Voting and Fairness 4

In this lecture we will cover the following topics.

- Breaking Ties
- Ranking

Breaking Ties

There are two types of ties that can occur in an election.

- *Essential ties.* No rational method can break an essential tie, only outside intervention.
- *Nonessential ties.* Tie-breaking rules and other rational voting methods can be used to break nonessential ties.

Example – an Essential Tie

Consider an election with two candidates, A and B, and the following preference schedule.

Number of voters	10	10
1st choice	A	B
2nd choice	B	A

If A and B were **interchanged**, the preference schedule would be exactly the same. That makes it an essential tie.

The tie can only be broken by something outside the voting method: chance (flip a coin), authority (ask the leader), etc.

Example – an Essential 3-Way Tie

An election with more than two candidates can also result in an essential tie.

Number of voters	10	10	10
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

There is no rational way to distinguish among the three candidates.

- They all have the same number of 1st, 2nd, and 3rd place votes.
- This will not change if any two of them are interchanged.

Again, the tie can only be broken by outside intervention.

Essential and Nonessential 3-Way Ties

Which, if any, of the following examples are essential 3-ways ties, and which are nonessential?

Example 1

Number of voters	10	10	10
1st choice	A	B	C
2nd choice	B	C	B
3rd choice	C	A	A

Example 2

Number of voters	10	10	10
1st choice	A	B	C
2nd choice	B	A	A
3rd choice	C	C	B

Example 3

Number of voters	10	10	10
1st choice	A	B	C
2nd choice	C	A	B
3rd choice	B	C	A

Example 4 E Example 2

For those that are not essential ties, what would be possible ways of breaking the tie, based upon the voters' preferences?

Example – a Nonessential Tie

The 20 students in a MA 110 study group conduct a preference ballot to elect a representative to the course management committee from among four of their members. They decide ahead of time to use Borda count.

Number of voters	7	6	3	4
1st choice	A	C	A	B
2nd choice	B	B	B	A
3rd choice	C	A	D	C
4th choice	D	D	C	D

A: _____

B: _____

C: _____

D: _____

Winner

We illustrate three ways suitable for breaking Borda count ties. (See textbook for other examples.) One should decide before the election what tie-breaking method(s) will be used, and in what order.

Number of voters	7	6	3	4
1st choice	A	C	A	B
2nd choice	B	B	B	A
3rd choice	C	A	D	C
4th choice	D	D	C	D

- *One-to-one comparison.* Do a one-to-one comparison of A and B.

- *Top-down comparison.* Compare first place votes of A and B. (If still tied, move *down* in places compared.)

- *Bottom-up comparison.* Compare last place votes of A and B. (If still tied, move *up* in places compared.)

Ranking

All the voting methods that we have discussed lend themselves very easily to *ranking* candidates.

This is useful, for instance, if we have several offices to fill.

In the MAC election, suppose that we want to fill the offices of President, Vice-President, and Treasurer with the top three finishers, in that order.

Rank	**Office**
Winner	President
2nd place	Vice-president
3rd place	Treasurer

We will cover only what are called *extended* ranking methods. The textbook discusses *recursive* ranking methods, but we will omit them.

Extended Plurality Method

Rank the candidates by the number of first place votes each received.

MAC Election – Preference Schedule

Number of voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Candidate	1st Place Votes	Rank	Office
A	14		
B	4		
C	11		
D	8		

Extended Borda Count Method

Rank the candidates by the number of total Borda count points each received.

MAC Election – Preference Schedule

Nbr of voters	14	10	8	4	1
1st choice 4	A: 56	C: 40	D: 32	B: 16	C: 4
2nd choice 3	B: 42	B: 30	C: 24	D: 12	D: 3
3rd choice 2	C: 28	D: 20	B: 16	C: 8	B: 2
4th choice 1	D: 14	A: 10	A: 8	A: 4	A: 1

Candidate	Borda Points	Rank	Office
A	79		
B	106		
C	104		
D	81		

Extended Plurality-with-Elimination Method

Rank the candidates in the reverse of the order in which they were eliminated.

MAC Election – Preference Schedule

Number of voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Round	Candidates			
	A	B	C	D
1	14	4	11	8
2	14		11	12
3	14			23

Candidate	Eliminated	Rank	Office
A	3rd		
B	1st		
C	2nd		
D			

Extended Method of Pairwise Comparisons

Rank the candidates by the number of pairwise comparison points each received.

MAC Election – Preference Schedule

Number of voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

Comp	Result	Points
AvsB	14-23	B: 1
AvsC	14-23	C: 1
AvsD	14-23	D: 1
BvsC	18-19	C: 1
BvsD	28-9	B: 1
CvsD	25-12	C: 1

Candidate	PWC Points	Rank	Office
A	0		
B	2		
C	3		
D	1		

Complications

Ties can also occur at other points in a voting method than first place. Beforehand, one should adopt a method for breaking ties within the method, if appropriate, and in ranking. In the following example, rank the candidates by the extended plurality-with-elimination method and break ties within the method by a pairwise comparison.

Number of voters	8	5	3	2
1st choice	A	B	C	D
2nd choice	B	D	B	C
3rd choice	C	A	D	B
4th choice	D	C	A	A

Round	**Candidates**			
	A	B	C	D
1				
2				
3				

Candidate	**Eliminated**	**Rank**
A		
B		
C		
D		