

Voting and Fairness 1

The very essence of democracy?

Majority Rule. In an election with two candidates, the candidate with more than half the votes wins.

We call more than half the votes a **majority**.

But what if there are more than two choices?

What is a **fair** way to decide the outcome of an election?

- 4 Voting Methods
- 4 Fairness Criteria

MAC Election

The 37 members of the Math Anxiety Club (MAC) cast ballots for President. There are four candidates:

Alisha (A) Boris (B)
Carmen (C) Dave (D)

Election Results

Number of voters	14	11	8	4
Favorite Candidate	A	C	D	B

Who has won?

Suppose we have more information on the voters' preferences to start with?

Election Results

Number of Voters	14	10	8	4	1
Favorite Candidate	A	C	D	B	C
2 nd Choice Candidate	B	B	C	D	D

Who has won?

Capture **all** the voters' preferences with a
preference ballot

BALLOT	
1 st :	
2 nd :	
3 rd :	
4 th :	

Textbook Figure 1.1, page 4

Textbook Figure 1.2, page 5

Assumptions

1. *Transitivity*. If a voter prefers X to Y and Y to Z, then the voter prefers X to Z.
2. *Stability*. Relative preferences are not altered if some choices are eliminated.
3. *Sincerity*. Each voter ranks the alternatives in the actual order in which the voter prefers them.

Our basic assumptions are that voters act *rationally* – (1) and (2) hold, and *sincerely* – (3) holds.

In actual voting situations, one or more of these assumptions may be violated. A **good** voting method will minimize the opportunities for violations.

[Beware! Value judgment made.]

Plurality Method

In the *plurality method*, the choice with the most first-place votes wins.

Number of voters	14	10	8	4	1
1 st choice	A	C	D	B	C
2 nd choice	B	B	C	D	D
3 rd choice	C	D	B	C	B
4 th choice	D	A	A	A	A

In the MAC election, the result by the plurality method is:

Candidates	A	B	C	D
1st place votes				

Winner

Math Lovers Club Election

The 11 members of the Math Lovers Club choose a president from among four candidates by preference ballot.

MLC Election – Preference Schedule

Number of voters	6	2	3
1 st choice	A	B	C
2 nd choice	B	C	D
3 rd choice	C	D	B
4 th choice	D	A	A

Candidate A has 6 first place votes, a majority of the 11 voters.

A majority (more than half the votes) is automatically a plurality (the most votes).

Majority Criterion

If there is an alternative that is the first place choice of a majority of voters, then that alternative should be the winner of the election.

The plurality method satisfies the majority criterion.

However, there are some problems with the plurality method.

Problems with Plurality Method

The Bowl Games

The 100 members of the University Band must decide in which of five bowl games to march in:

Rose Bowl (R)

Hula Bowl (H)

Cotton Bowl (C)

Orange Bowl (O)

Sugar Bowl (S)

The results of a preference ballot are tabulated below.

Number of voters	49	48	3
1 st choice	R	H	C
2 nd choice	H	S	H
3 rd choice	C	O	S
4 th choice	O	C	O
5 th choice	S	R	R

Most of the band members list the Rose Bowl last, yet it wins by the plurality method.

One-to-One Comparison

Compare the Hula Bowl on a one-to-one basis with every other alternative.

Number of voters	49	48	3
1 st choice	R	H	C
2 nd choice	H	S	H
3 rd choice	C	O	S
4 th choice	O	C	O
5 th choice	S	R	R

Comparison	Result
H versus R	
H versus C	
H versus O	
H versus S	

Note that the Hula Bowl wins **all** the one-to-one comparisons with other alternatives.

Insincere Voting (Strategic Voting)

Actual Preferences

Number of voters	49	48	3
1 st choice	R	H	C
2 nd choice	H	S	H
3 rd choice	C	O	S
4 th choice	O	C	O
5 th choice	S	R	R

Strategic Votes

Number of voters	49	48	3
1 st choice	R	H	H
2 nd choice	H	S	C
3 rd choice	C	O	S
4 th choice	O	C	O
5 th choice	S	R	R

Condorcet Candidate

An alternative that wins on a one-to-one comparison with every other alternative is called the *Condorcet candidate*.

In the Bowl Election, the Hula Bowl is the Condorcet candidate.

An election can have at most one Condorcet candidate.

An election may not have any Condorcet candidate.

Condorcet Criterion

If there is an alternative that wins in a one-to-one comparison between it and every other alternative, then that alternative should be the winner of the election.

The plurality method violates the Condorcet criterion.

Example with No Condorcet Candidate

The mathematics faculty at Southside University hold an election to select a chair of the department from among three candidates (A, B, and C). The 9 preference ballots are tabulated below.

SU Chair Election – Preference Schedule

Number of voters	4	3	2
1 st choice	A	B	C
2 nd choice	B	C	A
3 rd choice	C	A	B

Comparison	Result
A versus B	
A versus C	
B versus C	

Since no candidate won all of her one-to-one comparisons, there is no Condorcet candidate.

Paradox of Voting

Even though individual preferences are transitive, group preferences are not always transitive.

As we saw in the Southside University Election, the results of all the pairwise comparisons was

Comparison	Result
A versus B	6 to 3, A favored
A versus C	4 to 5, C favored
B versus C	7 to 2, B favored

Thus, as a group, the SU Math faculty prefer A to B and prefer B to C, but they do **not** prefer A to C! (Which they would if transitivity held.)

This violation of transitivity for group preferences is often called the *Paradox of Voting*.

Review 1

Majority Rule. In an election with only two alternatives, the alternative with the majority of votes (more than half) wins.

Preference Ballot. A ballot listing alternatives in the order (1st place, 2nd place, 3rd place, etc.) in which the voter prefers them.

Plurality Method. In an election (with any number of alternatives), the alternative with the most votes wins.

Majority Criterion. If there is an alternative that is the first choice of a majority of voters, then that alternative should be the winner of the election.

Condorcet Criterion. If there is an alternative that wins in a one-to-one comparison between it and every other alternative, then that alternative should be the winner of the election.

Paradox of Voting. Even though individual preferences are transitive, group preferences are not always transitive.