ALL QUESTIONS ARE WORTH 30 POINTS

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

I. Electric Force and Electric Charge

1. Coulomb's Law: \[ F = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot q'}{r^2} \]; like charges repel, unlike charges attract.

2. Permittivity constant: \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2) = 8.85 \times 10^{-12} \text{ F/m} \)


4. Charge of proton: \( e = 1.6 \times 10^{-19} \text{ C} \); charge of electron: \( -e = -1.60 \times 10^{-19} \text{ C} \)

5. Charge conservation: In any reaction or process, the net electric charge remains constant.

6. Conductor: Permits the motion of charge. Insulator: Does not permit the motion of charge.

II. The Electric Field

1. Definition of electric field: \( E = \frac{F}{q} \)

2. Electric field of point charge: \( E = \frac{1}{4\pi\varepsilon_0} \frac{q'}{r^2} \)

3. Electric field of large, uniformly charged flat sheet: \( E = \frac{\sigma}{2\varepsilon_0} \)

4. Electric field in the space between two flat sheets: \( E = \frac{\sigma}{\varepsilon_0} \)

5. Electric field of charged line: \( E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r} \)

6. Electric flux through a surface: \( \Phi = E \cdot A \); \( E_n \)-normal component of the electric field; \( E_n = E \cos \theta \); \( E_n \) is positive if the direction of the electric field is outward from the surface; and negative if it is inward, into the surface.

7. Gauss' Law (for closed surface): \( \Phi = \frac{Q}{\varepsilon_0} \)

8. Conductor in electrostatic equilibrium: the electric field within the conductor is zero; the charge resides on the surface; the electric field at the surface is perpendicular and of magnitude \( E = \frac{\sigma}{\varepsilon_0} \)

9. Torque on dipole: \( \tau = p \sin \theta \); Electric dipole moment: \( p = l\vec{Q} \)

III. Electrostatic Potential and Energy

1. Definition of electrostatic potential: \( V = \frac{U}{q} \)

2. Potential energy of two point charges: \( U = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot q'}{r} \)

3. Electrostatic potential of point charge: \( V = \frac{1}{4\pi\varepsilon_0} \frac{q'}{r} \)

4. Potential energy of a system of point charges: \( U = 1/2q_1V_{other\ at\ 1} + 1/2q_2V_{other\ at\ 2} + 1/2q_3V_{other\ at\ 3} + \cdots \)

5. Potential energy of a system of conductors: \( U = 1/2Q_1V_1 + 1/2Q_2V_2 + 1/2Q_3 + \cdots \)

6. Energy density in electric field: \( u = 1/2\varepsilon_0E^2 \)

7. Capacitance of a pair of conductors: \( C = Q/V \); Capacitance of parallel plates: \( C = \varepsilon_0A/d \)

8. Potential energy in capacitor: \( U = 1/2Q\Delta V = 1/2C(\Delta V)^2 = 1/2Q^2/C \)

9. Parallel combination of capacitors: \( C = C_1 + C_2 + C_3 + \cdots \)

10. Series combination of capacitors: \( 1/C = 1/C_1 + 1/C_2 + 1/C_3 + \cdots \)

11. Electric field in dielectric between parallel plates: \( E = 1/kE_{free} \); Capacitance \( C = kC_0 \)
IV. Currents and Ohm’s Law
1. Electric field in uniform wire: \( E = \Delta V / l \)
2. Electric current: \( I = \Delta q / \Delta t \)
3. Resistance in terms of resistivity: \( R = \rho l / A \)
4. Increase of resistance with temperature: \( \Delta R = \alpha R_0 \Delta T \)
5. Ohm’s Law: \( I = \Delta V / R \)
6. Series combination of resistors: \( R = R_1 + R_2 + R_3 + \ldots \)
7. Parallel combination of resistors: \( 1/R = 1/R_1 + 1/R_2 + 1/R_3 + \ldots \)

V. DC Circuits
1. Kirchhoff’s first rule: The sum of the currents leaving the junction must equal the sum of the currents entering.
2. Kirchhoff’s second rule: The sum of emfs and potential changes across resistors around any closed loop in a circuit must equal zero.
3. Power delivered by a source of emf: \( P = I \varepsilon \)
4. Power dissipated by a resistor: \( P = I \Delta V \)
5. Characteristic time for RC circuit: \( t = RC \)

VI. Magnetic Force and field
1. Magnetic force exerted on a point charge by a current: \( F = \pm \frac{\mu_0 q v I}{2 \pi r} \)
2. Magnetic field of a current: \( B = \frac{\mu_0 I}{2 \pi r} \) \{Unit 1 tesla = 1T = 1 N/(C-m/s)\}
3. Permeability constant: \( \mu_0 = 1.26 \times 10^6 \text{ N} \cdot \text{s}^2/\text{C}^2 = 1.26 \times 10^6 \text{ H/m} \)
4. Permittivity constant: \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) = 8.85 \times 10^{-12} \text{ F/m} \)
5. Force exerted by magnetic field: \( F = q v \times B \sin \alpha \)
   Direction: If you place the fingers of your right hand along the direction of the velocity \( v \) and curl the fingers toward the direction of the magnetic field \( B \) through the smallest angle between \( v \) and \( B \), the thumb will lie along the direction of the force \( F \)
6. Gauss’ Law for magnetism: \( \Phi_B = 0 \)
7. Ampere’s Law: \( \vec{B} \cdot \vec{l} = \mu_0 I \)
8. Magnetic field at center of ring of current: \( B = \frac{\mu_0 I}{2 R} \)
9. Magnetic field of ideal solenoid: \( B = \mu_0 n I_0 \)
10. Circular orbit in magnetic field: \( r = \frac{mv}{qB} \)
11. Cyclotron frequency: \( f = \frac{qB}{2 \pi m} \)
12. Force on wire segment: \( F = IB \Delta l \sin \alpha \)
13. Magnetic dipole moment of current loop: \( \mu = [\text{current}] \times [\text{area}] \)
14. Torque on current loop: \( \tau = \mu B \sin \theta \)

VII. Electromagnetic Induction
1. Motional emf in rod: \( \mathcal{E} = lvB \)
2. Emf of an electromagnetic generator: \( \mathcal{E} = -\Delta \Phi / \Delta t = NAB \omega \sin(\omega t) \)
3. Magnetic flux: \( \Phi_B = \vec{B} \cdot \vec{A} \)
4. Faraday’s Law: \( \mathcal{E} = -\Delta \Phi / \Delta t \)
5. Lenz’ Law: Induced emf opposes change that produced it
5. Mutual inductance: $\Phi_{m} = LI$
   
   $\mathcal{E}_2 = -L(\Delta I_1 / \Delta t)$

6. Self-inductance: $\Phi_n = LI$
   
   $\mathcal{E} = -L(\Delta I / \Delta t)$

7. Self-inductance of solenoid: $\mu_0 n^2 \pi R^2 l$, where $l$ is the length of solenoid

8. Magnetic energy in inductor: $U = \frac{1}{2} LI^2$

9. Energy density in magnetic field: $u = \frac{1}{2\mu_0} B^2$

10. Time constant of RL circuit: $\tau = L/R$

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VIII. REFLECTION, REFRACTION, AND OPTICS

1. Law of reflection: The angle of incidence equals the angle of reflection;

2. Index of refraction: $n = \frac{c}{v} = \frac{\lambda}{\lambda_m}$;

3. Law of refraction (Snell’s law): $\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$ or $n_1 \sin \theta_1 = n_2 \sin \theta_2$;

4. Critical angle for total internal reflection for boundary between two materials (where $n_1 > n_2$):
   
   $\sin \theta_c = \frac{n_2}{n_1}$;

5. Critical angle for material-air boundary (where $n$ is the index of refraction of the material):
   
   $\sin \theta_c = \frac{1}{n}$;

6. Focal length of spherical mirror: $f = \pm \frac{1}{2} R$ (if positive for concave mirror, negative for convex);

7. Mirror equation: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ (if positive if object or image is in front of the mirror, negative if behind; $f > 0$, concave; $f < 0$, convex);

8. Lens equation: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ (if positive if object is on near side of lens, negative if on far side; $s'$ is positive if image is on far side of the lens, negative if on near side; $f > 0$, convex; $f < 0$, concave);

9. Magnification: $M = -\frac{s'}{s}$ (when $M$ is positive image is upright; when $M$ is negative image is inverted);

10. Len’s maker equation: $\frac{1}{f} = (n-1) \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ (if positive for convex lens, $f < 0$ for concave);

11. Angular magnification of magnifier: $25cm/f$;

12. Angular magnification of microscope: $(25cm/f_0) \times (-s'/s) \approx \frac{(L-f_e)25cm}{f_0 f_e}$;

13. Angular magnification of telescope: $f/f_o$;
IX. INTERFERENCE AND DIFFRACTION

1. Constructive interference: path difference $PD + \Delta = n\lambda$ for $n=1,2,3,...; \Delta = 0$ or $\lambda/2$
2. Destructive interference: path difference $PD + \Delta = m\lambda/2$ for $m=1,3,5,...; \Delta = 0$ or $\lambda/2$
3. Two-slit interference pattern:
   maxima: $d\sin\theta = 0, \lambda, 2\lambda,...$
   minima: $d\sin\theta = 1/2\lambda, 3/2\lambda, 5/2\lambda,...$
4. Multiple slit interference pattern (grating): principal maxima: $d\sin\theta = n\lambda$ for $n=0,1,2,3,...$
5. Single slit diffraction pattern: $a\sin\theta = \lambda, 2\lambda, 3\lambda,...$
6. First minimum for circular aperture: $a\sin\theta = 1.22\lambda$
7. Raleigh’s criterion (resolution for a circular aperture): $\theta = 1.22\lambda / a$

X. Spectral lines and Bohr’s Theory

1. Spectral lines of hydrogen: $\frac{1}{\lambda} = R \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right); R = 1.09678 \times 10^7 \text{ m}^{-1}$
2. Quantization of angular momentum: $L = n\hbar; \hbar = \frac{\hbar}{2\pi} = 1.05 \times 10^{-34} \text{ J} \cdot \text{s}$
3. Bohr’s radius: $a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = 0.0529 \text{ nm}; r = n^2 a_o$
4. Energy of stationary states of hydrogen: $E_n = -\frac{m_e e^4}{2(4\pi\varepsilon_0)^2 \hbar^2} \frac{1}{n^2} = -\frac{13.6 eV}{n^2}$
5. Frequency and wavelength of photon emitted in transition: $f = \frac{E_i - E_f}{\hbar}; \lambda = \frac{c}{f} = \frac{E_i - E_f}{hc}$
6. De Broglie wavelength of waveicle: $\lambda = \frac{h}{p}$
7. Heisenberg uncertainty principle: $\Delta p \Delta \lambda \geq \hbar$

XI. Nuclear Physics

1. Radius of nucleus: $R = (1.2 \times 10^{15} \text{ m})xA^{1/3}$
2. Atomic mass unit: 1 u = $1.66 \times 10^{-27} \text{ kg};$ Energy equivalent of amu: 1 u x $c^2 = 931.5 \text{ MeV}$
3. Mass and energy: $E = mc^2$
5. Mass number: $A = N + Z$
6. Law of radioactive decay: $n = n_0 e^{-\lambda t} = n_0 \left( \frac{1}{2} \right)^{t/t_{1/2}} \lambda$ is decay constant, $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{t_{1/2}}$, half-life $t_{1/2}$ is the time required for one half of the nuclei present to disintegrate.
7. Decay rate: $\frac{\Delta n}{\Delta t} = -\lambda n = -0.693 \frac{n}{t_{1/2}}, \lambda$ is decay constant, $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{t_{1/2}}$
8. Becquerel: 1 Bq = 1 disintegration/s
9. Curie: 1 Ci = $3.7 \times 10^{10}$ disintegrations/s
1. When a hydrogen atom is bombarded, the atom is excited to its higher energy states. As it falls back to the lower energy levels, light is emitted. What are the three longest-wavelength spectral lines emitted by the hydrogen atom as it falls to the n=1 state from higher energy states? (\(h=6.63\times10^{-34}\) J-s; \(c=3\times10^8\) m/s; 1 eV=1.6\(\times10^{-19}\) J)

\[ \begin{align*}
  n=4 & \quad -0.85\text{eV} \\
  n=3 & \quad -1.51\text{eV} \\
  n=2 & \quad -\frac{13.6}{2^2} = -3.4\text{eV} \\
  n=1 & \quad -13.6\text{eV} \\
\end{align*} \]

Three longest wavelength spectral lines emitted by \(H_2^+\) as it falls to the \(n=1\) state mean that we are dealing with transitions:

\[ \begin{align*}
  a) & \quad 2 \rightarrow 1 \\
  b) & \quad 3 \rightarrow 1 \\
  c) & \quad 4 \rightarrow 1 \\
\end{align*} \]

\[ \begin{align*}
  a) \text{ According to Bohr\textquoteright}s postulate:} \\
  \lambda_{2-1} &= \frac{h}{\Delta E} \\
  &= \frac{\hbar c}{E_2-E_1} \\
  &= \frac{(6.63\times10^{-34})\,(8.8\times10^8)}{(-1.51+13.6)\times(1.6\times10^{-19})} \\
  &= 1.22 \times 10^{-7} \text{ m} = 0.122 \mu\text{m} = 22\text{nm} \\

  b) \lambda_{3-1} &= \frac{h c}{E_2-E_1} = \frac{(6.63\times10^{-34})\,(8.8\times10^8)}{-0.85+13.6} \times (1.6\times10^{-19}) = 1.03 \times 10^{-7} \text{ m} = 0.103 \mu\text{m} = 103\text{nm} \\

  c) \lambda_{4-1} &= \frac{h c}{E_4-E_1} = \frac{(6.63\times10^{-34})\,(8.8\times10^8)}{-0.85+13.6} \times (1.6\times10^{-19}) = (9.75 \times 10^{-8} \text{ m}) = 97.5\text{nm} \\
\end{align*} \]
2. What potential difference is required in an electron microscope to give electrons a wavelength of 0.05 nm? \((\hbar=6.63\times10^{-34} \text{ J}\cdot\text{s}; m_e=9.1\times10^{-31} \text{ kg})\)

The KE gained by an electron in an electron microscope \(\frac{1}{2}m_0v^2\) equals to the electric PE lost, \(V_q\).

\[
\Rightarrow \quad \frac{1}{2}m_0v^2 = V_q
\]

\[
\Rightarrow \quad \frac{m_0v^2}{m} = 2V_q
\]

\[
\Rightarrow \quad p^2 = 2V_q m \quad \Rightarrow \quad p = \sqrt{2V_q m} \quad \text{(1)}
\]

We know that \(\lambda = \frac{\hbar}{p}\) is given to us.

\[
\Rightarrow \quad p = \frac{\hbar}{\lambda} \quad \text{(2)}
\]

Combine (1) & (2) and solve with respect to \(V_q\):

\[
\sqrt{2V_q m} = \frac{\hbar}{\lambda} \quad \Rightarrow \quad 2V_q m = \frac{\hbar^2}{\lambda^2}
\]

\[
V = \frac{\hbar^2}{2qM\lambda^2} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})^2}{2\times(1.6\times10^{-19} \text{ C})(9.1\times10^{-31} \text{ kg}) \times (0.05\times10^{-2} \text{ m})^2} = 604V = 6.0\times10^2 \text{ V}
\]
3. Technetium-99 ($^{99}_{43}$Tc) has an excited state that decays by emission of gamma ray. The half life of the excited state is 360 min. What is the activity ($\Delta n/\Delta t$), in curies, of 1 mg of this excited isotope? (The atomic mass of Technetium is 99 g/mole).

\[ \text{1 mg of } {}^{99}_{43}\text{Tc} \text{ has } \frac{M}{\mu} \text{ N}_A \text{ atoms} = \]

\[ = \frac{1 \times 10^{-3} \text{ g}}{99 \text{ g/mole}} \times 6.02 \times 10^{23} \frac{\text{atoms}}{\text{mole}} = 6.08 \times 10^{18} \text{ atoms} \]

\[ \frac{\Delta n}{\Delta t} = 0.693 \frac{h}{t^{1/2}} = 0.693 \times \frac{6.08 \times 10^{18} \text{ atoms}}{360 \text{ min} \cdot \frac{60 \text{ s}}{1 \text{ min}}} = \]

\[ = 1.95 \times 10^{14} \frac{\text{ deciint}}{\text{ sec}} = 1.95 \times 10^{14} \text{ Bq} = \]

\[ = 1.95 \times 10^{19} \text{ Bq} \cdot \frac{1 \text{ Ci}}{3.7 \times 10^{10} \text{ Bq}} = 5.273 \text{ Ci} = \]

\[ = 5 \times 10^{3} \text{ Ci} \]
4. When a neutron is removed from a $^{43}_{20}\text{Ca}$ atom (of mass 42.958766 u), it will be transformed into $^{42}_{20}\text{Ca}$ atom (of mass 41.958618 u). What minimum energy must be provided to accomplish the removal? ($m_{\text{neutron}}=1.008665$ u).

\[
^{43}\text{Ca} \rightarrow n + ^{42}\text{Ca}
\]

\[
\Delta m = \left[ m_n + m_{^{42}\text{Ca}} \right] - \left[ m_{^{43}\text{Ca}} \right] =
\]

\[
= (1.008665u + 41.958618u) - (42.958766u) =
\]

\[
= 0.008517u
\]

**Energy to be provided**

\[
E = \Delta m c^2 = 0.008517u \times c^2 =
\]

\[
= \left(0.008517u \times c^2\right) \left(\frac{931.5\text{MeV}}{u \times c^2}\right) =
\]

\[
= 7.9335855\text{MeV} = 7.93\text{MeV}
\]
5. (a) What isotope is formed by the $\beta^-$ decay of $^{14}_6$C?
(b) What isotope is formed by the $\alpha$ decay of $^{234}_{94}$Pu?

a) $^{14}_6$C $\rightarrow$ $^{14}_{6-1}$ X + $e^-$ + $\gamma$

$\Rightarrow$ $^{14}_7$N is formed

6) $^{234}_{94}$Pu $\rightarrow$ $^4_AX + _2^2$He

from law of constancy of mass #

$A = 234 - 4 = 230$

from law of constancy of charge.

$Z = 94 - 2 = 92$

$\Rightarrow$ $^{230}_{92}$U is formed
6. The tiny ball at the end of the thread shown in Fig. has a mass of 0.60 g and is in the horizontal electric field of intensity 700 N/C. It is in equilibrium in the position shown. What are the magnitude and sign of the charge on the ball?

\[ E = 700 \text{ N/C} \]

\[ \theta = 20^\circ \]

\[ mg \]

\[ F \]

\[ \theta = \tan \theta \]

\[ F \]

\[ T \]

\[ \text{In order to arrange an equilibrium, } \theta \text{ should be negative and } F \text{ has the direction opposite to } E. \]

\[ T \sin \theta - F = 0 \]

\[ T \sin \theta = F \]

\[ F = mg \tan \theta \]

\[ T \cos \theta - mg = 0 \]

\[ T \cos \theta = mg \]

\[ |F| = |E| \times |Q| \]

\[ |Q| = \frac{|F|}{|E|} = \frac{mg \tan \theta}{E} = \frac{(0.60 \times 10^{-3} \text{ kg}) \times (9.8 \text{ m/s}^2) \times \tan 20^\circ}{(700 \text{ N/C})} \]

\[ = 3 \times 10^{-6} \text{ C, negative} \]
7. When a series combination of two uncharged capacitors is connected to a 12 V battery, 173 µJ of energy is drawn from the battery. If one of the capacitors has a capacitance of 4.0 µF, what is the capacitance of the other?

\[
C_1 \, ? \quad C_2 = 4.0 \mu F
\]

\[
\Sigma = 12 V
\]

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2} \quad ; \quad C = \frac{C_1 C_2}{C_1 + C_2}
\]

\( \text{(1) Energy} = \frac{1}{2} C \Sigma^2 = 173 \times 10^{-6} J \)

\[
\Rightarrow C = \frac{2 \times (173 \times 10^{-6} J)}{(12 V)^2} = 2.4 \times 10^{-6} F
\]

\( \text{(2) } \frac{1}{C_1} = \frac{1}{C} - \frac{1}{C_2} = \frac{C_2 - C}{C C_2} \)

\[
C_1 = \frac{C C_2}{C_2 - C} = \frac{2.4 \times 4.0}{4.0 - 2.4} = (6.0 \mu F)
\]
8. Four capacitors are connected in Fig. Find
(a) the charge on each of the capacitors
(b) the potential difference across each of the capacitors

\[ Q = CV = (0.4 \times 10^{-6}) \times 12 \text{V} = 4.8 \times 10^{-5} \text{C} = 4.8 \mu \text{C} \]

\[ Q_1 + Q_2 = 4.8 \mu \text{C} \]
\[ Q_3 + Q_4 = 4.8 \mu \text{C} \]
\[ C_1 = C_2 \]
\[ Q_1 = Q_2 = 2.4 \mu \text{C} \]

\[ Q_3 = \frac{4C_3V_{BC}}{C_3 + 3C_2V_{BC}} = 4.8 \mu \text{C} \]
\[ Q_4 = \frac{4C_3V_{BC}}{0.8 \mu \text{F}} = 3.6 \mu \text{C} \]

6. \[ V_{AB} = \frac{Q}{C} = \frac{4.8 \mu \text{C}}{0.8 \mu \text{F}} = 6.0 \text{V} \]
\[ V_{BC} = \frac{4.8 \mu \text{C}}{0.8 \mu \text{F}} = 6.0 \text{V} \]
\[ V_1 = V_2 = V_3 = V_4 = 6.0 \text{V} \]
9. A current of 3A flows through the wire shown in Fig. What will a voltmeter read when connected from:
(a) A to B
(b) A to C
(c) A to D

\( V_{AB} = V_B - V_A = V_B - (V_B - IR) = IR = 3A \cdot 6 \Omega = +18V \)

\( V_{AC} = V_C - V_A = V_C - (V_C + 8V - 18V) = 10V \)

\( V_{AD} = V_D - V_A = V_D - (V_D + 7V - 9V + 8V - 18V) =
= \Delta V_A + 7V + 9V - 8V + 18V = +26V \)
10. (a) Find the current in the three resistors shown in Fig.

(b) Find the power delivered by the battery $E_2$

\[ I_1 = 12 \text{ V}, \quad I_2 = 9 \text{ V}, \quad R_1 = 15 \Omega, \quad R_2 = 40 \Omega, \quad R_3 = 10 \Omega \]

\[
\begin{align*}
\Sigma E &= I_1 R_1 + I_2 R_2 + I_3 R_3 = 0 \\
E_1 &= I_1 R_1 = 12 - 15I_1 - 9 - 10I_2 + 10I_1 = 0 \\
E_2 &= I_2 R_2 = 9 - 6 - 40I_2 - 10I_2 + 10I_1 = 0 \\
E_3 &= I_3 R_3 = 3 - 25I = 0 \\
6 - 23I &= 0 \\
I_1 &= \frac{6}{23} = 0.261 \text{ A} = 3 \times 10^{-1} \text{ A} \\
I_2 &= \frac{3 + 2I_1}{10} = \frac{3 + 2 \times 0.261}{10} = 0.352 \text{ A} = 3 \times 10^{-1} \text{ A}
\end{align*}
\]

\[
I_{R_1} = I_1 = 0.261 \text{ A} \\
I_{R_2} = I_2 - I_1 = 0.352 - 0.261 = 0.091 \text{ A} \\
I_{R_2} = I_2 = 0.352 \text{ A}
\]

\[
(6) \quad P = \frac{E_2}{E_2} (I_2 - I_1) = (9 \text{ V}) (0.352 - 0.261) = 0.82 \text{ W}
\]
11. A flat circular coil with 10 loops of wire has a diameter of 2 cm and carries a current of 0.5 A. It is mounted inside a long solenoid that has 200 loops on its 25 cm length. The current in the solenoid is 2.4 A. Compute the torque required to hold the coil with its axis perpendicular to that of the solenoid.

Let the subscript "s" and "c" refer to the solenoid and coil, respectively.

Then

\[ \tau = N_c I_c A_c B_s \sin 90^\circ \]

But \[ B_s = \frac{\mu_0 n I_s}{L_s} \]

which gives

\[ \tau = \frac{\mu_0 N_c N_s I_c I_s (\pi L_c)^2}{L_s} \]

\[ = \frac{(4\pi \times 10^{-7} T m)(10)(200)(0.5A)(2.4A)(\pi (0.01m)^2)}{0.25m} \]

\[ = 3.8 \times 10^{-6} N \cdot m \]
12. The current through a coil changes from 300 mA to 150 mA in $5.00 \times 10^{-3}$ s. An induced emf of $2.00 \times 10^{-2}$ V is obtained. Find:
(a) the inductance of the coil
(b) the initial energy in the field
(c) the final energy in the field

\[(a) \quad \mathcal{E} = - \mathcal{L} \frac{dI}{dt} \]
\[\mathcal{L} = \left| \frac{\frac{\mathcal{E}}{1}}{\frac{dI}{dt}} \right| = \frac{2 \times 10^{-2}}{(300 - 150) \times 10^{-3}} \times \frac{1}{5.00 \times 10^{-3}} = \frac{(2 \times 10^{-2}) \times 5.00}{150} = \]
\[\mathcal{L} = 0.667 \text{ mH} \]

\[(b) \quad U_i = \frac{1}{2} \mathcal{L} I_i^2 = \frac{1}{2} \left(0.667 \times 10^{-3}\right) \times (300 \times 10^{-3})^2 = \]
\[U_i = 30 \mu J \]

\[(c) \quad U_f = \frac{1}{2} \mathcal{L} I_f^2 = \frac{1}{2} \left(0.667 \times 10^{-3}\right) \times (150 \times 10^{-3})^2 = \]
\[U_f = 7.5 \mu J \]
13. A long solenoid has 6000 turns per meter and radius of 2.0 cm. A circular coil of wire of radius 1.0 cm with 200 turns is placed in the solenoid.
(a) What is the mutual inductance?
(b) What is the rate of change of the current in the solenoid windings if the induced emf around the coil is equal to 20 mV.

\[
(a) \quad M = \frac{P_{\text{coil}}}{I_{\text{solenoid}}} = \frac{200 \times (\pi \times 0.1)^2 \times 6000 \cdot I_{\text{solenoid}}}{I_{\text{solenoid}}} = (200) \pi (0.01)^2 \times 6000 = 4.74 \times 10^{-4} H
\]

\[
(b) \quad |E_{\text{coil}}| = \frac{\Delta I}{M} \frac{\Delta t}{1 M} \quad \frac{\Delta I}{\Delta t} = \frac{E_{\text{coil}}}{M} = \frac{20 \times 10^{-3} V}{(4.7 \times 10^{-4} H)} = 4.2 \times 10^4 \frac{A}{s}
\]
14. What is the least thickness of a soap film \( d = ? \), placed on the polished surface of optical glass with \( n = 1.7 \), which will appear black (destructive interference) when viewed with sodium light \( (\lambda = 589.3 \text{ nm}) \) reflected practically perpendicular to the film? The refractive index for soap solution is \( n = 1.38 \).

\[ \lambda = 589.3 \text{ nm} \]

\[ \text{Air} \]

\[ a \]

\[ b \]

\[ d = \frac{\lambda}{2n} \]

\[ n = 1.38 \]

\[ \text{Glass, } n = 1.7 \]

1) Analysis of \( A \)

\( a \) is \( \frac{\lambda}{2} \) shifted with respect to \( c \).

\( b \) is \( \frac{\lambda}{2} \) shifted with respect to \( i \).

\( \rightarrow \) \( a \) is not shifted \( \Rightarrow \) \( b \)

2) Condition for destructive interference.

\[ 2d + \angle = \frac{\lambda}{2}, \quad \frac{3\lambda}{2}, \quad \frac{5\lambda}{2}, \ldots \]

\[ 2d_{\min} = \frac{\lambda}{2} \]

\[ \lambda_t = \frac{\lambda}{n_f} = \frac{589.3 \text{ nm}}{1.38} = 427 \text{ nm} \]

\[ d_{\min} = \frac{\lambda_t}{4} = \frac{427 \text{ nm}}{4} = \frac{107 \text{ nm}}{4} \]

\[ = 1.1 \times 10^2 \text{ nm} \]
15. A 4.0-cm object is placed 30 cm from a converging lens that has a focal length of 10 cm as shown in the diagram. A second converging lens is placed 20.0 cm to the right of the first lens. Determine the focal length of the second lens if an inverted image (relative to the object in the diagram) is formed 13.3 cm to the right of the first lens.

![Diagram of two converging lenses with object and image positions marked.]

a) Find image distance $s'_1$ of the first object.

\[ \frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f_1} \]

\[ \Rightarrow \quad \frac{1}{s'_1} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{s_1 - f_1}{s_1 f_1} \]

\[ s'_1 = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10 \text{ cm})(30 \text{ cm})}{(30 - 10) \text{ cm}} = 15 \text{ cm} \]

b) $s_2 = 20 \text{ cm} - s'_1 = 5 \text{ cm}$.

\[ s_2' = -6.7 \text{ cm} \]

\[ \frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \]

\[ f_2 = \frac{s_2 s_2'}{s_2' + s_2} = \frac{(5 \text{ cm})(-6.7 \text{ cm})}{(-6.7 + 5) \text{ cm}} = \frac{6.7}{-1.7} \approx -20.0 \text{ cm} \]
16. Without his contact lenses, Mr. Smith can focus from 0.80 m to infinity. What refractive power of the lenses does he require for normal reading 0.25 m from his eyes?

Contact lens should bring the image of the object positioned at the near point of normal eye (0.80 m).

\[ s = 0.25 \text{ m} \]
\[ s' = -0.80 \text{ m} \]

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]
\[ f = \frac{ss'}{s+s'} = \frac{(0.25 \times 0.80)}{(0.25 + 0.80)} = \]

\[ = 0.36 \text{ m} \]

Refractive Power = \[ \frac{1}{0.36 \text{ m}} = 2.75 \text{ Dipsles} \]
17. A beam of light which consists of a mixture of red light (\(\lambda=660\) nm in vacuum) and violet light (\(\lambda=410\) nm in vacuum) falls on the grating that contains \(1.0 \times 10^4\) lines/cm. Find the angular separation between the first-order maxima of the two wavelengths if the experiment takes place in a vacuum.

\[
\frac{d}{\lambda} = \frac{1\text{ cm}}{10^4 \text{ cm}} = \frac{1\text{ m}}{100 \text{ cm}} \times \frac{1\text{ cm}}{10^4 \text{ m}} = \frac{10 \times 10^{-3} \text{ m}}{1 \times 10^{-3} \text{ m}} = 10
\]

For red light (\(\lambda_R=660\) nm):

\[
d \sin \theta_R = n\lambda_R \Rightarrow \theta_R = \arcsin \left( \frac{\lambda_R}{d} \right) = \arcsin \left( \frac{660 \times 10^{-9}}{10^{-3}} \right) = 41.3^\circ
\]

For violet light (\(\lambda_B=410\) nm):

\[
d \sin \theta_B = \lambda_B \Rightarrow \theta_B = \arcsin \left( \frac{\lambda_B}{d} \right) = \arcsin \left( \frac{410 \times 10^{-9}}{10^{-3}} \right) = 24.2^\circ
\]

\[
\theta_R - \theta_B = 41.3^\circ - 24.2^\circ = 17.1^\circ
\]
18. Two candles are lit and separated by 0.10 m. The diameter of the pupil of the observer’s eye is 3.5 mm. What is the maximum distance that the candles can be away from the observer and be seen as two light sources? Use 545 nm for the wavelength of light in the eye.

\[
\frac{\theta_c}{2} = \frac{1.22 \lambda}{d}
\]

where \( d = 3.5 \times 10^{-3} \text{ m} \)

From the figure

\[
\theta_c = \frac{s}{L} \Rightarrow L = \frac{s}{\theta_c} = \frac{5}{\theta_c} = \frac{5 \cdot 9}{1.22} = \frac{(0.10 \text{ m}) (3.5 \times 10^{-3} \text{ m})}{1.22 \cdot (545 \times 10^{-9} \text{ m})} = 526.4 \text{ m} = 5.3 \times 10^2 \text{ m}
\]