Tentative Schedule:

	Date	Module	Topics
1	Aug. 25 (Mo)	Module 1. Spontaneous and Stimulated Transitions	Introduction, Spontaneous and Stimulated Transitions (Ch. 1)
2	Aug. 27 (We)		Spontaneous and Stimulated Transitions (Ch. 1) Homework 1: PH481 Ch.1 problems 1.4 &1.6. PH581 Ch.1 problems 1.4, 1.6 & 1.8 due Sep.3 before class
	Sep. 1 (Mo) No classes		**Labor Day Holiday**
3	Sep. 3 (We)	Module 2. Optical Frequency Amplifiers	Optical Frequency Amplifiers (Ch. 2.1-2.4) Problem solving for Ch.1
4	Sep. 8 (Mo)		Optical Frequency Amplifiers (Ch. 2.5-2.10)
5	Sep. 10 (We)		Optical Frequency Amplifiers (Ch. 2.5-2.10) Homework 2: PH481 Ch.2 problems 2.2 (a,b), 2.4 & 2.5 (a,b). PH581 Ch.2 problems 2.2 (a,b), 2.4 & 2.5 (a,b,c,d) due Sep.22 before class
6	Sep. 15 (Mo)	Module 3. Introduction to two practical Laser Systems	Problem solving for Ch.2 Introduction to two Practical Laser Systems (The Ruby Laser, The Helium Neon Laser) (Ch. 3)
7	Sep. 17 (We)		Review Chapters 1 & 2
8	Sep. 22 (Mo)		**Exam 1** Over Chapters 1-3; Grades for exam 1
9	Sep. 24 (We)	Module 4. Passive Optical Resonators	Exam 1 problem solving. Passive Optical Resonators – Lecture Notes
10	Sep. 29 (Mo)		Passive Optical Resonators – Lecture Notes.
11	Oct. 1 (We)		Passive Optical Resonators – Lecture Notes. Physical significance of χ' and χ'' (Ch.2.8-2.9). Homework 3: read Ch.2 & notes. Work out problems (see Canvas). Due Oct. 8
12	Oct. 6 (Mo)	Module 5. Optical Resonators Containing Amplifying Media	Optical Resonators Containing Amplifying Media (4.1-2).
13	Oct. 8 (We)		Optical Resonators Containing Amplifying Media (Ch.4.3-4.7) Homework 4: Ch. 4 problems 4.7 and 4.9. Due Oct 15.
14	Oct. 13 (Mo)	Module 6. Laser Radiation	Laser Radiation (Ch. 5.1-5.4)
15	Oct. 15 (We)	Module 7. Control of Laser Oscillations	Control of Laser Oscillators (6.1-6.3) Homework 5: Ch. 5 problems 5.1 and 5.5. Due Oct 29.
16	Oct. 20 (Mo)		Control of Laser Oscillators (6.4-6.5) and exam 2 review
17	Oct. 22 (We)	Module 8. Optically Pumped Solid State Lasers	Optically Pumped Solid State Lasers (7.1-7.11)
18	Oct. 27 (Mo)		Optically Pumped Solid State Lasers (7.1-7.11)
19	Oct. 29 (We)		**Exam 2 Over Chapters 4-6** Grades for exam 2 Exam 2 correct solution; Homework 6 Due Nov.5; see Canvas including article on Cr:CdSe
20	Nov. 3 (Mo)	Module 8. Optically Pumped Solid State Lasers	Optically Pumped Solid State Lasers (7.14-7.15)
21	Nov. 5 (We)		Optically Pumped Solid State Lasers (7.16-7.17) Homework 7 (see Canvas) Due Nov. 17
22	Nov. 10 (Mo)	Module 9. Spectroscopy of Common Lasers and Gas Lasers	Spectroscopy of Common Lasers and Gas Lasers (Ch. 8.1-8.10)
23	Nov. 12 (We)	Module10. Molecular Gas Lasers I	Molecular Gas lasers I (Ch. 9.1-9.5)
24	Nov. 17 (Mo)		Molecular Gas lasers I (Ch. 9.1-9.5) Homework 8 (see Canvas) Due Dec. 1
25	Nov. 19 (We)	Module 11. Molecular Gas Lasers II	Molecular Gas Lasers II (Ch. 10.1-10.8) and review for exam 3 (Ch. 10.1-0.8) Homework 9 (see Canvas) Due Dec. 1
	Nov. 24 (Mo) No classes		**Thanksgiving - no classes held**
	Nov.26 (We) No classes		**Thanksgiving - no classes held**
26	Dec. 1 (Mo)		**Exam 3** Over Chapters 7-10 Grades; Exam 3 Correct solution
27	Dec. 3 (We)		Review for Final
28	**Dec. 8 (We) in ESH 3160**		**FINAL EXAM Over Chapters 1-10 (4:15-6:45pm) in ESH 3160 Final Grades**

Laser Physics I

PH481/581-VT1 (Mirov)

Lecture 7. Review for chapters 1-3

Fall 2025

C. Davis, “Lasers and Electro-optics”

Problem. Find the relation between spontaneous emission lifetime and cross section for a simple atomic transition.

Cross-section of emission at frequency ν

$$\sigma(\nu) = \frac{c^2 A_{21}}{8\pi\nu_o^2} g(\nu,\nu_o) = \frac{\lambda_o^2}{8\pi n^2}\frac{g(\nu,\nu_o)}{\tau_{sp}} \quad (1)$$

where $\lambda_o = c/\nu_o$ is the wavelength (in vacuum) of an e.m. wave whose frequency corresponds to the center of the line.

Equation (1) can be used either to obtain the value of σ, when τ_{sp} is known, or the value of τ_{sp} when σ is known.

If $\sigma(\nu)$ is known, to calculate τ_{sp} from (1) we multiply both sides by dν and integrate. Since $\int g(\nu)d\nu = 1$ we get:

$$\tau_{sp} = \frac{\lambda_o^2}{8\pi n^2}\frac{1}{\int \sigma(\nu)d\nu}$$

Problem. Radiative lifetime and quantum yield of the ruby laser transition.

The R_1 laser transition of ruby has to a good approximation a Lorentzian shape of width (FWHM) 330 GHz at room temperature. The measured peak transition cross-section is $\sigma=2.5\times10^{-20}$ cm^2. Calculate the radiative lifetime (the refractive index is n=1.76). Since the observed room temperature lifetime is 3 ms, what is the fluorescence quantum yield.

From $\sigma(\nu) = \dfrac{c^2 A_{21}}{8\pi\nu_o^2} g(\nu,\nu_o) = \dfrac{\lambda_o^2}{8\pi n^2}\dfrac{g(\nu,\nu_o)}{\tau_{sp}}$ (1) we have

$$\tau_{sp} = \frac{\lambda_o^2}{8\pi n^2}\frac{g(\nu,\nu_o)}{\sigma} \quad (2)$$

For a Lorentzian lineshape at $\nu = \nu_o$ we have

$$g(0) = \frac{2}{\pi\Delta\nu_o}$$

For $\lambda_o = 694nm,$ we obtain from eq. (2) the spontaneous emission lifetime (i.e. the radiative lifetime)

$$\tau_{sp} = \frac{\lambda_o^2}{8\pi n^2}\frac{2}{\pi\Delta\nu_o\sigma} = 4.78ms$$

Fluorescence quantum yield ϕ is defined as the ratio of the number of emitted photons to the number of atoms initially raised to the level 2

$$\phi = \frac{\tau}{\tau_r} = 0.625$$

where observed lifetime τ given by

$$\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

Spectral radiancy

- **Thermal radiation: The radiation emitted by a body as a result of temperature.**
- **Blackbody : A body that surface absorbs all the thermal radiation incident on them.**
- **Spectral radiancy $R_T(\nu)$: The spectral distribution of blackbody radiation.**

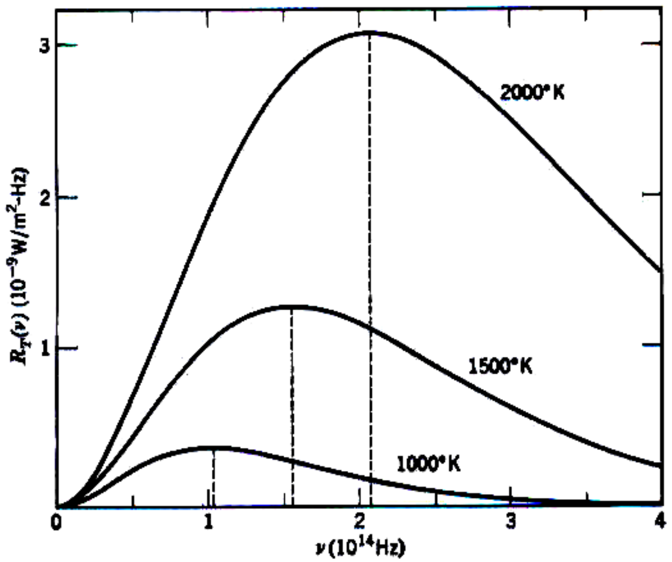
$R_T(\nu)d\nu$: represents the emitted energy from a unit area per unit time between ν and $\nu + d\nu$ at absolute temperature T.

radiancy

$$R_T = \int_0^\infty R_T(\nu)d\nu$$

- **Stefan's law (1879):**

$$R_T = \sigma T^4, \sigma = 5.67\times 10^{-8} W/m^2 - {}^oK^4$$



Energy Density and Spectral Radiancy

- energy density between ν and ν+dν: $\boxed{\rho_T(\nu) = \frac{8\pi\nu^2}{c^3} \times \frac{h\nu}{e^{-h\nu/kT} - 1}}$

$$\rho_T(\nu)d\nu = -\rho_T(\lambda)d\lambda$$

$$\Rightarrow \boxed{\rho_T(\lambda)} = -\rho_T(\nu)\frac{d\nu}{d\lambda} = \rho_T(\nu)\frac{c}{\lambda^2} \boxed{= \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}}$$

Ex: Show $\rho_T(\nu) = (4/c)R_T(\nu)$

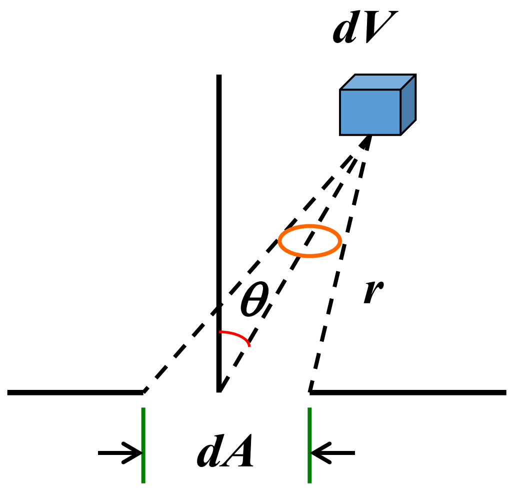
solid angle expanded by dA is $\Omega = \frac{d\vec{A}\cdot\hat{r}}{4\pi r^2} = \frac{dA\cos\theta}{4\pi r^2}$

spectral radiancy:

$$R_T(\nu) = \int \rho_T(\nu)dV\left(\frac{dA\cos\theta}{4\pi r^2}\right)/(dA\cdot\Delta t)$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \int_0^{c\Delta t} \rho_T(\nu)\frac{\cos\theta}{4\pi r^2\Delta t} r^2 \sin^2\theta dr$$

$$= \frac{c}{4}\rho_T(\nu)$$



Stefan's law

Ex: Use the relation $R_T(\nu)d\nu = (4/c)\rho_T(\nu)d\nu$ between spectral radiancy and energy density, together with Planck's radiation law, to derive Stefan's law $R_T = \sigma T^4, \sigma = 2\pi^5 k^4/15c^2h^3$

$$R_T = \int_0^\infty R_T(\nu)d\nu = \frac{c}{4}\int_0^\infty \rho_T(\nu)d\nu = \frac{2\pi}{c^2}\int_0^\infty \frac{h\nu^3}{e^{h\nu/kT}-1}d\nu$$

$$= \frac{2\pi}{c^2}\frac{(kT)^4}{h^3}\int_0^\infty \frac{x^3}{e^x-1}dx$$

$$x = h\nu/kT$$

$$\int_0^\infty x^3/(e^x-1)dx = \pi^4/15$$

$$= \frac{2\pi}{c^2}\frac{(kT)^4}{h^3}\times\frac{\pi^4}{15} = \sigma T^4$$

$$\Rightarrow \sigma = \frac{2\pi^5 k^4}{15c^2h^3}$$

$$\sigma = \frac{2\pi^5(1.38\times10^{-23})^4}{15(3\times10^8)^2(6.63\times10^{-34})^3} = 5.67\times10^{-8}\frac{W}{m^2K^4}$$

Problem. Compute the radiation flux or power in watts coming from a surface of temperature 300K (near room temperature) and area 0.02 m^2 over a wavelength interval 0.1 μm at a wavelength of 1.0 μm.

$$\rho_T(\nu)d\nu = -\rho_T(\lambda)d\lambda$$

$$\Rightarrow \rho_T(\lambda) = -\rho_T(\nu)\frac{d\nu}{d\lambda} = \rho_T(\nu)\frac{c}{\lambda^2} = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

$$R_T(\lambda,T) = \frac{c}{4}\rho(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}$$

The total radiancy emitted from the blackbody surface within a specific wavelength interval $\Delta\lambda$ would then be expressed as

$$R_T = R_T(\lambda,\Delta\lambda,T) = R_T(\lambda,T)\Delta\lambda$$

Specific values of radiancy $R_T(\lambda,\Delta\lambda,T)$ in units of power per unit area (intensity) can be obtained from the following empirical expression for

$$R_T(\lambda,T) = \frac{3.75\times10^{-25}}{\lambda^5\left(e^{0.0144/\lambda T}-1\right)}\frac{W}{m^2\cdot nm},$$

where λ in meters, $\Delta\lambda$ in nanometers, and T in K. Remember that these expressions are for the radiation into 2π steradian solid angle.

$$P = R_T(\lambda,T)\Delta\lambda\Delta A = \frac{3.75\times10^{-25}}{(0.000001)^5\left(e^{0.0144/(0.000001\times300)}-1\right)}\times100\times0.02 = 1.06\times10^{-15}W$$

We can see that at RT the power radiated from a blackbody within this wavelength region is almost too small to be measurable.

Problem 1.6

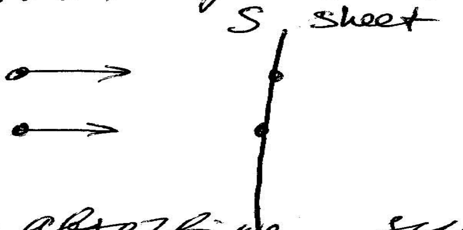
Estimate the total force produced by the photon pressure of the sun on an aluminium sheet of area $10^6 m^2$ situated on the surface of the earth. Use sensible values for the parameters of the problem in order to produce a numerical result.

$$\sigma = \frac{2\pi^5 (1.38\times10^{-23})^4}{15(3\times10^8)^2(6.63\times10^{-34})^3} = 5.67\times10^{-8}\,\frac{W}{m^2K^4}$$

for sun temperature $T = 5778K$

$$R_T = \sigma T^4 = 6.32\times10^7\,\frac{W}{m^2}$$

(2) Estimation of radiation pressure.



a) perfectly absorbing surface.

if Φ - photon flux => total momentum transferred to the surface S in the time interval Δt is

$$Q = \Phi q S \Delta t$$

where q - photon momentum $= \frac{h}{\lambda} = \frac{h\nu}{c}$

Force F can be calculated from impulse-momentum theorem

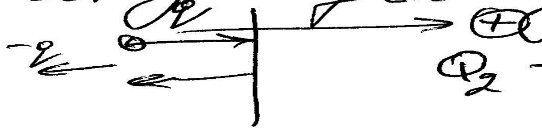
$$F\Delta t = \cancel{Q_2}^{\,0} - Q_1$$

=> radiation pressure $P = \frac{F}{S} = \frac{Q}{S\Delta t} = \frac{\Phi q \cancel{S \Delta t}}{\cancel{S\Delta t}} = \Phi \frac{h\nu}{c}$

Intensity of photon flux $I = \Phi h\nu$ =>

=> $\boxed{P = \frac{I}{c}}$

b) for perfectly reflecting surface (our case)



$$Q_2 - (-Q_1) = 2Q$$

=> $\boxed{P = \frac{2I}{c}} = \boxed{2\rho(\nu)}$

Total power radiated by the sun is

$$P = R_T \times A_{sun} = 6.32\times10^7 \times 4\pi(6.96\times10^8)^2 = 3.85\times10^{26} W$$

Power density at the surface of the earth

$$I_e = \frac{P}{4\pi D^2} = \frac{3.85\times10^{26}}{4\pi(1.5\times10^{11})^2} = 1.36\times10^3 \frac{W}{m^2}$$

ignoring losses due to absorption and scattering in atmosphere

Pressure on aluminum sheet of area $10^6 m^2$ situated on the surface of the earth

$$p = \frac{2I_e}{c} = 9\times10^{-6} \frac{N}{m^2}$$

The total force produced by the photon pressure of the Sun on aluminum sheet

$$F = p\times A_{sheet} = 9N$$

Problem 1.7.

Calculate the total stored energy in a $1 m^3$ box that lies between the wavelengths $10.5 \mu m$ and $10.7 \mu m$ at a temperature of 3000 K.

1) The energy density of the radiation within the cavity per volume, per frequency interval is

$$\rho(\nu) = \frac{8\pi h \nu^3}{c^3}\left(\frac{1}{e^{h\nu/KT}-1}\right)$$

2) Total energy stored is

$$E = V \times \int_{\nu_1}^{\nu_2} \rho(\nu)\,d\nu \quad \text{since } V = 1 m^3 \quad E = \int_{\nu_1}^{\nu_2} \rho(\nu)\,d\nu\,[J]$$

where $\nu_1 = \frac{c}{\lambda_1} = \frac{3\times10^{8}\ m/s}{10.7\times10^{-6}\ m} = 2.8\times10^{13}\ Hz$

$\nu_2 = \frac{c}{\lambda_2} = \frac{3\times10^{8}\ m/s}{10.5\times10^{-6}\ m} = 2.86\times10^{13}\ Hz$

3) let us estimate $\frac{h\nu}{KT} = \frac{6.62\times10^{-34}\ J\cdot s \times 2.8\times10^{13}\ s^{-1}}{1.38\times10^{-23}\ \frac{J}{K}\cdot 3000K} = 0.45$

$$e^{0.45} - 1 = 0.56$$

approximation $e^{a\nu} - 1 \simeq a\nu$ where $a = \frac{h}{KT}$

4) $$E = \int_{\nu_1}^{\nu_2} \frac{8\pi \cancel{h}\nu^{\cancel{3}\,2}}{c^3}\cdot\frac{KT}{\cancel{h\nu}}\,d\nu = \frac{8\pi KT}{c^3}\int_{\nu_1}^{\nu_2} \nu^2\,d\nu = \frac{8\pi KT}{c^3}\cdot\frac{\nu^3}{3}\Bigg|_{2.8\times10^{13}}^{2.86\times10^{13}} =$$

$$= 1 m^3 \times \left[\frac{8\cdot\pi\cdot(1.38\times10^{-23}\frac{J}{K})(3000K)}{(3\times10^{8}\ m/s)^3}\cdot\frac{(2.86\times10^{13}\ Hz)^3}{3} - 3.85\times10^{-44}\cdot\frac{(2.8\times10^{13})^3}{3}\right] =$$

$1,3.85\times10^{-44}$

$$= 3.002\times10^{-4} J - 2.817\times10^{-4} J = 1.85\times10^{-5} J = 19\mu J$$

Problem (2.1)

A homogeneously broadened laser amplifier, has a small-signal gain at line center of 0.6 m^{-1} and is at line center 3 m long. The saturation intensity of the medium at line center is 3 W/m^2. An input signal of intensity 2.5 W/m^2 enters the amplifier. What is the ~~output~~ ~~frequen~~ intensity if:

a) The input is at line center and I_s - neglected

b) The input is one linewidth (FWHM) from line center and gain saturation is neglected?

c) As in "a" but gain satur. is included

d) As in (b) but gain satur. is included

$\gamma_0(\nu_0) = 0.6\ m^{-1}$

$z = 3\ m$

$I_s(\nu_0) = 3\ W/m^2$

$I(0) = 2.5\ W/m^2$

a) $$I_{\nu_0} = I_{\nu_0}(0) \cdot e^{\gamma_0(\nu_0) z} = (2.5\ W/m^2) \cdot e^{(0.6\ m^{-1})(3m)} = 15\ W/m^2$$

b) $$\gamma(\nu_0) = \left(N_2 - \frac{g_2}{g_1} N_1\right) \frac{c^2 A_{21}}{8\pi \nu_0^2} \cdot \frac{2/\pi\Delta\nu}{1 + [2(\nu_0 - \nu_0)/\Delta\nu]^2} =$$

$$= \left(N_2 - \frac{g_2}{g_1} N_1\right) \frac{c^2 A_{21}}{8\pi \nu_0^2} \cdot \frac{2}{\pi \Delta\nu}$$

$$\gamma(\nu_0 - \Delta\nu) = \left(N_2 - \frac{g_2}{g_1} N_1\right) \underbrace{\frac{c^2 A_{21}}{8\pi (\nu_0 - \Delta\nu)^2}}_{(\nu_0-\Delta\nu)^2 \approx \nu_0^2} \cdot \frac{2/\pi\Delta\nu}{\underbrace{1 + \left(\frac{2\Delta\nu}{\Delta\nu}\right)^2}_{=5}} = \frac{\gamma(\nu_0)}{5}$$

$$I_{\nu_0 - \Delta\nu} = I_{\nu_0 - \Delta\nu}(0)\, e^{\frac{\gamma_0(\nu_0)}{5} \cdot z} = 2.5 \cdot e^{\frac{0.6}{5} \cdot 3} = 3.6\ W/m^2$$

c) $I_{\nu_0} = I_{\nu_0}(0)\, e^{\gamma_0(\nu_0) z - \frac{I - I_0}{I_s}} =$

$= 2{,}5 \cdot e^{0{,}6 \times 3 - \frac{I - 2{,}5}{3}} = 2{,}5\, e^{\frac{7{,}9 - I}{3}}$

$2.5 < I < 15$

Guess	Calculate
5	6.5
4	9,17
6	4.7
5.5	5.56

$\Rightarrow I \simeq 5.5\ W/m^2$

d) $I = I_{(0)}\, e^{\frac{\gamma_0(\nu_0)}{5} \cdot z - \frac{I - I_0}{I_s}} =$

$= 2.5 \cdot e^{0{,}12 \cdot 3 - \frac{I - 2{,}5}{3}} = 2{,}5\, e^{\frac{3{,}58 - I}{3}}$

Guess	Calculate
2	4.2
3	3.03

$\Rightarrow I = 3\ W/m^2$

Problem 2.6

What is the saturation intensity one FWHM from line center in a naturally broadened amplifier if the amplifier has the following parameters: $A_{21} = 10^8 s^{-1}$; $\tau_1 = 1 ns$; $\tau_2 = 5 ns$; $\lambda_0 = 1 \mu m$

$$I_s(\nu) = \frac{8\pi h \nu^3}{c^2 \phi\, g(\nu_0 \nu)}$$

$$\phi = A_{21}\tau_2 \left[1 + (1 - A_{21}\tau_2)\frac{\tau_1}{\tau_2}\right] = (10^8 s^{-1})(5\times10^{-9} s)\left[1 + (1 - 10^8 \cdot 5\times10^{-9})\frac{10^{-9}}{5\times10^{-9}}\right] =$$

$$= 0.5[1 + (0.5)\times(0.2)] = 0.55$$

$$\phi = 0.55$$

$$\lambda_0 = 10^{-6} m \Rightarrow \nu_0 = \frac{c}{\lambda_0} = \frac{3\times10^8 m/s}{10^{-6} m} = 3\times10^{14} Hz$$

$$\Delta\nu_L = \frac{A_{21}}{2\pi} = \frac{10^8}{2\pi} = 1.6\times10^7 Hz$$

$$g(\nu_0, \nu) = \frac{(2/\pi\Delta\nu_L)}{1 + [2(\nu - \nu_0)/\Delta\nu_L]^2} = \frac{2}{\pi\Delta\nu_L\, 5} = 8\times10^{-9} s$$

$\nu - \nu_0 = \Delta\nu_L$; $\nu = \nu_0 + \Delta\nu_L = 3.00000016\times10^{14} Hz$

$$I_s(\nu) = \frac{8\pi\cdot(6.6\times10^{-34} J\cdot s)\cdot(3\times10^{14} Hz)^3}{(3\times10^8 m/s)^2\cdot 0.55\cdot 8\times10^{-9} s} = 1.1\times10^3 \frac{W}{m^2}$$

Problem 2.7

The lifetime of a particular excited argon level varies with pressure as

$\tau = 10^{-8}/(1+P)$ s at 2000K, where P is pressure measured in atmospheres.

At what Pressure will the collisional broadening be as great as the Doppler broadening? Neglect the effect of the lower level lifetime.

Take $\lambda_0 = 488$ nm

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1) $\Delta\nu_D = 2\nu_0 \sqrt{\dfrac{2KT \ln 2}{MC^2}}$

$\nu_0 = \dfrac{c}{\lambda_0} = \dfrac{3\times10^8 \text{ m/s}}{488\times10^{-9} \text{ m}} = 6.15\times10^{14}$ Hz

$T = 2000$ K ; $k = 1.38\times10^{-23}$ J/K ; $N_A = 6.02\times10^{23} \dfrac{\text{atoms}}{\text{mole}}$

$M = \dfrac{\mu}{N_A} = \dfrac{40 \text{ g/mole}}{6\times10^{23} \text{ atoms/mole}} = 6.6\times10^{-23}$ g $= 6.6\times10^{-26}$ kg

$\Delta\nu_D = 2\cdot(6.15\times10^{14} \text{ Hz}) \sqrt{\dfrac{2\times(1.38\times10^{-23} \text{ J/K})\times2000\text{K}\cdot\ln 2}{(6.6\times10^{-26} \text{ kg})\cdot(3\times10^8 \text{ m/s})^2}} =$

$= 3.12\times10^{10}$ Hz

2) $\Delta\nu_L = \dfrac{A_1 + A_2}{2\pi}$; A_2 & A_1 – Einstein coeff. of the upper and lower levels

A_1 – neglect

$\Rightarrow \Delta\nu_L = \dfrac{A_2}{2\pi} = \dfrac{1}{2\pi\tau_2} = \dfrac{(1+P)}{10^{-8}\cdot2\pi} = 3.12\times10^{10}$

$\Rightarrow P = 3.12\times10^{10}\times10^{-8}\times2\pi - 1 = 1960$ atm

LASER PHYSICS I PH 481/581 VT1 (MIROV)

Sample Exam 1

STUDENT NAME: ______________________ STUDENT id #: ______________________

Opened textbook

PH 581 ALL QUESTIONS ARE WORTH 50 POINTS. WORK OUT ANY 3 PROBLEMS OUT OF 6

PH481 ALL QUESTIONS ARE WORTH 75 POINTS. WORK OUT ANY 2 PROBLEMS OUT OF 6

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

1. For a cavity volume $V=1\ cm^3$ calculate the number of modes that fall within a bandwidth $\Delta\lambda=10$ nm centered at $\lambda=600$ nm.

- # of modes per unit volume, per frequency range is $\rho_\nu = \frac{1}{V}\frac{dN}{d\nu} = \frac{8\pi\nu^2}{c^3}$ (1) (for an empty cavity $n=1$)
- # of modes for a cavity volume V that falls within a frequency bandwidth $\Delta\nu$ is:
  $N = \rho_\nu \cdot V |\Delta\nu|$ (2) assume $\rho_\nu =$ const over $\Delta\nu$
- to solve the problem find relationship between $\Delta\nu$ and $\Delta\lambda$
  since $\nu = \frac{c}{\lambda}$; $\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$ (3)
- using (1) and (3) in Eq. (2)
  $N = \frac{8\pi \cancel{c^2}}{\cancel{c^3}\lambda^2} V \cdot \frac{\cancel{c}\Delta\lambda}{\lambda^2} = \frac{8\pi \cdot \Delta\lambda}{\lambda^4} V =$
  $= \frac{8\pi \cdot (10\times10^{-9} m)\cdot(1\times10^{-6} m^3)}{(600\times10^{-9} m)^4} = 1.9\times10^{12}$ modes

Intensity on the retina of the sun light and of a He-Ne laser beam. At the surface of the earth the intensity of the sun is approximately 1 kW/m^2. Calculate the intensity at the retina that results when looking directly at the sun. Assume that: (i) the pupil of a bright-adapted eye is 2 mm in diameter; (ii) the focal length of the eye is 22.5 mm; (iii) the Sun subtends an angle of 0.5°. (Hint: first calculate sun power passing through the pupil and diameter of the image of the sun on the retina). Compare this intensity with that resulting when looking into a 1-mW He-Ne laser (λ=632.8 nm) with a 2 mm diameter. (Hint: the diameter of the laser beam in the focus of a lens of focal length f can be calculated as $D_F = \dfrac{4 f \lambda}{\pi D_o}$, where D_o is the beam diameter on the lens and λ is the laser wavelength).

- The pupil area of bright-adapted eye is $A = \frac{\pi D^2}{4} = 3.14\,mm^2$.
- The sun power passing through the pupil $P = I \cdot A = \frac{\cancel{1000}\,W}{\cancel{m^2}} \cdot \frac{\cancel{m} \cdot \cancel{m}}{\cancel{1000}\,mm \cdot 1000\,mm} \cdot 3.14\,mm^2 = 3.14\,mW$.
- Assuming that the focal length of the eye is $f = 22.5\,mm$ and that the total angle subtended by the sun is $\theta_s = 0.5°$, the image of the sun on the retina has a diameter D_s

$$\frac{D_s}{2 f_E} = \tan\frac{\theta_s}{2}$$

$$\Rightarrow D_s = 2 f_E \tan\frac{\theta_s}{2} = 0.2\,mm$$

- $\Rightarrow$ the intensity at the retina resulting from looking directly at the sun is $I_s = \frac{4P}{\pi D_s^2} = 10^5\,W/m^2$
- In case of a 1-mW He-Ne laser beam ($\lambda = 632.8\,nm$) with 2 mm diameter D_o

$$D_F = \frac{4 f_E \lambda}{\pi D_o} = 9\,\mu m$$

- $\Rightarrow$ the intensity at the retina $I_{He\text{-}Ne} = \frac{4 P_{He\text{-}Ne}}{\pi D_F^2} = 1.6 \times 10^7\,W/m^2$. This is 160 times the intensity resulting when looking directly at the sun.

2. A He-Ne laser, operating at 632.8 nm has an output power of P=1.0 mW with a 1–mm beam diameter. Power in the cavity is $99P$ since the output mirror has 1% transmission. The beam diameter is also 1 mm inside the laser cavity and the power is uniform over the beam cross-section. The laser linewidth is $1.5\text{x}10^{8}$ Hz.

a) What is the ratio of stimulated and spontaneous emission rates $[B_{21}\rho(\nu)/A_{21}]$ (Hint: $I(\nu)$=power/(beam cross-sectional area)x(frequency width of the beam); $\rho(\nu)= I(\nu)/c$

b) What is the effective blackbody temperature of the laser beam near the output mirror in the cavity.

a) 1) $\nu = \frac{c}{\lambda} = \frac{3\times10^{8}\,m/s}{6.328\times10^{-7}\,m} = 4.74\times10^{14}\,Hz$

2) $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} = \frac{(8\pi)(6.63\times10^{-34}\,J.s)(4.74\times10^{14}\,Hz)^3}{(3\times10^{8}\,m/s)^3} = 6.57\times10^{-14}\,\frac{J.s}{m^3}$

$\Rightarrow \frac{B_{21}}{A_{21}} = 1.52\times10^{13}\,\frac{m^3}{J.s}$

3) $\rho(\nu) = \frac{I(\nu)}{c} = \frac{P}{A\cdot\Delta\nu\cdot c} = \frac{99\cdot(1.0\times10^{-3}\,W)}{\pi(5\times10^{-4}\,m)^2\cdot(1.5\times10^{8}\,Hz)\cdot3\times10^{8}\,m/s} =$

$= 2.80\times10^{-12}\,\frac{J.s}{m^3}$

$\Rightarrow \frac{B_{21}\cdot\rho(\nu)}{A_{21}} = (1.52\times10^{13}\,\frac{m^3}{J.s})\cdot(2.80\times10^{-12}\,\frac{J.s}{m^3}) = 42.6$

b) $\frac{B_{21}\cdot\rho(\nu)}{A_{21}} = \underbrace{\frac{c^3}{8\pi h\nu^3}}_{=\frac{B_{21}}{A_{21}}}\cdot\underbrace{\frac{8\pi h\nu^3}{c^3}\cdot\frac{1}{e^{h\nu/kT}-1}}_{=\rho} = \frac{1}{e^{h\nu/kT}-1}$

$\Rightarrow \frac{1}{e^{h\nu/kT}-1} = 42.6$

$e^{\frac{h\nu}{kT}} - 1 = \frac{1}{42.6} = 0.0235$

$\frac{h\nu}{kT} = \ln(1.0235) = 2.32\times10^{-2}$

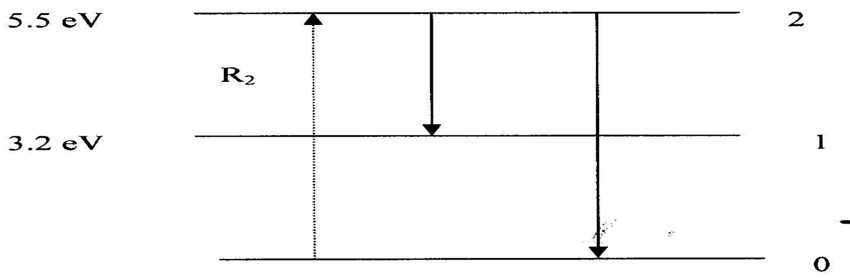
$T = \frac{h\nu}{(2.32\times10^{-2})k} = \frac{(6.63\times10^{-34}\,J.s)(4.74\times10^{14}\,Hz)}{(2.32\times10^{-2})(1.38\times10^{-23}\,J/k)} = 9.8\times10^{5}\,K$

3. Consider the ideal laser medium shown below. The pump excites the atoms to state 2 at a rate R_2, which then decays to state 1 at a rate τ_{21}^{-1} and back to state 0 at a rate τ_{20}^{-1}. State 1 decays back to state 0 so fast that the approximation $N_1 \approx 0$ is appropriate. The radiative rate for the $2 \rightarrow 1$ transition is $6\times10^6 sec^{-1}$, and its width is 10 GHZ. (assume Lorentzian profile and steady state.)

(a) What is the stimulated emission cross section for the $2 \rightarrow 1$ transition?
[Hint: $\sigma_e(\nu_o,\nu)=\gamma(\nu_o,\nu)/\Delta N$]

(b) What must be the pump rate R_2 in order to obtain a small-signal gain coefficient of 0.01 cm^{-1}?

(c) What is the saturation intensity for the $2 \rightarrow 1$ transition?



$A_{21}=6\times10^6 sec^{-1}$
$\tau_{21}=100$ ns
$\tau_{20}=200$ ns
Hint: $1eV=1.6\times10^{-19}$ J

$$\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}} = \frac{1}{100} + \frac{1}{200} = \frac{3}{200}$$

$$\Rightarrow \tau_2 = 67\ ns$$

a) $$\nu_o = \frac{(5.5-3.2)\cdot 1.6\times10^{-19}}{6.62\times10^{-34}} = 5.56\times10^{14}\ Hz$$

$$g(\nu_o,\nu_o) = \frac{\frac{2}{\pi\Delta\nu_L}}{1+\left[\frac{2(\nu_o-\nu_o)}{\Delta\nu_L}\right]^2} = \frac{2}{\pi\Delta\nu_L} = \frac{2}{\pi\cdot 10\times10^{9}\ Hz} = 6.4\times10^{-11}\ s$$

$$\sigma(\nu_o) = \frac{c^2 A_{21}}{8\pi\nu^2}\, g(\nu_o) = \frac{(3\times10^{10}\frac{cm}{s})^2 (6\times10^6 s^{-1})}{8\pi(5.56\times10^{14} Hz)^2} * (6.4\times10^{-11} s) = 4.4\times10^{-14}\ cm^2$$

c) $$N_2 - N_1 = \frac{\gamma}{\sigma} = \frac{0.01\ cm^{-1}}{4.4\times10^{-14} cm^2} = 2.27\times10^{11}\ cm^{-3}$$

$$N_2 - N_1 = \frac{R_2\tau_2 - \cancel{R_1\tau_1} - \cancel{R_2\tau_1\tau_2A_{21}}}{1+W\tau_2\left(1+\cancel{\frac{\tau_1}{\tau_2}} - \cancel{A_{21}\tau_1}\right)} = \frac{R_2\tau_2}{1+W\tau_2}\ ; \text{ where } W = \frac{A_{21}c^2 \cdot I_\nu \cdot g(\nu,\nu_o)}{8\pi h\nu^3}$$

$$I_s = \frac{8\pi h\nu^3}{c^2\phi\, g(\nu_o,\nu_o)} = \frac{8\pi\cdot(6.62\times10^{-34})(5.56\times10^{14})^3}{(3\times10^{10})^2\cdot(0.4)\cdot(6.4\times10^{-11})} = 124\ \frac{W}{cm^2} = 1\times10^2\ \frac{W}{cm^2}$$

$$\phi = A_{21}\tau_2\left[1+\cancel{(1-A_{21}\tau_2)\cdot\frac{\tau_1}{\tau_2}}\right] = (6\times10^6)(67\times10^{-9}) = 0.4$$

b) since we are speaking about a small signal gain $W=0$

$$\Rightarrow R_2\tau_2 = N_2 - N_1 \quad \Rightarrow R_2 = \frac{N_2-N_1}{\tau_2} = \frac{2.27\times10^{11}}{67\times10^{-9}} = 3.4\times10^{18} = 3\times10^{18}\ \frac{atoms}{s}$$

4. A homogeneously broadened laser transition at λ=10.6 μm (CO_2) has the following characteristics: A_{21}=0.34 s^{-1}; J_2=21; J_1=20; $\Delta\nu_h$=1GHz.

a) What is the stimulated emission cross section (gain coefficient/population inversion) at line center?

b) What must be the population inversion N_2-(g_2/g_1)N_1 to obtain a gain coefficient of 2 cm^{-1}?

c) If the lifetime of the upper state is 10μs and that of the lower state 0.1μs, what is the saturation intensity?

(Hint: For a simple atom degeneracy is related to the total angular momentum quantum number "J" by $g=2J+1$)

a) $$\gamma(\nu) = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^2 A_{21}}{8\pi\nu_0^2}\,\frac{2/\pi\Delta\nu}{1+[2(\underbrace{\nu_0-\nu_0}_{=0})/\Delta\nu]^2}$$

$$\sigma(\nu_0) = \frac{\gamma(\nu_0)}{\left(N_2 - \frac{g_2}{g_1}N_1\right)} = \frac{c^2A_{21}}{8\pi\nu_0^2}\cdot\frac{2}{\pi\Delta\nu} = \frac{(3\times10^8\,m/s)^2\cdot(0.34\,s^{-1})\cdot 2}{8\pi(2.83\times10^{13}\,Hz)^2\cdot\pi\cdot10^9\,Hz}$$

$$\left[\nu_0 = \frac{c}{\lambda_0} = \frac{3\times10^8\,m/s}{10.6\times10^{-6}\,m} = 2.83\times10^{13}\,Hz\right] = 9.7\times10^{-22}\,m^2 = 9.7\times10^{-18}\,cm^2$$

b) $$\left(N_2 - \frac{g_2}{g_1}N_1\right) = \frac{\gamma(\nu_0)}{\sigma(\nu_0)} = \frac{2\,cm^{-1}}{9.7\times10^{-18}\,cm^2} = 2\times10^{17}\,cm^{-3}$$

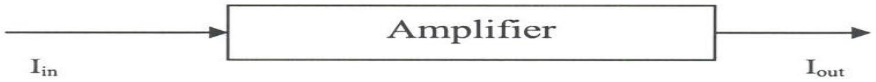
c) $$I_s(\nu_0) = \frac{8\pi h\nu^3}{c^2\phi\, g(\nu_0,\nu_0)} =$$

$$\left[\phi = A_{21}\tau_2\left[1+(1-A_{21}\tau_2)\frac{\tau_1}{\tau_2}\right] = (0.34\,s^{-1})(10\times10^{-6}\,s)\left[1+(1-0.34\times10\times10^{-6})\frac{0.1}{10}\right]\right.$$

$$\left. = 3.4\times10^{-6}\left[1+(1-3.4\times10^{-6})\,0.01\right] \approx 3.43\times10^{-6}\right]$$

$$= \frac{8\pi(6.62\times10^{-34}\,J\cdot s)\cdot(2.83\times10^{13}\,Hz)^3}{(3\times10^8\,\frac{m}{s})^2\cdot 3.43\cdot10^{-6}\cdot\frac{2}{\pi\cdot10^9}} = 1.92\times10^6\,\frac{W}{m^2}$$

5. An experiment involving a homogeneously broadened amplifier is depicted in the diagram below. For an input intensity of 1 W/cm^2, the gain (output/input) is 10 dB (i.e., $G_o(dB)=10\log[I_{out}/I_{in}]$). If the input intensity is doubled to 2W/cm^2, the gain is reduced to 9 dB.



(a) What is the small–signal gain (i.e. $I_{in}\rightarrow 0$) of this amplifier (in dB)?
(b) What is the saturation intensity?

- $10 = 10\log\frac{I_{out_1}}{1} \Rightarrow I_{out_1} = 10\,W$
- $G_{o2} = 9\,dB = 10\log\frac{I_{out_2}}{2} \Rightarrow I_{out_2} = 2\cdot(10^{0.9}) = 15.89\,W$
- Use $I_{out} = I_{in}\, e^{G_{ss} - \frac{I_{out}-I_{in}}{I_s}}$

$$\left.\begin{aligned} 10 &= 1\cdot e^{G_{ss} - \frac{10-1}{I_s}} \\ 15.89 &= 2e^{G_{ss} - \frac{15.89-2}{I_s}} \end{aligned}\right\} \rightarrow \left.\begin{aligned} 10 &= e^{G_{ss} - \frac{9}{I_s}} \\ 7.945 &= e^{G_{ss} - \frac{13.89}{I_s}} \end{aligned}\right\} \rightarrow$$

$$\left.\begin{aligned} 2.3 &= G_{ss} - \frac{9}{I_s} \quad (1) \\ -\;2.073 &= G_{ss} - \frac{13.89}{I_s} \quad (2) \end{aligned}\right\}$$

Subtract (2) from (1)

$$0.227 = \frac{4.89}{I_s} \Rightarrow I_s = 21.5\ W/cm^2$$

Substitute I_s in (1) $G_{ss} = 2.3 + \frac{9}{21.5} = 2.72$

Convert G_{ss} in dB

$e^{G_{ss}} = \frac{I_{out}}{I_{in}}$; $10\log\frac{I_{out}}{I_n} = G_{ss(dB)}$

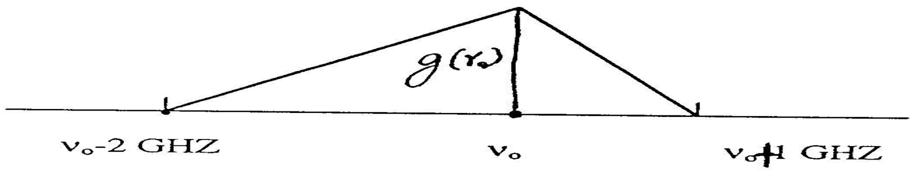
$\Rightarrow G_{ss(dB)} = 10\log e^{G_{ss}} = 10\, G_{ss}\log e = 11.9\,dB$

For a simple atom degeneracy is related to the total angular momentum quantum number "J" by $g = 2J+1$

6. The following question refer to an atomic system with $J_2=1$ and $J_1=2$.

(a) What is the ratio B_{12}/B_{21}?

(b) If the lineshape function could be approximated by the graph shown below, $A_{21}=10^6\ s^{-1}$, $\lambda=640.1$ nm and $N_2=N_1=10^{12}\ cm^{-3}$, what is the small-signal gain coefficient for the $2\rightarrow1$ transition at $\nu=\nu_o$?



a) $$\frac{B_{12}}{B_{21}} = \frac{g_2}{g_1} = \frac{(2J_2+1)}{(2J_1+1)} = \frac{2\cdot1+1}{2\cdot2+1} = \frac{3}{5} = 0.6$$

b) $$\gamma(\nu_0) = \left(N_2 - \frac{g_2}{g_1}N_1\right)\frac{c^2 A_{21}}{8\pi\nu_0^2}\, g(\nu_0,\nu)$$

1) $$N_2 - \frac{g_2}{g_1}N_1 = 10^{12} - 0.6\times10^{12} = 4\times10^{11}\ cm^{-3}$$

2) $$\int_{-\infty}^{+\infty} g(\nu_0,\nu)\,d\nu = 1$$

$$\int_{-\infty}^{+\infty} g(\nu_0,\nu)\,d\nu = \frac{1}{2}g(\nu_0)(\nu_0-\nu_0+2) + \frac{1}{2}g(\nu_0)(\nu_0+1-\nu_0) =$$

$$= \frac{1}{2}g(\nu_0)\cdot 3(GHz) = 1$$

$$\Rightarrow g(\nu_0) = \frac{2}{3}(GHz)^{-1} = \frac{2}{3\times10^9}\ s^{-1} = \frac{2}{3}\times10^{-9}$$

$$\gamma(\nu_0) = (4\times10^{11}\ cm^{-3})\cdot\frac{(640.1\times10^{-7}\ cm)^2\cdot 10^6\ s^{-1}}{8\pi}\times\frac{2}{3}\times10^{-9} =$$

$$= 4.3\times10^{-2}\ cm^{-1}$$