ANSWERS

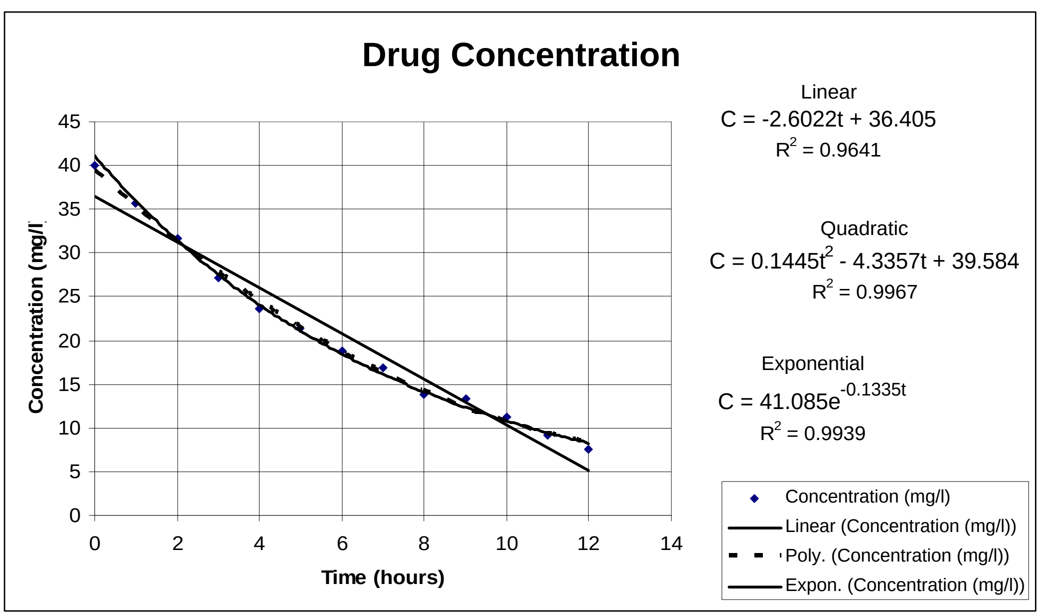
MA 261/419/519 Test #1 Given February 22, 2006

Copy this test, **Test1Spr06.doc**, and the Excel file **drug.xls** from the course website (http://www.math.uab.edu/mayer/Test1Spr06.doc) to your computer and diskette. In responding to the questions below, be sure you follow the previously stated standards for submitting reports and graphs. Problem (1) requires the use of EXCEL; problem (2) requires the use of STELLA. Do not open any files other than those you create as part of this test, or as directed below. The remaining problems refer to STELLA, but do not require you to use it. Insert (copy or type) your responses to the questions in the places provided below in this document. There are five questions on this test. Questions count points indicated [nn]. If you have trouble copying, formatting, or typing anything, ask us for help. When you have completed the test, print it out with your name on it and turn in the paper copy. (Alternately, you may email me your completed test, labeled LastName_Initials_Test1, or copy it to MY diskette.) One hour, fifteen minutes is allotted for the test.

Save your work and test to your computer and diskette often as you work.

(1) [25] Open the Excel speadsheet **drug.xls**. It contains a table of data collected in an experiment with a drug in a person's bloodstream. A quantity of the drug is injected into the bloodstream and the drug concentration in milligrams/liter is measured at the times (in hours) indicated. Use Excel to generate a **disconnected scatter graph** of the data with time on the horizontal axis. Place **three** trendlines, with equations and R^2 values, on the graph: one linear, one quadratic (polynomial of order 2), and one exponential. (a) [10] Copy the graph with trendlines, equations, and R^2 values. (b) [10] Based upon the foregoing, type a paragraph explaining which model (trendline) is the better fit to the data and why. (c) [5] Now suppose you receive additional information: the concentration of the drug is measured at 20 hours and is found to be 2.5 mg/l. Would you change your mind about which model (trendline) is best? Explain your answer.

(a) [10]



(b) [10] The quadratic and the exponential both fit the data very well because the R^2 value is over 0.99. The closer R^2 is to the maximum of 1, the better the fit. Either the quadratic or exponential model accounts for 99% of the variability of concentration over time. There are two reasons to reject the linear: a lower R^2 value of about 0.96 and the fact that the data trends above the straight line at both ends and below in the middle, suggesting a concave up curve would fit better. Because the R^2 value is slightly higher, I'll pick quadratic.

(c) [5] The predictions of concentration at 20 hours for each model are as follows:
Linear: $C = -2.6022(20) + 36.405 = -15.639$
Quadratic: $0.1445(20)^2 - 4.3357(20) + 39.584 = 10.67$
Exponential: $C = 41.085e^{-0.1335(20)} = 2.761$
The exponential model extrapolates better. The linear is impossible, since the drug concentration can't be negative. The quadratic has the concentration rising with time, which does not seem right if no more drug is administered. So I pick the exponential model based on the new information.

(2) [25] Using STELLA, create a model of your choice exhibiting linear growth (or decay). (a) [15] Copy the model diagram, equations with units, and Behavior Over Time graph of the stock into the space below. (Copy from a selection in any STELLA window to the clipboard, and paste the selection into this Word document.) (b) [10] Write a paragraph explaining your model, how it works, and what the numbers mean (paying attention to starting values, calculation of one or two time steps, ending values, and assumptions). Say where the feedback cycle is, if any, what kind of feedback it is (positive or negative), and why the system grows (decays) linearly.

(a) [5] Paste diagram here. No crossing elements.
[5] Paste equations here. Units included.
[5] Paste graph here. Score by Graph Scoring Guide.

(b**)** [10] The explanation should address all of the following:

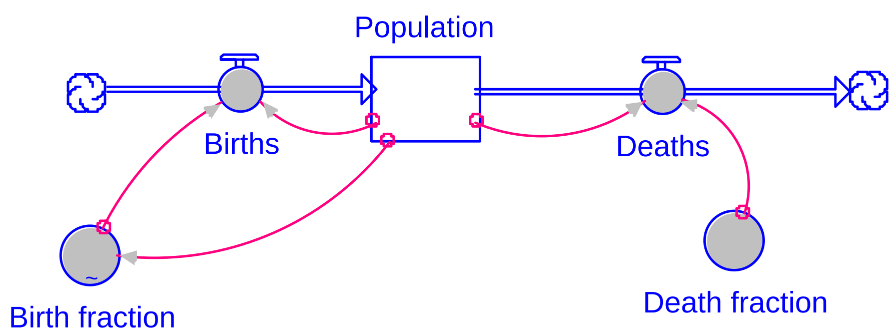
1. [1] Tell what the starting values are for each component.
2. [1] Tell how the model calculates its values for one or two time cycles, showing that you get the same numbers shown in the table. For example, starting with the stock value at time 3 explain how the computer calculated the value for time 4. Then indicate that you checked your calculation against the value in the table and it is as expected.
3. [2] Explain that, by the end of the simulation, the stock value will end at value xxxx, assuming yyyy. Always consider the simplification of your model as compared to a real-world situation. Those are the assumptions you are making when you create and run your simulation.
4. [5] Explain what you did in your model that caused the stock value to grow (or decay) linearly. This is extremely important, and worth half the points for the summary.
5. [1] No feedback in the model.

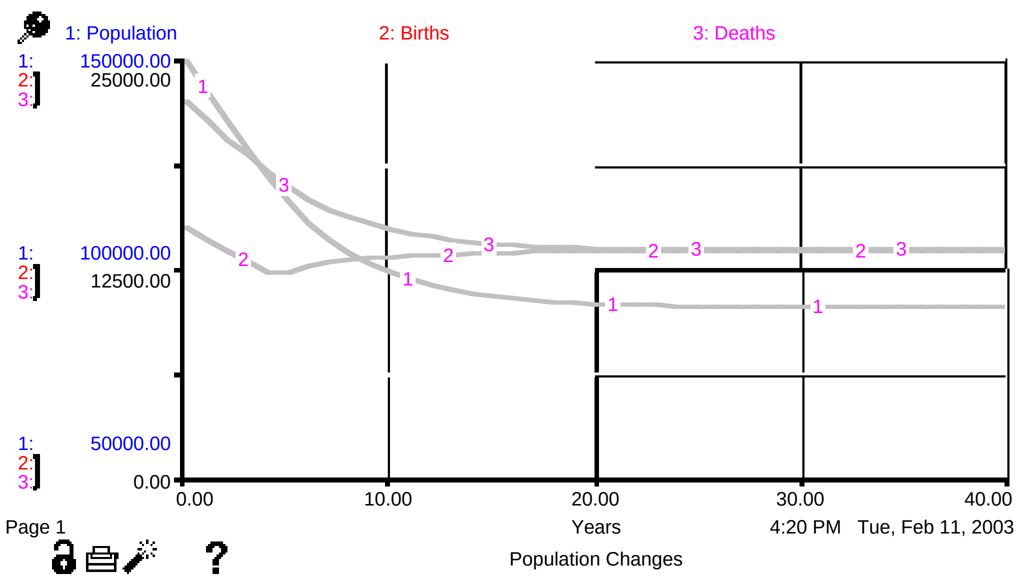
(3) [20] Below is the STELLA diagram of a population model. Accompanying it is the graph of the stock **Population** and the flows **Births** and **Deaths**. It may help you to know that the **Death fraction** is a constant (0.15), but the **Birth fraction** is a graphical converter depending upon Population (with a Birth fraction range of 0.10 to 0.20, decreasing continuously as population increases from 70,000 to 120,000). The flows are defined by the equations:

Births = Birth_fraction*Population

Deaths = Death_fraction*Population

Based on your understanding of the diagram, the equations, and the graph, write a paragraph addressing what happened to the population over time, why it leveled off, and what the leveling-off means. What does it mean that the Births and Deaths graphs coincide over the last 15 years or so? What does it mean that the Population graph crosses the Births and Deaths graphs?

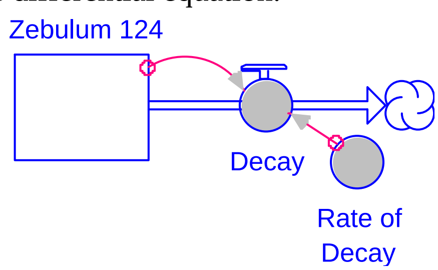




[15] Over time, the population decreased at a decreasing rate from 150,000 at time 0, and appeared to level off after time 25 or so to a value of about 90,000. The leveling off means the population is becoming constant. Since Births and Deaths are on the same scale, the fact that they coincide means that they become nearly equal around Year 20. That means the population is not changing because the inflow and outflow are the same. It means nothing that the population graph crossed births and deaths because it is on a different scale. [5] At a deeper level of understanding, we see that it must have been the birth fraction that changed to make the birth inflow equal the death outflow. That is because the death fraction is constant, so the deaths are a constant multiple of the population. However, the birth fraction reacts to the population by increasing as the population decreases. At about year 6, when the population had fallen below 120,000, the birth fraction began to increase. Births become a larger fraction of the population. So the number of births, which had been falling with the

population, began to increase. Since the birth fraction varies from 0.10 to 0.20 as population decreases, it eventually increases to match the death fraction of 0.15 and births balance deaths for the last 15 years or so. The exact value of the stable population depends upon what population corresponds to a birth fraction of 0.15. It is somewhere between 70,000 and 120,000 according to the description of the graphical converter, but by the graph is near 90,000.

(4) [20] Refer to the STELLA diagram and equations below. (a) [15] Given the equations and constants in the model, the Behavior Over Time graph of the stock **Zebulum 124** will exhibit which of the following shapes: increasing at a constant rate, increasing at an increasing rate, increasing at a decreasing rate, decreasing at a constant rate, decreasing at an increasing rate, or decreasing at a decreasing rate? Explain your answer in terms of the feedback loop(s), mathematical relationships, and specific numbers in the model. What kind of feedback (positive or negative) is exhibited by the outflow of this model? Explain your feedback answer. (b) [5] From the Stella difference equation in the model, derive the differential equation that Stella numerically simulates (more and more closely as **dt** goes to 0). State, but do not solve, the differential equation.



Zebulum_124(t) = Zebulum_124(t - dt) + (- Decay) * dt
INIT Zebulum_124 = 100 {grams}

OUTFLOWS:
Decay = Rate_of_Decay*Zebulum_124 {grams/year}

Rate_of_Decay = 35/10000 {grams/gram/year}

(a) [15] The BOT graph of Zebulum 124 will decrease at a decreasing rate. To see this, note that at time 0 there are 100 grams of Zeb124. So in the next time step, 35/10000*100 = 0.35 grams will flow out, leaving 99.65 grams at time 1. But in the next time step, 35/10000*99.65 = 0.348775 grams will flow out, less than the previous step. Though the stock will decrease further, to 99.301225 grams at time 2, it will have decreased less than in the first time step. As the stock decreases, the same sort of thing will happen in further time steps. So the stock will decrease, but at a decreasing rate. Hence, the feedback of information from stock to outflow counteracts the decrease (though it doesn't balance it). Thus, the single feedback loop from Zeb124 to Decay is negative feedback.

(b) [5]

Zebulum_124(t) = Zebulum_124(t - dt) + (- Decay) * dt
Z(t) – Z(t-dt) = -Decay*dt
(Z(t) – Z(t-dt))/dt = -Decay
(Z(t) – Z(t-dt))/dt = -0.0035*Z(t-dt)
Letting dt → 0

$$dZ(t)/dt = -0.0035\ Z(t)$$

This is the differential equation.

(5) [10] Below is the graph of the flow (Flow_A) into a stock (Stock_A). The equation that defines the flow is

Flow_A = 4 + STEP(8,3) - STEP(12,6) + STEP(8,9)

You are given that the initial value of Stock_A is 12. Carefully sketch the corresponding graph of Stock_A on the axes below. You will need to sketch the stock graph on a paper copy of the flow graph. Be sure to put an appropriate scale for the stock on the vertical axis.