MA 125 8C, CALCULUS I
September 25, 2014

Name (Print last name first): ....

TEST I

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 6 points each.

1. Show, **using the definition**, that the derivative of $y = f(x) = x^2$ is $2x$.

2. Evaluate $\displaystyle\lim_{x \to 3} \frac{x-3}{x^2 - 8x + 15}$

3. Evaluate $\lim_{x\to\pi} \sqrt{\sin(x)+4}$

4. Find the derivative of $y = f(x) = \cos(x)\sin(x)$

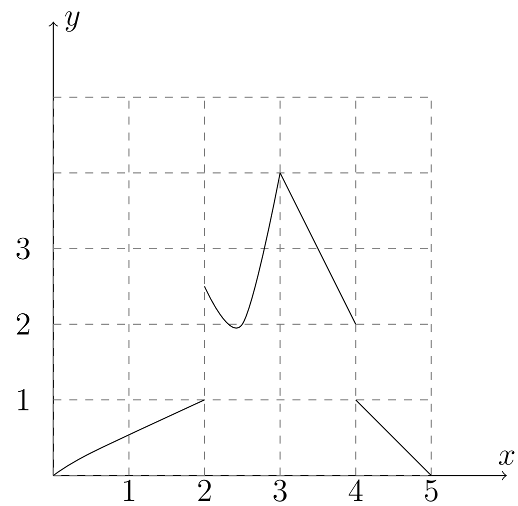
5. Find the derivative of $y = f(x) = \dfrac{x^3 + x^2 + x}{\sqrt{x}}$

6. Find the derivative of $y = f(x) = \dfrac{x^3 + 1}{x^2 + 5}$

7. Find the derivative of $y = f(x) = \sqrt[3]{x^2 + 3x + 1}$

8. Find the equation of the tangent line to the graph of $y = f(x) = \sin(x) + 2x$ at the point $x = \pi/2$.

9. Given the graph of the function below, state (a) where it is continuous and (b) where the derivative exist.



10. Evaluate $\lim_{x\to\infty} \dfrac{3x^4+\sqrt{x}}{x^4+5x-100}$

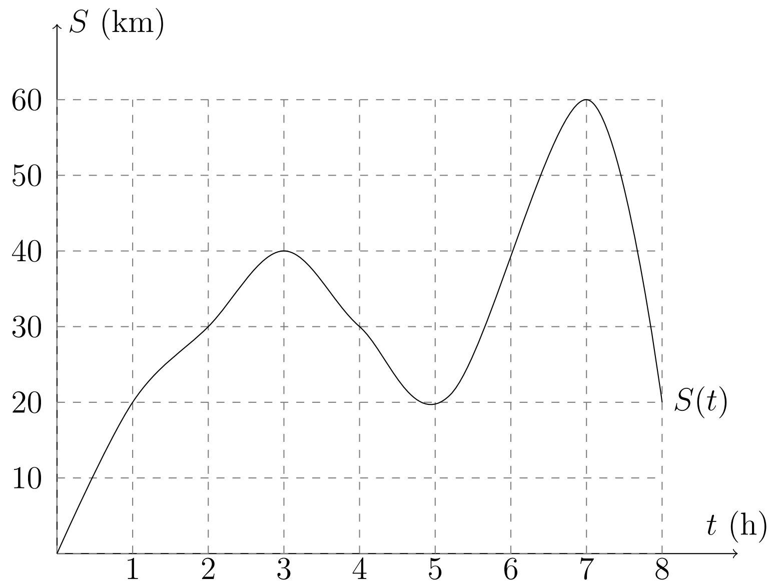
PART II

Points for each problem are as indicated.

1. [**10 points**] Find the x-coordinates of all points where the graph of $y = f(x) = (5x-1)^2(3x+1)^3$ has a horizontal tangent line

2. **[10 points]** If the position of a particle is given by $S(t) = 5\sec(t)$, find the velocity and acceleration at time $t = \pi/4$.

3. [**10 points**] A rail road car travels along a straight track. The graph below gives its position (in km) as a function of time (in h).



(a) Estimate the velocity at time $t = 3$. [Show your work!!]

(b) At the above time was the velocity increasing or decreasing? [Explain!!]

(c) At the above time was the car accelerating or decelerating? [Explain!]

4. You can (and should) use your calculator in the following problem (but do not use the derivative of this function, even if you know it; in the latter case you can of course check to see if your answer is reasonable). Suppose that $C(x) = 400+3x+2x^2$ is the cost of producing x-items. We are interested in the derivative of the this function.

   (a) [**4 points**] Using the definition of the derivative, write $C'(2)$ as a limit $(\lim_{h\to 0} ......)$. (Use the definition of $C(x)$ given above.)

   (b) [**3 points**] Find the derivative $C'(500)$ with properties of derivatives.

   (c) [**3 points**] Estimate $C'(2)$ using your calculator (you only need to use $h = \pm\frac{1}{10}$ to get your approximate answer) .