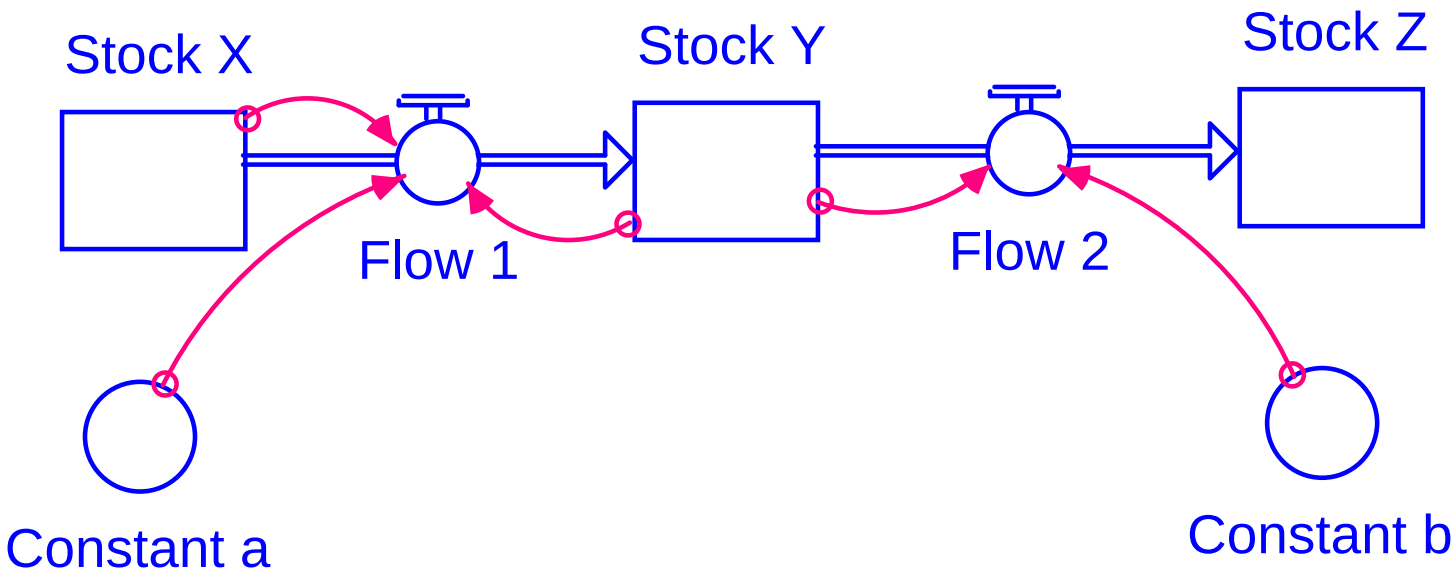
- (Stock_X(t) - Stock_X(t - dt))/dt = Flow_1 - Flow_2
- (Stock_Y(t) - Stock_Y(t - dt))/dt = Flow_2 - Flow_3
- dX/dt = aX - bXY
- dY/dt = bXY - cY

(A pair of differential equations)

Differential Equations 6

- $dX/dt = -aXY$
- $dY/dt = aXY - bY$
- $dZ/dt = bY$

STELLA Diagram 6



STELLA Equations 6

- Stock_X(t) = Stock_X(t - dt) + (- Flow_1) * dt
  - Flow_1 = Constant_a*Stock_X*Stock_Y /dt
- Stock_Y(t) = Stock_Y(t - dt) + (Flow_1 - Flow_2) * dt
  - Flow_1 = Constant_a*Stock_X*Stock_Y
  - Flow_2 = Constant_b*Stock_Y
- Stock_Z(t) = Stock_Z(t - dt) + (Flow_2) * dt
  - Flow_2 = Constant_b*Stock_Y