Voting and Fairness 3

In this lecture we will cover the following voting method and fairness criterion.

- Method of Pairwise Comparisons
- Independence of Irrelevant Alternatives Criterion

Method of Pairwise Comparisons

Pairwise Comparisons

- Match each candidate on a one-to-one basis with every other candidate.

Points – Suppose X is compared with Y**.**

- If X wins the comparison, then X gets 1 point and Y gets 0.
- If Y wins the comparison, then Y gets 1 point and X gets 0.
- In case of a tie, both X and Y get ½ point.

Winner. After all pairwise comparisons have been made, the candidate with the most points is the winner.

This is similar to a *round-robin tournament*, where each player plays every other player to decide the winner.

The method of pairwise comparisons is one of several so-called *Condorcet* methods. It is also called *Copeland's method*.

MAC Election – Pairwise Comparisons

Number of voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

There are four candidates in the MAC election (A, B, C, and D).

There are 6 pairwise comparisons that must be made, as one can see by listing them in a systematic fashion.

A vs B

A vs C

A vs D

B vs C

B vs D

C vs D

MAC Election – Comparisons and Results

Nbr of voters	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

Comparison	**Result**	**Points**
A vs B		
A vs C		
A vs D		
B vs C		
B vs D		
C vs D		

Candidate	A	B	C	D
Points				

Winner

Summary of MAC Election Results

In the MAC election, we have used four voting methods and have four different results!

Voting Method	Winner
Plurality	Alisha
Borda Count	Boris
Plurality-with-Elimination	Dave
Pairwise Comparisons	Carmen

As the playwright, Tom Stoppard said, “It’s not the voting that’s a democracy; it’s the counting.”

Have we arrived at the fairest method now?

Problems with Pairwise Comparisons

Consider an election with 5 candidates, 9 voters, and the following preference schedule.

Number of voters	1	4	1	3
1st choice	A	C	E	E
2nd choice	B	D	A	A
3rd choice	C	B	D	B
4th choice	D	E	B	D
5th choice	E	A	C	C

Comparison	Result	Points
A vs B		
A vs C		
A vs D		
A vs E		
B vs C		

Comparison	Result	Points
B vs D		
B vs E		
C vs D		
C vs E		
D vs E		

Winner:

Because of an election irregularity, the votes have to be recounted. Meanwhile, B, C, and D drop out of the election. The new preference schedule is much simpler.

Nbr of voters	1	4	1	3
1st	A	C	E	E
2nd	B	D	A	A
3rd	C	B	D	B
4th	D	E	B	D
5th	E	A	C	C

Nbr of voters	1	8
1st	A	E
2nd	E	A

Winner

How does this compare to the previous outcome?

We eliminated some of the candidates, recounted the ballots, and the winner under the method of pairwise comparisons changed, from A to E.

The three "irrelevant" candidates who dropped out, and who were not winners, should not affect the outcome of the election.

Independence of Irrelevant Alternatives Criterion

If an alternative X is the winner of an election, and one or more of the other alternatives are removed and the ballots recounted, then X should still be the winner of the election.

- The method of pairwise comparisons violates the independence of irrelevant alternatives criterion.
- The method of pairwise comparisons satisfies the majority, Condorcet, and monotonicity criteria. (See exercises.)

Conflict with Borda Count

An even more serious problem with pairwise comparisons is revealed by the preceding example. Let us apply the Borda count method to it.

Number of voters	1	4	1	3
1st choice: 5	A	C	E	E
2nd choice: 4	B	D	A	A
3rd choice: 3	C	B	D	B
4th choice: 2	D	E	B	D
5th choice: 1	E	A	C	C

Borda Totals

A: ______________________________

B: ______________________________

C: ______________________________

D: ______________________________

E: ______________________________

Winner

Observations

- A comes in last by Borda count, exactly the opposite order as with pairwise comparisons.
  - Pairwise comparisons: A, B-C-D tied, E.
  - Borda count: E, B-C-D tied, A.
- The Borda count winner E is preferred to the pairwise comparisons winner A by 8 of the 9 voters.
  - This overwhelming margin of 8 to 1 is picked up by Borda count, but is missed by the method of pairwise comparisons.
  - We call A a *disliked winner* since 8 of the 9 voters prefer E.

More Problems with Pairwise Comparisons

Example: Deadlock

Consider the following preference schedule with 9 voters and three candidates.

Number of voters	3	2	4
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

Comparison	Result	Points
A vs B		
A vs C		
B vs C		

Winner

Note that in this election group preferences are not transitive.

Observations

Advantages of the method of pairwise comparisons

- Pairwise comparisons satisfies
  - The Condorcet criterion
  - The majority criterion
  - The monotonicity criterion
- Pairwise comparisons uses all the information provided by the voters.

Disadvantages of the method of pairwise comparisons

- Violates the independence of irrelevant alternatives criterion.
- Often is indecisive, producing a deadlock.
- Can conflict spectacularly with Borda count, producing a disliked winner.
- The number of comparisons mounts fast as the number of candidates increases.

Summary of Voting Methods and Fairness Criteria

Voting Method	Fairness Criterion	
	Satisfied	**Violated**
Plurality	Majority Monotonicity	Condorcet IIA
Borda Count	Monotonicity	Majority Condorcet IIA
Plurality-with-Elimination	Majority	Monotonicity Condorcet IIA
Pairwise Comparisons	Majority Condorcet Monotonicity	IIA

Note: IIA = Independence of Irrelevant Alternatives

Is there a voting method that satisfies all four fairness criteria?

Arrow's Impossibility Theorem

Kenneth J. Arrow (1921 –) is a mathematician, economist, and winner of the Nobel Prize (1972) in economics. The prize was given partly for the following theorem that he proved in 1949.

Theorem. It is mathematically impossible for a rational voting method to satisfy all four of the stated fairness criteria.

More precisely, the two individual rationality conditions (transitivity, stability) and the four fairness criteria (majority, Condorcet, monotonicity, and independence of irrelevant alternatives) constitute a set of **inconsistent** statements – they contain an internal contradiction.

There cannot be a perfectly fair voting method, no matter how hard we try to find one.