Motion Along a Straight Line

Lecture 2

Chapter 2
(Halliday/Resnick/Walker, Fundamentals of Physics 9th edition)
Motion along a straight line

**Key Concepts**

1. Ideal particle – a point-like mass of infinitesimal size
2. Rectangular coordinates and origin point
3. Reference frame – coordinate grid with suitably adjusted clocks
Motion Along a Straight Line

In this chapter we will study kinematics i.e. how objects move along a straight line.

The following parameters will be defined:

Displacement
Average velocity
Average Speed
Instantaneous velocity
Average and instantaneous acceleration

For constant acceleration we will develop the equations that give us the velocity and position at any time. In particular we will study the motion under the influence of gravity close to the surface of the earth.

Finally we will study a graphical integration method that can be used to analyze the motion when the acceleration is not constant.
Kinematics is the part of mechanics that describes the motion of physical objects. We say that an object moves when its position as determined by an observer changes with time.

In this chapter we will study a restricted class of kinematics problems.

Motion will be along a straight line.

We will assume that the moving objects are “particles” i.e. we restrict our discussion to the motion of objects for which all the points move in the same way.

The causes of the motion will not be investigated. This will be done later in the course.

Consider an object moving along a straight line taken to be the x-axis. The object’s position at any time $t$ is described by its coordinate $x(t)$ defined with respect to the origin O. The coordinate $x$ can be positive or negative depending whether the object is located on the positive or the negative part of the x-axis.
Displacement. If an object moves from position \( x_1 \) to position \( x_2 \), the change in position is described by the displacement

\[
\Delta x = x_2 - x_1
\]

For example if \( x_1 = 5 \text{ m} \) and \( x_2 = 12 \text{ m} \) then \( \Delta x = 12 - 5 = 7 \text{ m} \). The positive sign of \( \Delta x \) indicates that the motion is along the positive x-direction

If instead the object moves from \( x_1 = 5 \text{ m} \) and \( x_2 = 1 \text{ m} \) then \( \Delta x = 1 - 5 = -4 \text{ m} \). The negative sign of \( \Delta x \) indicates that the motion is along the negative x-direction

Displacement is a vector quantity that has both magnitude and direction. In this restricted one-dimensional motion the direction is described by the algebraic sign of \( \Delta x \)

Note: The actual distance for a trip is irrelevant as far as the displacement is concerned

Consider as an example the motion of an object from an initial position \( x_1 = 5 \text{ m} \) to \( x = 200 \text{ m} \) and then back to \( x_2 = 5 \text{ m} \). Even though the total distance covered is 390 m the displacement then \( \Delta x = 0 \)
One afternoon, a couple walks three-fourths of the way around a circular lake, the radius of which is 1.50 km. They start at the west side of the lake and head due south at the beginning of their walk.

a) What is the distance they travel?
b) What are the magnitude and direction (relative to due east) of the couple’s displacement?
Average Velocity

One method of describing the motion of an object is to plot its position $x(t)$ as function of time $t$. In the left picture we plot $x$ versus $t$ for an object that is stationary with respect to the chosen origin $O$. Notice that $x$ is constant. In the picture to the right we plot $x$ versus $t$ for a moving armadillo. We can get an idea of “how fast” the armadillo moves from one position $x_1$ at time $t_1$ to a new position $x_2$ at time $t_2$ by determining the average velocity between $t_1$ and $t_2$.

$$\textbf{v}_{\text{avg}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Here $x_2$ and $x_1$ are the positions $x(t_2)$ and $x(t_1)$, respectively.

The \textbf{time interval} $\Delta t$ is defined as: $\Delta t = t_2 - t_1$

The units of $v_{\text{avg}}$ are: m/s

\textbf{Note:} For the calculation of $v_{\text{avg}}$ both $t_1$ and $t_2$ must be given.
Graphical determination of $v_{avg}$

On an $x$ versus $t$ plot we can determine $v_{avg}$ from the slope of the straight line that connects point $(t_1, x_1)$ with point $(t_2, x_2)$. In the plot below $t_1=1$ s, and $t_2 = 4$ s. The corresponding positions are: $x_1 = -4$ m and $x_2 = 2$ m.

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{2 - (-4)}{4 - 1} = \frac{6}{3} = 2 \text{ m/s}$$

Average Speed $s_{avg}$

The average speed is defined in terms of the total distance traveled in a time interval $\Delta t$ (and not the displacement $\Delta x$ as in the case of $v_{avg}$).

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

Note: The average velocity and the average speed for the same time interval $\Delta t$ can be quite different.
A person who walks for exercise produces the position-time graph

a) Without doing any calculations, decide which segments of the graph (A, B, C or D) indicate positive, negative and zero average velocities.
b) Calculate the average velocity for each segment to verify your answers to part A

a) The sign of the average velocity during a segment corresponds to the sign of the slope of the segment. The slope, and hence the average velocity, is positive for segments A and D, and negative for C, zero for segment B

b)

\[
V_A = \frac{1.00 \text{ km} - 0 \text{ km}}{0.20 \text{ h} - 0.00 \text{ h}} = 5.0 \text{ km/h}
\]
\[
V_B = \frac{1.00 \text{ km} - 1.00 \text{ km}}{0.40 \text{ h} - 0.20 \text{ h}} = 0.0 \text{ km/h}
\]
\[
V_C = \frac{0.25 \text{ km} - 1.00 \text{ km}}{0.60 \text{ h} - 0.40 \text{ h}} = -3.5 \text{ km/h}
\]
\[
V_D = \frac{1.25 \text{ km} - 0.25 \text{ km}}{1.00 \text{ h} - 0.60 \text{ h}} = 2.5 \text{ km/h}
\]
A runner runs 100m in 10s, rests 60s and returns at a walk in 80s. What is the average speed for the complete motion? What is the average velocity?

- The runner moved a total distance \( d = 100 + 0 + 100 = 200 \text{ m} \)
- The round trip took \( t = 10 + 60 + 80 = 150 \text{ s} \)
  \[ \text{V}_{\text{av}} = \frac{d}{t} = \frac{200\text{m}}{150\text{s}} = 1.3 \text{ m/s} \]
- After the motion, the runner is precisely located at the starting point
  \( \Rightarrow \) His position didn’t change
  \( \Rightarrow \text{V}_{\text{av}} = \Delta x/t = 0/t = 0 \)
Problem 5. The position of an object moving along an x axis is given by \( x = 3t - 4t^2 + t^3 \), where \( x \) is in meters and \( t \) is in seconds. Find the position of the object at the following values of \( t \): (a) 1 s, (b) 2 s, (c) 3 s, (d) 4 s. (e) What is the object’s displacement between \( t = 0 \) and \( t = 4 \) s? (f) What is the average velocity for the time interval from \( t = 2 \) s and \( t = 4 \) s? (g) Graph \( x \) versus \( t \) for \( 0 \leq t \leq 4 \) s and indicate how the answer for (f) can be found on the graph.

Using \( x = 3t - 4t^2 + t^3 \) with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write

\[
x = (3 \text{ m/s})t - (4 \text{ m/s}^2)t^2 + (1 \text{ m/s}^3)t^3.
\]

We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

(a) Plugging in \( t = 1 \) s yields \( x = 3 - 4 + 1 - 0 \).

(b) With \( t = 2 \) s we get \( x = 3(2) - 4(2)^2 + (2)^3 = -2 \) m.

(c) With \( t = 3 \) s we have \( x = 0 \) m.

(d) Plugging in \( t = 4 \) s gives \( x = 12 \) m.

For later reference, we also note that the position at \( t = 0 \) is \( x = 0 \).

(e) The position at \( t = 0 \) is subtracted from the position at \( t = 4 \) s to find the displacement \( \Delta x = 12 \) m.

(f) The position at \( t = 2 \) s is subtracted from the position at \( t = 4 \) s to give the displacement \( \Delta x = 14 \) m. Eq. 2-2, then, leads to

\[
\bar{v} = \frac{\Delta x}{\Delta t} = \frac{14 \text{ m}}{2 \text{ s}} = 7 \text{ m/s}.
\]

(g) The horizontal axis is \( 0 \leq t \leq 4 \) with SI units understood.

A straight line drawn from the point at \((t, x) = (2, -2)\) to the highest point shown (at \( t = 4 \) s) would represent the answer for part (f).
Instantaneous Velocity

The average velocity $v_{\text{avg}}$ determined between times $t_1$ and $t_2$ provide a useful description on "how fast" an object is moving between these two times. It is in reality a “summary” of its motion. In order to describe how fast an object moves at any time $t$ we introduce the notion of instantaneous velocity $v$ (or simply velocity). Instantaneous velocity is defined as the limit of the average velocity determined for a time interval $\Delta t$ as we let $\Delta t \to 0$.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

From its definition instantaneous velocity is the first derivative of the position coordinate $x$ with respect to time. Its is thus equal to the slope of the $x$ versus $t$ plot.

Speed

We define speed as the magnitude of an object’s velocity vector.
1. [1pt] The following graphs show the position ($x$) of a car as a function of time $t$. The east is chosen as $+x$ direction. Which graphs are consistent with the following descriptions? If more than one graphs match to the description, enter two or more letters without space. For example, AC.

(a) The car is initially at rest and travels due east.

(b) The car is traveling due west and increasing its speed.

(c) The car is initially traveling due west and stops.

4. [1pt] Amy is walking along a straight line. The following graph shows her velocity ($v$) as a function of time ($t$).

5. [1pt] A commuter airplane, starting from rest on an airport runway, accelerates for 22.5 s before taking off. Its speed at takeoff is 51.8 m/s (116 mi/hr). (a) Calculate the acceleration of the plane, assuming it remains constant.

(b) How far did the plane move while accelerating for 22.5 s?

6. [1pt] A Boeing 767 jet taking off from an airport accelerates from rest for 31.0 s before leaving the ground. Its acceleration is 2.16 m/s$^2$. (a) Assuming that the acceleration is constant, calculate the plane's speed at takeoff in m/s.

What is the take off speed in km/hr?

8. [1pt] An object is moving in a straight line with a constant
Problem 15. (a) If a particle's position is given by \( x = 4 - 12t^2 + 3t^2 \) (where \( t \) is in seconds and \( x \) is in meters), what is the velocity at \( t = 1 \) s? (b) Is it moving in the positive or negative direction of \( x \) just then? (c) What is the speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation) (e) Is there ever an instant when the velocity is zero? If so, give the time \( t \); if not, answer no. (f) Is there a time after 3 s when the particle is moving in the negative direction of \( x \)? If so, give the time \( t \); if not, answer no.

(a) The velocity of the particle is

\[
v = \frac{dx}{dt} = \frac{d}{dt} \left( 4 - 12t + 3t^2 \right) = -12 + 6t.
\]

Thus, at \( t = 1 \) s, the velocity is \( v = (-12 + (6)(1)) = -6 \) m/s.

(b) Since \( v < 0 \), it is moving in the negative \( x \) direction at \( t = 1 \) s.

(c) At \( t = 1 \) s, the speed is \( |v| = 6 \) m/s.

(d) For \( 0 < t < 2 \) s, \( |v| \) decreases until it vanishes. For \( 2 < t < 3 \) s, \( |v| \) increases from zero to the value it had in part (c). Then, \( |v| \) is larger than that value for \( t > 3 \) s.

(e) Yes, since \( v \) smoothly changes from negative values (consider the \( t = 1 \) result) to positive (note that as \( t \to +\infty \), we have \( v \to +\infty \)). One can check that \( v = 0 \) when \( t = 2 \) s.
### Average Acceleration

We define as the average acceleration \( a_{\text{avg}} \) between \( t_1 \) and \( t_2 \) as:

\[
a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}
\]

Units: m/s\(^2\)

### Instantaneous Acceleration

If we take the limit of \( a_{\text{avg}} \) as \( \Delta t \to 0 \) we get the instantaneous acceleration \( a \) which describes how fast the velocity is changing at any time \( t \):

\[
a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}
\]

\[
a = \frac{dv}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2 x}{dt^2}
\]

\( \Delta t \to 0 \)

The acceleration is the slope of the \( v \) versus \( t \) plot

**Note:** The human body does not react to velocity but it does react to acceleration
The average acceleration is defined to provide a measure of how much the velocity changes per unit of elapsed time.

- $a_{av}$ is a vector that points in the same direction as $\Delta V$
- + and – indicate two possible directions for the acceleration vector

$\Rightarrow a_{av}$ directed to the left
$\Rightarrow$ it's component on the positive x direction is negative

**Instantaneous acceleration**

$$\overrightarrow{a} = \lim_{\Delta t \to 0} \frac{\Delta \overrightarrow{V}}{\Delta t}$$
Equations of kinematics for constant acceleration

• Assume \( x_0 = 0 \) when \( t_0 = 0 \) \( \Rightarrow \Delta x = x - x_0 = x \)

• Since motion is along a straight line all the vectors of displacement, velocity and acceleration are along this line and we will substitute them with their magnitudes having plus or minus signs conveying the direction of these vectors.

• Assume \( v = v_0 \) at \( t_0 = 0 \) and \( v \) at \( t \)

• Assume \( a = \) constant

\[ a_{av} = a = (v-v_0)/t \text{ or } v = v_0 + at \]

• From the definition of the \( v_{av} \)

\[ v_{av} = (x-x_0)/(t-t_0) = x/t \text{ or } x = v_{av}t \]

• Because \( a = \) constant, \( v \) increases at a constant rate.

\[ v_{av} \text{ is midway between } v_0 \text{ and } v_{\text{final}} \]

\[ v_{av} = \frac{1}{2}(v_0 + v) \Rightarrow x = \frac{1}{2}(v_0 + v)t \]

\[ x = v_{av}t = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v_0 + v_0 + at)t = v_0 t + at^2/2 \]

\[ x = v_0 t + at^2/2 \]

\[ x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v_0 + v)[(v-v_0)/a] = (v^2 - v_0^2)/2a \Rightarrow v^2 = v_0^2 + 2ax \]

\[ x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v-at+v)t = vt - at^2/2 \Rightarrow x = vt - at^2/2 \]
Equations of kinematics for constant acceleration

<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Equation</th>
<th>Missing Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$v = v_0 + at$</td>
<td>$x - x_0$</td>
</tr>
<tr>
<td>2</td>
<td>$x - x_0 = \frac{1}{2}(v_0 + v)t$</td>
<td>$a$</td>
</tr>
<tr>
<td>3</td>
<td>$x - x_0 = v_0t + \frac{1}{2}at^2$</td>
<td>$v$</td>
</tr>
<tr>
<td>4</td>
<td>$v^2 = v_0^2 + 2a(x - x_0)$</td>
<td>$t$</td>
</tr>
<tr>
<td>5</td>
<td>$x - x_0 = vt - \frac{1}{2}at^2$</td>
<td>$v_0$</td>
</tr>
</tbody>
</table>
Another look at Constant Acceleration

\[ a = \frac{dv}{dt} \rightarrow dv = adt \quad \text{If we integrate both sides of the equation we get:} \]

\[ \int dv = \int adt = a \int dt \rightarrow v = at + C \quad \text{Here } C \text{ is the integration constant} \]

C can be determined if we know the velocity \( v_o = v(0) \) at \( t = 0 \)

\[ v(0) = v_o = (a)(0) + C \rightarrow C = v_o \rightarrow \quad v = v_o + at \quad \text{(eqs.1)} \]

\[ v = \frac{dx}{dt} \rightarrow dx = vdt = (v_o + at)dt = v_o dt + atdt \quad \text{If we integrate both sides we get:} \]

\[ \int dx = \int v_o dt + a \int t dt \rightarrow x = v_o t + \frac{at^2}{2} + C' \quad \text{Here } C' \text{ is the integration constant} \]

C' can be determined if we know the position \( x_o = x(0) \) at \( t = 0 \)

\[ x(0) = x_o = (v_o)(0) + \frac{a}{2} (0) + C' \rightarrow C' = x_o \]

\[ x(t) = x_o + v_o t + \frac{at^2}{2} \quad \text{(eqs.2)} \]
\[ v = v_0 + at \quad \text{ (eqs.1) } \; ; \; \quad x = x_0 + v_0 t + \frac{at^2}{2} \quad \text{ (eqs.2)} \]

If we eliminate the time \( t \) between equation 1 and equation 2 we get:

\[ v^2 - v_0^2 = 2a \left(x - x_0\right) \quad \text{ (eqs.3)} \]

Below we plot the position \( x(t) \), the velocity \( v(t) \) and the acceleration \( a \) versus time \( t \):

\[ x = x_0 + v_0 t + \frac{at^2}{2} \]

The \( x(t) \) versus \( t \) plot is a parabola that intercepts the vertical axis at \( x = x_0 \).

\[ v = v_0 + at \]

The \( v(t) \) versus \( t \) plot is a straight line with Slope = \( a \) and Intercept = \( v_0 \).

The acceleration \( a \) is a constant.
a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a speed of 8.0 m/s when going down a slope for 5.0s?
b) How far does the skier travel in this time?

5. Solve the equation with respect to the unknown kinematic variable

\[ a_{av} = \frac{v - v_0}{t} = \frac{8.0 \text{ m/s} - 0}{5.0 \text{ s}} = 1.6 \text{ m/s}^2 \]

\[ x = \frac{1}{2} (v_0 + v) t = \frac{1}{2} (8.0 \text{ m/s} + 0) (5.0 \text{ s}) = 20 \times 10 \text{ m} \]

3. Fill up the table of given kinematic variables with appropriate + or – signs.

4. Verify that given information contains at least three of the five kinematics variables and choose one kinematic equation relating three given kinematic variables with the unknown one.
The driver of an automobile traveling at 95 km/h perceives an obstacle on the road and slams on the brakes. Calculate the total stopping distance in meters. Assume that the reaction time of the driver is 0.75s (so there is a time interval of 0.75s during which the automobile continues at constant speed while the driver gets ready to apply the brakes and that the deceleration of the automobile is 7.8 m/s² when the brakes are applied.

Often the motion of an object is divided into segments, each with a different acceleration. !!! When the motion of an object is divided into segments, remember that the final velocity of one segment is the initial velocity for the next segment.
Two soccer players start from rest 48m apart. They run directly toward each other, both players accelerating. The first player has an acceleration whose magnitude is 0.50m/s². The second player’s acceleration has a magnitude of 0.30m/s².

a) How much time passes before they collide?
b) At the instant they collide, how far has the first player run?
Reasoning:
Since the spacecraft is slowing down, $\mathbf{a}$ has a direction opposite to the velocity.
$\Rightarrow a = -10.0 \text{m/s}^2$

Solution:
$v^2 = v_0^2 + 2ax = (3250 \text{m/s})^2 +$
$2(-10.0 \text{m/s}^2)(215000 \text{m}) = 6.3 \times 10^6 \text{ m}^2/\text{s}^2$

$v = \pm \sqrt{6.3 \times 10^6 \text{ m}^2/\text{s}^2} = \pm 2500 \text{ m/s}$

Both of these answers correspond to the same displacement $x = +215 \text{km}$, but each arises in a different part of the motion.
Galileo Galilei was an Italian scientist who formulated the basic law of falling bodies, which he verified by careful measurements. In the absence of air resistance, he found that all bodies at the same location above the earth fall vertically with the same acceleration.
Freely Falling Bodies

• The effect of gravity causes objects to fall downward
• In the absence of air resistance, it is found that all bodies at the same location above the earth fall vertically with the same acceleration
• If the distance of the fall is small compared to the radius of the earth, the acceleration remains constant throughout the fall

![Diagram showing gravitational force and free fall equations]

• Idealized motion, in which air resistance is neglected is known as free fall
• Since the acceleration is nearly constant in free fall, the equations of kinematics can be used
• Since the motion occurs in vertical or “y” direction we simply replace “x” with “y” in kinematics equations

\[
\begin{align*}
v &= v_o + at \\
y &= \frac{1}{2} (v_o + v)t \\
y &= v_o t + \frac{1}{2} at^2 \\
v^2 &= v_o^2 + 2ay
\end{align*}
\]

\[
\begin{align*}
v &= v_o - gt \\
y &= \frac{1}{2} (v_o + v)t \\
y &= v_o t - \frac{1}{2} gt^2 \\
v^2 &= v_o^2 - 2gy
\end{align*}
\]
Free Fall

Close to the surface of the earth all objects move towards the center of the earth with an acceleration whose magnitude is constant and equal to 9.8 m/s$^2$. We use the symbol $g$ to indicate the acceleration of an object in free fall.

If we take the y-axis to point upwards then the acceleration of an object in free fall $a = -g$ and the equations for free fall take the form:

$$v = v_0 - gt \quad \text{(eqs.1)}$$;

$$x = x_o + v_o t - \frac{gt^2}{2} \quad \text{(eqs.3)}$$

$$v^2 - v_o^2 = -2g \left( x - x_o \right) \quad \text{(eqs.4)}$$

**Note:** Even though with this choice of axes $a < 0$, the velocity can be positive (upward motion from point A to point B). It is momentarily zero at point B. The velocity becomes negative on the downward motion from point B to point A.

**Hint:** In a kinematics problem always indicate the axis as well as the acceleration vector. This simple precaution helps to avoid algebraic sign errors.
A Falling Stone. The Velocity of a Falling Stone

A stone is dropped from rest from the top of a tall building. After 3.00s of free fall

a) What is the displacement of the stone?

b) What is the velocity of the stone?

b) Because of the acceleration due to gravity, the magnitude of the stone’s downward velocity increases by 9.80 m/s during each second of free fall \[ v = v_0 + at \]

• Since the stone is moving downward in the negative direction the value for \( v \) should be negative

\[ v = v_0 + at = (-9.80 \text{ m/s}^2)(3.00 \text{s}) = -29.4 \text{ m/s} \]
How High Does It Go?

A golf ball rebounds from the floor and travels straight upward with a speed of 5.0 m/s. To what max height does the ball rise?

\[ v^2 = v_0^2 + 2ay \]
\[ y = \frac{v^2 - v_0^2}{2a} = \frac{(0 - (5.0 \text{ m/s})^2)}{2 (-9.80 \text{ m/s}^2)} = 1.3 \text{ m}. \]
How Long Is It in the Air?

An arrow is fired from ground level straight upward with an initial speed of 15 m/s. How long is the arrow in the air before it strikes the ground?

- During the time the arrow travels upward, gravity causes its speed to decrease to 0.
- On the way down, gravity causes the arrow to regain the lost speed.
- $\Rightarrow$ the time for the arrow to go up is equal to the time to go down.

\[
\begin{align*}
\text{up:} & \quad \frac{v - v_0}{g} = \frac{0 - 15}{-9.8} \approx 1.54 \text{ s} \\
\text{down:} & \quad \frac{v - v_0}{g} = \frac{-15 - 0}{-9.8} = 1.54 \text{ s} \\
\text{total:} & \quad t_{\text{up}} + t_{\text{down}} = 3.08 \text{ s}
\end{align*}
\]
Problem 55. A ball of moist clay falls 15.0 m to the ground. It is in contact with the ground for 20.0 ms before stopping. (a) what is the magnitude of the average acceleration of the ball during the time it is in contact with the ground? (Treat the ball as a particle.) (b) Is the average acceleration up or down?

(a) We first find the velocity of the ball just before it hits the ground. During contact with the ground its average acceleration is given by

\[ a_{\text{avg}} = \frac{\Delta v}{\Delta t} \]

where \( \Delta v \) is the change in its velocity during contact with the ground and \( \Delta t = 20.0 \times 10^{-3} \) s is the duration of contact. Now, to find the velocity just before contact, we put the origin at the point where the ball is dropped (and take +y upward) and take \( t = 0 \) to be when it is dropped. The ball strikes the ground at \( y = -15.0 \) m. Its velocity there is found from \( v^2 = -2gy \). Therefore,

\[ v = -\sqrt{-2gy} = -\sqrt{-2(9.8 \text{ m/s}^2)(-15.0 \text{ m})} = -17.1 \text{ m/s} \]

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

\[ a_{\text{avg}} = \frac{0 - (-17.1 \text{ m/s})}{20.0 \times 10^{-3} \text{ s}} = 857 \text{ m/s}^2. \]

(b) The fact that the result is positive indicates that this acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.
Graphical Integration in Motion Analysis (non-constant acceleration)

When the acceleration of a moving object is not constant we must use integration to determine the velocity $v(t)$ and the position $x(t)$ of the object.

The integration can be done either using the analytic or the graphical approach.

$$a = \frac{dv}{dt} \rightarrow dv = adt \rightarrow \int_{t_o}^{t_1} dv = \int_{t_o}^{t_1} adt \rightarrow v_1 - v_o = \int_{t_o}^{t_1} adt \rightarrow v_1 = v_o + \int_{t_o}^{t_1} adt$$

$$\int_{t_o}^{t_1} adt = \left[ \text{Area under the } a \text{ versus } t \text{ curve between } t_o \text{ and } t_1 \right]$$

$$v = \frac{dx}{dt} \rightarrow dx = vdt \rightarrow \int_{t_o}^{t_1} dx = \int_{t_o}^{t_1} vdt \rightarrow$$

$$x_1 - x_o = \int_{t_o}^{t_1} vdt \rightarrow x_1 = x_o + \int_{t_o}^{t_1} vdt$$

$$\int_{t_o}^{t_1} vdt = \left[ \text{Area under the } v \text{ versus } t \text{ curve between } t_o \text{ and } t_1 \right]$$
To solve this problem, we note that velocity is equal to the time derivative of a position function, as well as the time integral of an acceleration function, with the integration constant being the initial velocity. Thus, the velocity of particle 1 can be written as

\[ v_1 = \frac{dx_1}{dt} = \frac{d}{dt} \left( 6.00t^2 + 3.00t + 2.00 \right) = 12.0t + 3.00. \]

Similarly, the velocity of particle 2 is

\[ v_2 = v_{20} + \int a_2 dt = 20.0 + \int (-8.00t) dt = 20.0 - 4.00t^2. \]

The condition that \( v_1 = v_2 \) implies

\[ 12.0t + 3.00 = 20.0 - 4.00t^2 \implies 4.00t^2 + 12.0t - 17.0 = 0 \]

which can be solved to give (taking positive root) \( t = (3 + \sqrt{26})/2 = 1.05 \) s.

Thus, the velocity at this time is \( v_1 = v_2 = 12.0(1.05) + 3.00 = 15.6 \) m/s.