Voting and Fairness 2

In this lecture we will cover the following voting methods and fairness criterion.

- Borda Count Method
- Plurality-with-Elimination Method
- Monotonicity Criterion

Borda Count Method

In the *Borda Count Method* each place on the ballot is assigned points. The alternative receiving the most points wins.

Borda point assignment for *n* alternatives

Last place	1 point
Next-to-last place	2 points
Third place	n-2 points
Second place	n-1 points
First place	n points

MAC Election – Borda Count

Nbr of voters	14	10	8	4	1
1st choice:___	A:	C:	D:	B:	C:
2nd choice:___	B:	B:	C:	D:	D:
3rd choice:___	C:	D:	B:	C:	B:
4th choice:___	D:	A:	A:	A:	A:

Totals

A: _____

B: _____

C: _____

D: _____

Winner

Problems with Borda Count

Math Lovers Club Election

The 11 members of the Math Lovers Club choose a president from among four candidates by preference ballot.

MLC Election – Borda Count

Nbr of voters	6	2	3
1st choice: 4	A:	B:	C:
2nd choice: 3	B:	C:	D:
3rd choice: 2	C:	D:	B:
4th choice: 1	D:	A:	A:

What does A think of this?

Totals:

A: _____

B: _____

C: _____

D: _____

Winner:

Observations

- Disadvantages of the Borda Count Method
  - Borda count method violates the majority criterion.
  - Consequently, Borda count method violates the Condorcet criterion. (Why? Explain.)
- Advantages of the Borda Count Method
  - Borda Count Method uses all the available voter preferences, not just first choices.
  - Borda Count Method often produces the best compromise winner.

So far, we have two voting methods applied to the MAC election and two different winners!

Voting Method	**Winner**
Plurality	Alisha
Borda Count	Boris

Plurality-with-Elimination Method

Round 1

- Count 1st place votes for each candidate.
- If a candidate has a majority, then that candidate is the winner.
- Otherwise, eliminate candidate(s) with fewest 1st place votes and simplify preference schedule.

Round 2

- Count 1st place votes for each candidate.
- If a candidate has a majority, then that candidate is the winner
- Otherwise, eliminate candidate(s) with fewest 1st place votes and simplify preference schedule.

Rounds 3, 4, etc.: Repeat above steps; eventually, some candidate will have a majority of 1st place votes.

MAC Election – Plurality-with-Elimination

Number of voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

1. Candidates	A	B	C	D
1st place votes				

2. Candidates			
1st place votes			

3. Candidates		
1st place votes		

Winner:

Example: Young Liberals Election

The UAB Young Liberals elect a president of their club from among five candidates A, B, C, D, and E using the plurality-with-elimination method. There are 24 preference ballots.

YL Election – Preference Schedule

Number of voters	8	6	2	3	5
1st choice	A	B	C	D	E
2nd choice	B	D	A	E	A
3rd choice	C	E	E	A	D
4th choice	D	C	B	C	B
5th choice	E	A	D	B	C

What is the maximum number of rounds that might be needed to decide the winner of this election by the plurality-with-elimination method?

YL Election – Preference Schedule

Number of voters	8	6	2	3	5
1st choice	A	B	C	D	E
2nd choice	B	D	A	E	A
3rd choice	C	E	E	A	D
4th choice	D	C	B	C	B
5th choice	E	A	D	B	C

1. Candidates	A	B	C	D	E
1st place votes					

2. Candidates				
1st place votes				

3. Candidates			
1st place votes			

4. Candidates		
1st place votes		

Winner:

Problems with Plurality-with-Elimination

The UAB Young Conservatives elect a president of their club from among three candidates A, B, and C using the plurality-with-elimination method. There are 29 preference ballots.

YC Election – Preference Schedule

Number of voters	7	8	10	4
1st choice	A	B	C	A
2nd choice	B	C	A	C
3rd choice	C	A	B	B

1. Candidates	A	B	C
1st place votes			

2. Candidates		
1st place votes		

Winner:

Because of election irregularities, the original election is declared void. Meanwhile, candidate C convinces the 4 voters represented by the last column of the preference schedule that she is better than candidate A. They switch their preference order to C, A, B. The re-election results are as follows.

YC Re-Election – Preference Schedule

Number of voters	7	8	14
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

1. Candidates	A	B	C
1st place votes			

2. Candidates		
1st place votes		

Winner:

This is quite a shock to C!

- C had the original YC election won
- All changes in votes were only in C's favor
- C loses the re-election!
- Is this fair?

Monotonicity Criterion

If an alternative X is the winner of an election, and, in a re-election, all the voters who change their preferences do so in a way that is favorable only to X, then X should still be the winner of the election.

- The plurality-with-elimination method violates the monotonicity criterion.
- The plurality-with-elimination method also violates the Condorcet criterion. (Exercise)

Summary of Voting Methods and Fairness Criteria

Voting Method	Fairness Criterion	
	Satisfied	**Violated**
Plurality	Majority Monotonicity	Condorcet
Borda Count	Monotonicity	Majority Condorcet
Plurality-with-Elimination	Majority	Monotonicity Condorcet

In the MAC election, we have used three voting methods and have three different winners!

Voting Method	Winner
Plurality	Alisha
Borda Count	Boris
Plurality-with-Elimination	Dave