**Tentative Schedule:**

<table>
<thead>
<tr>
<th>Date</th>
<th>Place &amp; Time</th>
<th>Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Aug.24 (Mo) 394; 4:00-5:15</td>
<td>Introduction, Spontaneous and Stimulated Transitions (Ch. 1) – Lecture Notes</td>
<td></td>
</tr>
</tbody>
</table>
| 2 Aug.26 (We) 394; 4:00-5:15 | Spontaneous and Stimulated Transitions (Ch. 1) – Lecture Notes  
Homework 1: PH481 Ch.1 problems 1.4 & 1.6  
PH581 Ch.1 problems 1.4, 1.6 & 1.8 due Sep.2 before class |
| 3 Aug.31 (Mo) 394; 4:00-5:15 | Optical Frequency Amplifiers (Ch. 2.1-2.4) – Lecture Notes  
Problem solving for Ch.1 |
| 4 Sep.2 (We) 394; 4:00-5:15 | Optical Frequency Amplifiers (Ch. 2.5-2.10) – Lecture Notes  
Homework 2: PH481 Ch.2 problems 2.2 (a,b), 2.4 & 2.5 (a,b)  
PH581 Ch.2 problems 2.2 (a,b), 2.4 & 2.5 (a,b,c,d) due Sep.16 before class |
| Sep.7 (Mo) No classes | Labor Day Holiday |
| 5 Sep.9 (We) 394; 4:00-5:15 | Problem solving for Ch.2  
Introduction to two Practical Laser Systems  
(The Ruby Laser, The Helium Neon Laser) (Ch. 3) – Lecture Notes |
| 6 Sep.14 (Mo) 394; 4:00-5:15 | Review Chapters 1 & 2 – Lecture Notes |
| 7 Sep.16 (We) 394; 4:00-5:15 | Exam 1 Over Chapters 1-3; Grades for exam 1 |
| 8 Sep.21 (Mo) 394; 4:00-5:15 | Exam 1 problem solving. Passive Optical Resonators (Lecture notes) |
| 9 Sep.23 (We) 394; 4:00-5:15 | Passive Optical Resonators (Lecture notes). |
| 10 Sep.28 (Mo) 394; 4:00-5:15 | Passive Optical Resonators (Lecture notes). Physical significance of $\chi'$ and $\chi''$ (Ch.2.8-2.9), Homework 3: read Ch.2 & notes. Work out problems. Due Oct. 5 |
| 11 Sep.30 (We) 394; 4:00-5:15 | Optical Resonators Containing Amplifying Media (4.1-2). |
| 12 Oct. 5 (Mo) 394; 4:00-5:15 | Optical Resonators Containing Amplifying Media (Ch.4.3-4.7) Homework 4: Ch. 4 problems 4.7 and 4.9. Due Oct 12. |
| 13 Oct. 7 (We) 394; 4:00-5:15 | Laser Radiation (Ch. 5.1-5.4) |
| 14 Oct. 12 (Mo) 394; 4:00-5:15 | Control of Laser Oscillators (6.1-6.3) Homework 5: Ch. 5 problems 5.1 and 5.5. Due Oct 19. |
| 15 Oct. 14 (We) 394; 4:00-5:15 | Control of Laser Oscillators (6.4-6.5) and exam 2 review |
| 16 Oct. 19 (Mo) 394; 4:00-5:15 | Optically Pumped Solid State Lasers (7.1-7.11) |
| 17 Oct. 21 (We) 394; 4:00-5:15 | Optically Pumped Solid State Lasers (7.1-7.11) |
| 18 Oct. 26 (Mo) 394; 4:00-5:15 | Exam 2 Over Chapters 4-6 Grades for exam 2  
Exam 2 correct solution; Homework 6 Due Nov. 2; Article on Cr: CdSe |
| 19 Oct. 28 (We) 394; 4:00-5:15 | Optically Pumped Solid State Lasers (8.15-8.16) |
| 20 Nov. 2 (Mo) 394; 4:00-5:15 | Optically Pumped Solid State Lasers (8.15-8.16) Supplemental material for Homework 6–diode pumped LiF:F$^*$ laser/ Homework 7 Due Nov.9 |
| 21 Nov. 4 (We) 394; 4:00-5:15 | Optically Pumped Solid State Lasers (8.15-8.16) |
| 22 Nov. 9 (Mo) 394; 4:00-5:15 | Spectroscopy of Common Lasers and Gas Lasers (Ch. 9.1-9.10 and class material) |
| 23 Nov. 11 (We) 394; 4:00-5:15 | Gas lasers (Ch. 9.4 -9.10); Molecular Gas lasers I (Ch. 10.1-10.5) |
| 24 Nov. 16 (Mo) 394; 4:00-5:15 | Molecular Gas lasers I (Ch. 10.1-10.5) Homework 8 Due Nov. 30 |
| 25 Nov. 18 (We) 394; 4:00-5:15 | Molecular Gas Lasers II (Ch. 11.1-11.8) and review for exam 3 (Ch. 11.1-11.8) Homework 9 Due Dec 2 |
| Nov.23 (Mo) No classes | Thanksgiving - no classes held |
| Nov.25 (We) No classes | Thanksgiving - no classes held |
| 26 Nov. 30 (Mo) 394; 4:00-5:15 | Exam 3 Over Chapters 8-11 Grades; Exam 3 Correct solution |
| 27 Dec. 2 (We) 394; 4:00-5:15 | Review for Final |
| 28 Dec. 9 (Wed) in CH 394 | FINAL EXAM Over Chapters 1-11 (4:15-6:45pm) in CH 394 Final Grades |
Laser Physics I

PH481/581-VT (Mirov)

Optically Pumped Solid State Lasers

Lectures 16,17 chapter 7

Fall 2015

C. Davis, “Lasers and Electro-optics”
Optically Pumped Solid State Lasers

- Operating principles, characteristics & design features
- Characteristics of the radiation emitted by such lasers and how the radiation can be modified and controlled in time.

**Optical Pumping in Three and Four-Level Lasers**

- Light from the pumping lamps excites ground state particles into an absorption band.
- Particles from the state 3 should transfer rapidly into the upper laser level 2.
- Population inversion will result between levels 2 and 1 and laser action can be obtained.
- The drain transition from level 1 back to the ground state should be fast.

![Diagram of energy levels and transitions](image)

**Effective lifetime of the levels involved**

- The length of time a particle can remain in an excited level is governed by its effective lifetime, which is influenced by both radiative and nonradiative processes.
- 3 → 2 nonradiative. Particles dump their excess energy into the lattice and heat up the medium.
- A nonradiative process from level $i$ to level $j$ can be described by a rate coefficient $X_{ij}$, similar to spontaneous emission coefficient $A_{ij}$.
- The overall rate at which particles leave level $i$ is:

$$\frac{dN_i}{dt} = -\sum_j N_i N_j A_{ij} - \sum_j N_i X_{ij}$$

- The effective lifetime of level $i$ is:

$$\tau_i = \frac{1}{\sum_j (A_{ij} + X_{ij})}$$

### Threshold Inversion in Three & Four-Level Lasers

- If the lower laser level is very close to the ground state ($E_i \ll kT$), or is the ground state, the system is a three-level laser.

- If $E_i \gg kT$ - Four level laser.

- If the total # of particles per unit volume participating in pumping & lasing process is $N$

$$N = N_3 + N_2 + N_1 + N_{gs}$$

- For a three-level laser, level 2 is essentially the ground state so

$$N = N_3 + N_2 + N_1$$

Since level 3 transfers its excitation to level 2 rapidly and pseudo, $N_3 = N_2 - \frac{g_3}{g_2} N_1$.

$$N = N_2 + N_1$$

$N_4 = N_2 - \frac{g_2}{g_1} N_1$

$$(N_2)_3 = \frac{g_2 (g_2 + 1) N + N_2}{(g_3 + 1)} \quad \text{if} \; g_2 = g_1$$
• In a good 4-level laser in which level 1 remains depopulated, 
  \[ N = N_2 + N_1, \]
  \[ N_2 = N_2 - \frac{2}{3} N. \]

• The relative rate at which level 2 must be excited to produce an inversion
  \[ \frac{(N_2)_{3e}}{(N_2)_{4e}} = \frac{(2/3)N + N_4}{(N_2)_{4e}} \]
  \[ \approx \frac{N}{2N_4}, \]
  Usually \( N >> N_4. \) (For \( \gamma_3 = 3, \))

  **Quantum Efficiency**

• If the average energy of photons from the pumping lamp is \( h\nu \text{pump} \) and the laser photon is \( h\nu, \) the intrinsic quantum efficiency of the pumping process is \( \eta = \frac{h\nu}{h\nu \text{pump}}. \)

  **Pumping Power**

• The rate at which particles must be excited to the upper laser level to sustain a population inversion is
  \[ R_2 = \frac{N_2}{\gamma_2} \] (particles, \( m^{-3} s^{-1} \))

• If the average probability factor for a particle in level 3 transferring to level 2 is \( D \) (branching factor), then the rate at which level 3 should be pumped is corresponding abs. power
  \[ R_3 = \frac{N_3}{\gamma_3} \]
  \[ P_4 = \frac{N_4}{\gamma_4 \text{pump}}. \]
Threshold Lamp Power

To determine the power and spectral characteristics of the lamps needed to create an inversion, we must relate the threshold pumping rate \( R_2 \) to the absorption coefficient in the pump band, and electrical and geometrical factors that determine how efficiently the lamp generates pump light and couples this into the medium.

- The abs. band has a lineshape function \( g_3 (v) \).
  - If the energy density of the pump radiation is \( S_\nu (\nu) \), then the rate at which level 3 is excited is
    \[
    R_3 = \int N_0 B_0 g_3 (\nu) S_\nu (\nu) \, d\nu
    \]
  - If we assume plane wave illumination \( S_\nu (\nu) = I_\nu (\nu)/c \), then
    \[
    R_3 = \int N_0 \frac{C^2 A_2 \lambda_0}{8\pi \nu^2} \frac{I_\nu (\nu) g_3 (\nu)}{h\nu} \, d\nu
    \]

- Absorption coefficient of the laser medium \( \chi (\nu) \Rightarrow \)
  \[
  R_3 = \int \frac{I_\nu (\nu) \chi (\nu)}{h\nu} \, d\nu
  \]
- \( R_2 = \int \frac{I_\nu (\nu) \chi (\nu) P_\nu (\nu)}{h\nu} \, d\nu \) - if there is a possibility of a frequency dependent factor.

- If the abs. band is narrow, of width \( \Delta \nu \)
  \[
  R_2 = \bar{I} (\nu) \bar{\chi} (\nu) \bar{P} (\nu) \Delta \nu / h\nu_{\text{pump}}
  \]
Pulsed Versus CW Operation

- If the pumping lamp is a flashlamp of duration \( t_p << t_2 \) then at the end of the flash all the excited particles will still be in level 2.

\[
N_2 = t_p \frac{I(x)}{2(y)} \frac{\sigma(x)}{\Delta \nu} \Delta \nu \frac{1}{4} V_{pump} \]

- The flashlamp energy needed to achieve \( N_2 \) depends on the following factors:
  1) Electrical efficiency \( e \) of the lamp

\[
e = \frac{\text{joules of light energy out}}{\text{capacitor joules in}} \approx \text{ten of } \%
\]

  2) The fraction \( f \) of this light in the spectral region that will pump the abs. band

\[
f = \frac{\text{light energy within the abs. band}}{\text{total light energy}}
\]

  3) Geometrical efficiency \( g \) with which the light is coupled to the laser medium

\[
g = \frac{\text{light energy within abs. band reaching laser med.}}{\text{total light energy within abs. band}}
\]

- The total energy that must reach the laser crystal in the right spectral region to reach threshold can be written as

\[
N_4 = t_p \frac{I(x)}{2(y)} \frac{\sigma(x)}{\Delta \nu} \Delta \nu \frac{1}{4} V_{pump}
\]

\[
\frac{N_2}{t_p} = \frac{I(x)}{2(y)} \frac{\sigma(x)}{\Delta \nu} \Delta \nu \frac{1}{4} V_{pump}
\]
Threshold for Pulsed Operation of a Ruby Laser

- Typical Cr³⁺ concentration is ~ 0.05% by weight, equivalent to ~ 10²⁵ Cr³⁺/m³.
- 3-level system ⇒ \( N_2 \approx \frac{N}{2} = 5 \times 10^{-24} \text{ m}^{-3} \)
- \( \lambda (\nu) \) in 350–600nm is ~ 100m⁻¹.
- Assume efficient transfer \( Q(\nu) = 1 \).
- Average pump wavelength is ~ 475nm.

![Graph showing emission spectrum of a ruby crystal, with peaks at 694 and 695nm.]

\[
U_t = \frac{N_2 \cdot h \nu_{\text{pump}}}{\int \lambda (\nu) Q(\nu) \, d\nu} = \frac{5 \times 10^{-24} \times 6.66 \times 10^{-34} \times 3.10^8}{100 \times 4.75 \times 10^{-9}} = 21 \text{ kJ/m}^2
\]

- For a cylindrical ruby crystal 10 x 20 mm
- If the lamp energy reaches the crystal uniformly, the threshold input energy is ~ 13 J.
- Electrical efficiency \( \epsilon = 50\% \)
• The spectral output of the lamp will approximate a black body with superimposed spectral features. 

~25% of the emitted light is in the ruby pump band.

Fig. 8.2. Examples of output spectra from various flashlamps at different flashlamp current densities; $f_\text{w}$ is the fraction of explosion energy at which each lamp was operated: (a) 1.3 cm-bore lamp; (b) 4.2 cm-bore lamp.[8.5]

Fig. 8.3. Ellipsoidal cavity for solid-state laser pumping that provides axi-symmetric illumination of a cylindrical laser rod by a cylindrical lamp.[8.5][8.4]

• The electrical input to reach the threshold is

$$\frac{U_t}{e_\text{f.g.}} = \frac{13}{0.5 \cdot 0.25 \cdot 0.5} = 208 \text{W}$$

• To achieve CW operation the pumping rate of the upper level should be sufficient to maintain $N_2/N_1$ in the face of all the spontaneous relaxation processes.

$$P = \frac{N_0 \cdot h \cdot \nu_{\text{pump}}}{\sqrt{2 \cdot \pi \cdot \gamma \cdot (V)}}$$ — the threshold absorbed power per unit volume.
Threshold for CW Operation of a Ruby laser

\[
P = \frac{N_4 \hbar \nu_{pump}}{\tau_2 \bar{E}(\nu)} = \frac{5 \times 10^{-24} \times 6.6 \times 10^{-3} \times 2 \times 10^8}{3 \times 10^{-3} \times 4 \times 10^5 \times 10^{-9}} \approx 7 \times 10^8 \text{ W/m}^3
\]

Threshold population inversion and stimulated emission cross section

The small signal gain \( g \) at the center of the line can be written in the form

\[
g_0 = (N_2 - \frac{\nu_2}{\nu_1} N_1) \delta_0 = (N_2 - \frac{\nu_2}{\nu_1} N_1) \frac{k^2 A_21}{8 \pi r^2} \delta(\nu_0, \nu)
\]

\[\delta_0 = \frac{C^2 A_21}{4 \pi^2 V^2 \Delta \nu}\]

\[C_0 = \frac{1}{4 \pi^2 V^2 \Delta \nu}\]

for a homogeneously broadened line \( g(\nu_0, \nu) = \frac{2}{\pi \Delta \nu} \)

\[
N_4 = (N_2 - \frac{\nu_2}{\nu_1} N_1) = \frac{1}{\delta_0} \left( 2 - \frac{1}{2} C_0 \nu_2 \nu_1^2 \right)
\]

\[
\delta_0 = \frac{C^2 A_21}{8 \pi V^2} \frac{k^2 A_21}{\Delta \nu}
\]

\[\delta_0 = \frac{C^2 A_21}{8 \pi V^2} \frac{\frac{2}{\pi \Delta \nu}}{1 + \frac{(\nu - \nu_0)^2}{\Delta \nu}^2}\]
Paramagnetic Ion Solid State lasers

- a large # of paramagnetic ions from the iron, rare-earth, and actinide groups of the periodic table exhibit laser action when doped into a large # of host crystals or glasses.

- Nd\(^{3+}\) lasers
  - \(Y_2Al_5O_{12}\) (YAG) yttrium aluminum garnet
  - CaWO\(_4\), calcium tungstate
  - LiYF\(_4\) (YLF), lithium yttrium fluoride
  - \(YAlO_3\) (YALO), yttrium aluminum oxide
  - GGG, gadolinium gallium garnet
  - GSAG, gadolinium scandium gallium garnet
  - glasses

- Ho\(^{3+}\) lasers
- Er\(^{3+}\) lasers
- Cr\(^{3+}\) lasers (BeAl\(_2\)O\(_4\)), (LiCAF), (LiSAF)
- Ti\(^{3+}\) lasers (Al\(_2\)O\(_3\))
Edge Dislocations

Screw Dislocations

LINE IMPERFECTIONS

DEFECTS CLASSIFICATION

POINT DEFECTS

Impurity-Vacancy Dipoles

Impurity

Cation-Vacancy
\( F_A; F_B; (F_2^+) A; \)
\( \text{Me}^+ - F; \text{Me}^{2+} - F_2^-; \)
\( \text{Zcenters}: \)
\( F - \text{Me}^{2+} V_c^- \)

Anion-Vacancy
\( O^- V_a^+ \)

Intrinsic defects

Anion Vac.

Cation Vac.

Electron centers
(Color Centers)

Hole Trapping Centers

 Aggregate
\( F_2; F_2^+; F_2^-; F_3; F_3^+; F_3^-; F_4 \)

Simple
\( F \)

Anion
\( \text{OH}^-; O_2^- \)

Cation
\( \text{Cr}^{3+}; \text{Cr}^{4+}; \text{Ti}^{3+}; \text{Co}^{2+}; \text{Ni}^{2+}; \text{V}^{2+}; \text{Nd}^{3+}; \text{Er}^{3+}; \text{Ho}^{3+} \)

Cation, perturbed by ionizing treatment
\( \text{Sm}^{3+} \rightarrow \text{Sm}^{2+}; \text{Nd}^{3+} \rightarrow \text{Nd}^{2+}; \text{Ag}^+ \rightarrow \text{Ag}^0 \rightarrow \text{Ag}^-; \text{Sc}^{3+} \rightarrow \text{Sc}^{2+} \)
Table 1 Main physicochemical, mechanical and optical characteristics of the most promising crystal hosts with CCs

<table>
<thead>
<tr>
<th>Crystal</th>
<th>(\rho) (g/cm(^3))</th>
<th>(d) (Å)</th>
<th>(H) (kg/mm(^2))</th>
<th>Solubility (g/100g H(_2)O)</th>
<th>(T_m) (°C)</th>
<th>(K) (W/m °C)</th>
<th>(E) (kg/mm(^2))</th>
<th>(S_T) (kg/mm(^2))</th>
<th>(R_T^*) (W/m)</th>
<th>(n)</th>
<th>(n_2)</th>
<th>(dn/dT) (10(^{-22}) m(^2)/V(^2))</th>
<th>(dn/dT) (10(^{-5})/°C)</th>
<th>Range of a transparency (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiF</td>
<td>2.60</td>
<td>4.03</td>
<td>99.0</td>
<td>0.1</td>
<td>870.0</td>
<td>14.2</td>
<td>32.0</td>
<td>8820.0</td>
<td>0.28</td>
<td>1.38</td>
<td>2.7</td>
<td>–</td>
<td>–</td>
<td>0.1–7.0</td>
</tr>
<tr>
<td>NaF</td>
<td>2.79</td>
<td>4.63</td>
<td>60.0</td>
<td>4.2</td>
<td>992.0</td>
<td>9.2</td>
<td>33.0</td>
<td>8780.0</td>
<td>0.19</td>
<td>–</td>
<td>–</td>
<td>1.32</td>
<td>1.0</td>
<td>0.2–14.0</td>
</tr>
<tr>
<td>NaCl</td>
<td>2.17</td>
<td>5.64</td>
<td>15.0</td>
<td>36.0</td>
<td>801.0</td>
<td>6.3</td>
<td>39.2</td>
<td>4360.0</td>
<td>0.20</td>
<td>–</td>
<td>–</td>
<td>1.53</td>
<td>7.2</td>
<td>0.2–16.0</td>
</tr>
<tr>
<td>KF</td>
<td>2.50</td>
<td>5.35</td>
<td>–</td>
<td>94.9</td>
<td>857.0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>KCl</td>
<td>1.99</td>
<td>6.29</td>
<td>7.0</td>
<td>36.0</td>
<td>776.0</td>
<td>6.9</td>
<td>37.4</td>
<td>3810.0</td>
<td>0.14</td>
<td>–</td>
<td>–</td>
<td>1.18</td>
<td>3.7</td>
<td>0.2–20.0</td>
</tr>
<tr>
<td>KBr</td>
<td>2.75</td>
<td>6.60</td>
<td>8</td>
<td>68.1</td>
<td>730.0</td>
<td>2.9</td>
<td>37.6</td>
<td>3290.0</td>
<td>0.32</td>
<td>–</td>
<td>–</td>
<td>1.54</td>
<td>16.0</td>
<td>0.2–25.0</td>
</tr>
<tr>
<td>KI</td>
<td>3.12</td>
<td>7.07</td>
<td>–</td>
<td>144.0</td>
<td>686.0</td>
<td>2.1</td>
<td>41.7</td>
<td>2550.0</td>
<td>0.14</td>
<td>–</td>
<td>–</td>
<td>1.64</td>
<td>–</td>
<td>0.3–35.0</td>
</tr>
<tr>
<td>RbCl</td>
<td>2.76</td>
<td>6.58</td>
<td>–</td>
<td>94.2</td>
<td>717.0</td>
<td>32.8</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>MgF(_2)</td>
<td>3.13</td>
<td>4.64</td>
<td>576.0</td>
<td>&lt;0.1</td>
<td>1213.0</td>
<td>3.1</td>
<td>8.8</td>
<td>17,625.0</td>
<td>0.27</td>
<td>5.4</td>
<td>470</td>
<td>13.1</td>
<td>–</td>
<td>0.1–7.0</td>
</tr>
<tr>
<td>CaF(_2)</td>
<td>3.18</td>
<td>5.46</td>
<td>158.0</td>
<td>&lt;0.1</td>
<td>1400.0</td>
<td>9.7</td>
<td>19.5</td>
<td>14,000.0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.43</td>
<td>3.1</td>
<td>0.1–9.0</td>
</tr>
<tr>
<td>SrF(_2)</td>
<td>4.24</td>
<td>5.79</td>
<td>144.0</td>
<td>&lt;0.1</td>
<td>1190.0</td>
<td>9.6</td>
<td>19.6</td>
<td>3290.0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.00</td>
<td>–</td>
<td>0.1–10.0</td>
</tr>
<tr>
<td>(\alpha)-Al(_2)O(_3)</td>
<td>3.96</td>
<td>a. 4.76</td>
<td>2100.0</td>
<td>0</td>
<td>2050.0</td>
<td>35.0</td>
<td>5.3c</td>
<td>35,230.0</td>
<td>0.27</td>
<td>55.0</td>
<td>10,000</td>
<td>1.76</td>
<td>1.5</td>
<td>0.2–6.5</td>
</tr>
<tr>
<td>Diamond</td>
<td>3.51</td>
<td>3.57</td>
<td>8820.0</td>
<td>0</td>
<td>3500.0</td>
<td>55.0</td>
<td>0.9</td>
<td>58,000.0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>2.40</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Y(_3)Al(_5)O(_12)</td>
<td>4.55</td>
<td>12.01</td>
<td>1380.0</td>
<td>0</td>
<td>1930.0</td>
<td>13.0</td>
<td>7.0</td>
<td>31,725.0</td>
<td>–</td>
<td>20.0</td>
<td>790</td>
<td>1.82</td>
<td>3.9</td>
<td>0.3–5.5</td>
</tr>
<tr>
<td>ED-2 glass</td>
<td>2.539</td>
<td>–</td>
<td>–</td>
<td>582.0</td>
<td>1.4</td>
<td>8.0</td>
<td>0.24</td>
<td>9190.0</td>
<td>0.24</td>
<td>14.0</td>
<td>10.0</td>
<td>1.56</td>
<td>1.7</td>
<td>0.3–5.0</td>
</tr>
</tbody>
</table>


* The thermal shock parameter, \(R\), characterizes the thermo-mechanical strength of the material. For instance, for a plate element with a thickness \(t\), thermal power \(P_v\), which may be absorbed by the medium with a parameter \(R_T\) without being damaged, \(P_v = 12R_T/t^2\), where \(R_T = S_T(1 - \mu)k/\alpha E\) [49]. Symbols: \(\rho\), density (g/cm\(^3\)); \(d\), lattice constant (Å); \(H\), Knupp hardness (kg/mm\(^2\)); \(T_m\), melting temperature (°C); \(K\), coefficient of thermal conductivity (W/m °C); \(\alpha\), coefficient of linear expansion (10\(^{-6}\)/°C); \(E\), Young’s modulus (kg/mm\(^2\)); \(\mu\), Poisson’s coefficient; \(S\), compression (tension) or bending strength (kg/mm\(^2\)); \(R\), thermal shock parameter (W/m); \(n\), refractive index; \(n_2\), nonlinear index (m\(^2\)/V\(^2\)); \(dn/dT\), temperature derivative refractive index (10\(^{-5}\)/°C).
In the rare earths, the active electron participating in the optical transition is one of the 4f electrons that is shielded by electrons in the larger N = 5 and 6 shells (or orbits).

One of the consequences - the energy levels of rare earths are only weakly dependent on the host lattice.

The characteristics of the quantum states are usually specified by the spectroscopic name according to the following scheme.

Superscript = number

\[ \text{letter} \]

Subscript = number

where the letter symbol indicates the orbital angular momentum quantum number \( \ell \) according to the following prescription:

<table>
<thead>
<tr>
<th>letter</th>
<th>S</th>
<th>P</th>
<th>D</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>etc</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ell )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>etc</td>
</tr>
</tbody>
</table>

- the numerical value of the superscript = \( 2S+1 \)
- the numerical value of the subscript = \( \ell \)

(total angular momentum quantum number)
A level such as \( ^4I_{9/2} \) (the ground state of \( \text{Nd} \)) is referred to as “quarter-I-spin-values” and from its name we immediately know that the degeneracy of that level is \( g = 2 \cdot \frac{1}{2} + 1 = 2 \).

\( \text{Nd}^{3+} \) ions experience the local electric field generated by the host lattice and thus these levels are shifted by the electric field at the site where the \( \text{Nd} \) ions are. Since the Stark effect is bilinear, a positive and negative \( v \) to \( 1 \cdot \mu \), not its direction, a positive and negative electric field yield the same shift for \( \text{Nd}: \text{YAG} \). The lone manifold splits into \( (2 \cdot \frac{1}{2})^2 \) distinct doubly degenerate levels. 

Semiconductor laser pumping route of \( \text{Nd}: \text{YAG} \)
Thresholds for CW and Pulsed Operation of Flashlamp Pumped Nd Glass Lasers

\[ \Delta r = 200 \text{ cm}^{-1} \]
\[ n = 1.5 \]
\[ t_{\text{spont}} \approx t_2 = 3 \times 10^{-5} \]
\[ L = \text{length of the cavity} = 20 \text{ cm} \]
\[ \tau = \text{loss per pass} = 2 \% \]
\[ \tau \approx \frac{4e}{c} \approx 5 \times 10^{-8} \]

1) Finally, \( N_t \):
\[ N_t = \frac{\Delta r}{c} - \frac{8 \pi n^2 t_{\text{spont}}}{\sigma V} \left( \frac{1}{2} \right) \frac{1}{2} \]
\[ \frac{1}{\tau} = \frac{1}{2} e^{-2 \pi t_{\text{spont}}} \]

Since \( \frac{1}{2} e^{-2 \pi t_{\text{spont}}} \approx 1 \)
we can use the relation \( 1 - x \approx -\tau x \), \( x \approx 1 \)
to write
\[ \frac{1}{\tau} = \frac{1}{2} \left( \frac{1}{2} \right) \frac{1}{2} \left( \frac{1}{2} \right) \]

\[ \Rightarrow N_t = \frac{8 \pi n^2 V^2 t_{\text{spont}}}{c^2 \sigma V} \approx \frac{8 \pi t_{\text{spont}} V^3}{c \tau \sigma} \approx 9 \times 10^{15} \text{ atoms/cm}^3 \]

2) The power at threshold:
\[ P = \frac{N_t \hbar V}{t_{\text{spont}}} = 5.65 \text{ W} \text{ in a crystal V}=1\text{cm}^3 \]

3) Assume 4) that only 10% of the pump light lies within the useful abs. bands within the useful abs. bands is absorbed.
5) 10% of the energy within the abs. bands is absorbed.
6) \( 2 \gamma e = 40 \% \)
The lamp output at threshold is

\[
\frac{2 \times 5.65}{0.1 \times 0.1 \times 0.4} = 2825 \text{ W}
\]

if the efficiency of the lamp in converting electrical to optical energy is 50%.

\( \Rightarrow \) CW operation requires \( \approx 5.5 \text{ kW} \) of power.

(100 kW for Nd:YAG)

Pulse operation:

\( L = 20\% \)

\( \Rightarrow \) \( N_L = \frac{8\pi \times \text{spectral \ intensity} \times \lambda^3 \text{A} \lambda}{C \times \left( \frac{\lambda}{2} \right)^2} = 9 \times 10^{16} \text{ atoms/cm}^3 \)

Energy to pump \( N_L \) atoms into level 2 is

\[ \frac{E_{\text{in}}}{V} = N_L (h\nu) = 1.7 \times 10^{-2} \text{ J/cm}^3 \]

Assuming a crystal volume \( V = 10 \text{ cm}^3 \) and the same eff. factor:

\( V \) to the flash lamp: \( \approx \frac{2.1 \times 10^{-2} \times 10}{0.1 \times 0.1 \times 0.4} = 52 \)
The Nd:YAG Laser

- Nd:YAG has a combination of desirable properties as a host medium for Nd³⁺ ions:
  - High thermal conductivity
  - High mechanical strength
  - Can be grown as crystals of large size with good optical properties (quality).
- Nd³⁺ ions subl. 1e⁻ in a single site ⇒ emission & absorption lines are homogeneously broadened.
- Doping densities range up to 1%.

Energy level diagram for Nd³⁺ ions in YAG:

- Primary pumping process is absorption of lamp energy from the ground state 4F⁵/₂ into the 4F⁷/₂.
- Transfer to the upper laser level 4I⁵/₂.
- 4I⁵/₂ consists of a set of closely spaced levels R₂ & R₁ serve as upper levels for trans. C₂ & C₁.
  - \( \lambda_2 = 1.06415 \) μm strong
  - \( \lambda_1 = 1.0646 \) μm
  - \( \Delta \gamma \approx 2 \times 10^{-9} \) Hz
- Gain near 1.06415 μm.

Gain curve with asymmetric lineshape.
Effective Spontaneous Emission Coefficient

at ambient T the populations of \( R_2 \) and \( R_1 \) are in thermal equilibrium

\[
\frac{N_{R_2}}{N_{R_1}} = \frac{g_{R_2}}{g_{R_1}} = \frac{2}{3}
\]

the effective spontaneous emission coefficient \( A_{21} \) is related to the coefficients for \( E_2 \) and \( E_1 \) according to

\[
A_{21} = A_{E_2} \frac{N_{E_1} + N_{E_2}}{N_{E_1}}
\]

\( A_2 = 14405 \text{ s}^{-1} \);
\( A_1 = 2505 \text{ s}^{-1} \);
\( A_{21} = 1815 \text{ s}^{-1} \)

\[
\sigma_0 = \frac{c^2 A_{21}}{4\pi^2 \nu^2} \approx 9 \times 10^{-23} \text{ m}^2
\]

Threshold Pump Energy of a Pulsed Nd:YAG Laser

- YAG 5 x 50 mm.
- \( R_1 = 100 \% \); \( R_2 = 92 \% \)
- \( d \approx 0 \) for a good crystal
- threshold gain \( \Delta \nu = d - \Delta \nu_{\text{gain}} = 0.833 \text{ m}^{-1} \)
- \( N_2 = \left( \frac{\nu_2}{\nu_1} \right)^2 \left( \frac{g_{E_2}}{g_{E_1}} \right) = \frac{1}{2} \left( d - \Delta \nu_{\text{gain}} \right) = \frac{0.833 \text{ m}^{-1}}{9 \times 10^{-23} \text{ m}^2} \approx 9 \times 10^{-21} \text{ m}^{-1} \)
- it is \( \sim 500 \) times smaller than for ruby
- absorbed energy \( \Delta E = \frac{N_2 h \nu_{\text{pump}}}{\xi} = \frac{9.3 \times 10^{21} \text{ m}^{-1} \times 6.626 \times 10^{-34} \text{ J} \cdot \text{m} \cdot \text{s}^{-1}}{1} \approx 2.25 \times 10^3 \text{ J}^{-1} \)
- for any crystal \( \Delta E \times V = 2.24 \text{ mJ} \)
- electrical energy \( \sim 15 \text{ mJ} \)
- output up to \( 50 \text{ mJ} \)
Diode-pumped Solid State Lasers

1. The development of efficient, high power semiconductor lasers using GaAlAs operating near 800 nm has created a new class of optically pumped lasers in which a crystalline laser is optically pumped by a semiconductor laser.

2. Coincidence of output wavelengths at ~809 nm and the absorption of Nd\textsuperscript{3+} lasers at ~809 nm provides a very high efficiency of the system (over 10%).

3. If the diode laser pump radiation is injected along the axis and matched to the transverse mode geometry of the solid state laser, then very stable, narrow linewidth laser oscillation can be obtained.


5. Faraday rotation within the laser crystal produced by integral permanent magnet, leads to unidirectional oscillation and single longitudinal mode oscillation.

6. Faraday rotation takes place along the paths AB and DA.
Nd:YAG Pumped by a Semiconductor Laser

1) Oscillation is constrained to the counterclockwise direction at $\lambda \approx 1.064 \mu m$. $M_1$ ($R_{808} = 0$; $R_{1064} = 100$)

2) CW operation and hence $\frac{d \psi}{dt} = 0$

3) Ignore the significant problem of optimizing the overlap between the pump beam (at $\lambda = 808 \mu m$) and the laser (at $\lambda = 1.064 \mu m$)

4) YAG crystal is long enough so that pump intensity at $z = l_2$ is very small to that at $z = 0$.

5) The pumping will be to one or more levels of the $4F_{7/2}$ or $4H_{15/2}$ manifold from which a fast relaxation to the $4F_{9/2}$ takes place.

To avoid a separate rate equation define $\eta$, the pumping efficiency, or branching factor, also called quantum yield, as being the fraction $\psi(\leq 1)$ of the atoms promoted to the $4F_{9/2}$ or $4H_{15/2}$ manifolds that relax to the $4F_{9/2}$ group.
6) the lifetime of the \( ^4I_{11/2} \) manifold is so short (30ns) compared to the \( ^4F_{3/2} \) (255us) that we set any changes in \( N_{11/2} \) to \( 0 \).

7) There are two laser intensities, frequencies, wavelengths, stimulated emission cross sections, and saturated intensities. We will use the letters \( P \) and \( L \) to distinguish between them.

8) Assume all states have the same degeneracy \( g = 2 \). Ignore the small losses associated with scattering from crystal imperfections.

Our goal is to predict the intensity and the laser wavelength in the vicinity of 1.06um of the output. There are two closely spaced transitions which fit this specification. They are so close that the gain on one will contribute to the other.
The states contributing to the gain at 1.064 \mu m and the overlapping line shapes.

The composite line shape includes the fraction of the density in the 4F_{9/2} manifold in each state and maximizes at \( -1.23 \text{ cm}^{-1} \) from the center of the \( A' \) transition and \( 0.94 \text{ cm}^{-1} \) from that of the A.

- If we assume that the population of \( {}^1I_{1/2} \) is negligible then the gain coefficient is given by

\[
\mathcal{J}(r) = N_{2e} \sigma_{A'} J_{A'}(r) + N_{2a} \sigma_{A} J_{A}(r)
\]

where \( N_{2a} + N_{2e} = \left[ \left( 4F_{3/2} \right) \right] \) - the population density of the upper state manifold.

- There is a fast thermal process maintaining a Boltzmann distribution in the \( 4F_{3/2} \) states.

\[
\frac{N_{2e}}{N_{2a}} = \exp \left[ -\frac{\Delta E}{kT} \right] = 0.668 \text{ (at 300K)}
\]

\[
\left[ kT = 208 \text{ cm}^{-1} \right. \text{, } \Delta E = 11.507 - 11.423 = 84 \text{ cm}^{-1} \]

• gain coefficient in terms of the $[^4F_{3/2}]$ density:

$$f_a = \frac{1}{1 + \exp (-\frac{\Delta E}{kT})} = 0.6 \quad f_b = 1 - f_a = 0.4 \quad \text{for } kT = 208 \text{ cm}^{-1}$$

where $f_a, f_b$ is fraction of $[^4F_{3/2}]$ in $a$ or $b$.

• normalized combined line shape function has linewidth $\approx 6.8 \text{ cm}^{-1}$ and $\Gamma_{\text{eff}} = 1.6 \times 10^{-19} \text{ cm}^2$ at $\lambda = 1.0642 \mu\text{m}$.

• Another "new" issue arising in the YAG system is the use of $[^4F_{3/2}]$ population by stimulated emission. The rate equations for the $(a, b)$ levels are

$$\frac{dN_{2a}}{dt} = R_{2a} - \frac{N_{2a}}{\tau_a} - N_{2a} \sigma_a g_a(v) \frac{I_v}{h \nu} - \Gamma a N_{2a} + \Gamma b N_{2b}$$

$$\frac{dN_{2b}}{dt} = R_{2b} - \frac{N_{2b}}{\tau_b} - N_{2b} \sigma_b g_b(v) \frac{I_v}{h \nu} + \epsilon b N_{2a} - \epsilon a N_{2a}$$

where $\Gamma_a, \Gamma_b$ represent the fast intermanifolds relaxation process.

$$\sigma_a = \frac{1}{B_a \nu^2} \int f_a(x) \nu^2 dx$$

$$W_{2i} = B_{2i} g(x) \sigma_a \frac{I_v}{h \nu}$$

where $B_{2i}$, $g(x)$ represent the line shapes.
if rates $2ab$ and $2ca$ are fast, much faster than the natural decay rate $\frac{1}{\tau_a}$, or the stimulated rate $\frac{h\nu}{kT}$, or the sum, then the population ratio $\frac{N_{a}}{N_{a}} = \exp\left[\frac{-AE}{kT}\right]$ even when lasering, and the stimulated emission can use the population in both states.

if we add eq. $\frac{dN_{a}}{dt} = ...$ and $\frac{dN_{a}}{dt} = ...$, the sum $N_{2a} + N_{a} = \left[ \frac{dF_{3/2}}{dt} \right]$ becomes a rate equation for the entire manifold.

$$\frac{d \left[ \frac{dF_{3/2}}{dt} \right]}{dt} = R_2 - \frac{\left[ \frac{dF_{3/2}}{dt} \right]}{\tau_a} - \frac{h\nu}{kT} \left[ N_{a} \sigma_a g_a (v) + N_{2a} \sigma_a g_a (v) \right]$$

$$= R_2 - \frac{\left[ \frac{dF_{3/2}}{dt} \right]}{\tau_a} (1 + \frac{h\nu}{kT}) = R_2 - \frac{\left[ \frac{dF_{3/2}}{dt} \right]}{\tau_a} (1 + L)$$

where $L = \frac{h\nu}{kT}, \text{the laser intensity divided by } I_s$. 

$I_s = \left( \frac{4\pi}{\sigma_a} \right) = \text{Saturation intensity for the laser at } 1.0643\mu m$ 

$L = \frac{h\nu}{\sigma_a} I_s$, the laser intensity divided by $I_s$. 

$I_s = \frac{8\pi h\nu^3}{c^2 \sigma_a g_a (v, \nu)} = \frac{8\pi h\nu^2 x h\nu}{c^2 \sigma_a g_a (v, \nu) \tau_a} = \frac{h\nu}{\sigma_a g_a (v, \nu) \tau_a}$ 

$\Phi = A_{21} \frac{h\nu}{\sigma_a g_a (v, \nu) \tau_a} \left[ 1 + (1 - A_{21}) \frac{\tau_a}{\tau_2} \right] = A_{21} \frac{h\nu}{\sigma_a g_a (v, \nu) \tau_a}$
\[ R_2 = J_0 \Delta \rho (x) \frac{I_P}{h \nu_p} \]

where \( \Delta \rho (x) \) is the absorption coefficient of the pump.

* the fact that the pump is attenuated with \( z \)
  indicates that the \([^4F_{3/2}] \) density and thus the small signal gain coefficient will also be a function of \( z \).

\[
\frac{d[^4F_{3/2}]}{dt} = N_0 \Delta \rho \frac{I_P(z)}{h \nu_p} - \frac{[^4F_{3/2}]}{N_2}[1 + L(2)]
\]

\( \Delta \rho = f_0 N_0 \sigma_p (x) \)

\( \sigma_p = \) absorption cross section = \( 1.16 \times 10^{-19} \) cm\(^2\) at 808.5 nm

\( f_0 = \) fraction of \( N_0 =[^4I_{15/2}] \) manifold in the lowest state

\( N_2 = \) total population in \([^4F_{3/2}] \) manifold; upper state density

\( N_0 +[^4F_{3/2}] = [Nd]^0; \) the doping density of Nd\(^{3+}\) in YAG

\( I_{sp} = \frac{1}{f_0 \sigma_p \Delta \rho} \) \( \); the pump saturation intensity

\( I_{sp} = 18 \) kW/cm\(^2\) for 808.5 nm (\( f_0 = 1 \))

\( P(z) = \frac{I_P}{I_{sp}} \) \( \); the normalized pump intensity

\( L(z) = \frac{I_F}{I_s} \) \( \); the 1064 nm intensity normalized to its saturation value

\( I_s = \frac{h \nu}{\sigma_{eff} T_2} \) \( \); the saturation intensity at 1064 nm.
Equation \( \frac{d [^4\!F_{3/2}]}{dt} = \nu_0 \frac{d P(\nu)}{dt} - \frac{[^4\!F_{3/2}]}{2 \Gamma_2} \left[ 1 + \mathcal{L}(\nu) \right] \)

can be rewritten in terms of the above abbreviations:

\[ \frac{d N_2}{dt} = \frac{P(\nu) N_0}{\Gamma_2} - \frac{N_2}{\Gamma_2} \left[ 1 + \mathcal{L}(\nu) \right] \]

\[ \Rightarrow \text{for } \text{CW} \text{ operation} \]

\[ N_2 = \frac{P(\nu) N_0}{1 + \mathcal{L}(\nu)} \]

- The sum of the densities in \(^{4}I_{9/2}\) and \(^{4}F_{3/2}\) must be equal to the doping density \([\text{Nd}]^0\)

\[ N_2 + N_0 = [\text{Nd}]^0 \]

which leads to

\[ N_0 = \frac{[\text{Nd}]^0}{1 + \frac{P(\nu)}{[1 + \mathcal{L}(\nu)]}} \]

\[ N_2 = \frac{P(\nu)[\text{Nd}]^0}{1 + \frac{P(\nu)}{[1 + \mathcal{L}(\nu)]}} \times \frac{1}{1 + \mathcal{L}(\nu)} \]

- The variation of the pump with \( \nu \) is described by

\[ \frac{d P(\nu)}{d \nu} = - \frac{d}{d \nu} N_0 \delta P(\nu) \]

\[ P(\nu) = \frac{d P(\nu)}{d \nu} [\text{Nd}]^0 \]

Superscript \(^0\) indicates a value in the absence of a strong intensity.

Combine \( \nu \) and \( \delta \) to obtain the variation of the laser intensity with \( \nu \).
\[
\frac{dL(z)}{dz} = N_2 \text{ eff} \ L(z) = \left( N_0 \text{ eff} \ \frac{P(z)}{1 + \frac{P(z)N(z)}{1 + L(z)}} \right) \cdot \frac{1}{L(z)}
\]

or
\[
\frac{dL(z)}{dz} = \gamma_m \ \frac{P(z)}{1 + \frac{P(z)N(z)}{1 + L(z)}} \cdot \frac{1}{L(z)} \cdot L(z)
\]

where \( \gamma_m = N_0 \text{ eff} \) - maximum possible gain coefficient with all of the Nd atoms in the \( 4F_{9/2} \) manifold.

- if we divide eq. \( \frac{dL(z)}{dz} \) by \( \frac{dL(z)}{dz} \)
- cancel common factors, arrange all of the laser terms on the left, the pump on the right
- integrate from \( z = 0 \) to \( z = L \)

Simplification is possible

\[
\int \frac{1}{L(z)} \ dL = -\gamma_m \int \frac{dp}{L(z)}
\]

where the subscript 1 refers to the parameters at \( z = 0 \), the entrance of the gain medium, and \( z = L \), the exit plane at \( z = L \).

The subscript 2 refers to the medium at \( z = L \) is small compared to the input, but in any case, we define

\[
\alpha = \text{ absorption efficiency} = \frac{P_0 - P_2}{P_1} = \frac{I_p(1) - I_p(2)}{I_p(1)}
\]
Equation \( \int \frac{C_1 + L C_2}{L C_2} \, dl = -\frac{\partial m}{\partial p} \) can be easily integrated:

\[
\ln \left( \frac{I_2}{I_1} \right) + \frac{I_2}{I_1} (1 - \frac{I_1}{I_2}) = \frac{\partial m}{\partial p} \left[ r - \frac{n_a}{I_{sp}} \right] = \frac{\partial m}{\partial p} \frac{n_a}{I_{sp}} \frac{I_0 (c)}{I_{sp}}
\]

Follow the intensity \( I_2 \) around the loop and back to the entrance of the gain medium to time

\[
I_1 = \left[ I_2, R_2, R_3, I_{u}, I_{a}, T_a, T_{e}, \exp \left[ -\Delta l \Delta n \right] \right] (1 - T_2) \frac{I_2}{I_{sp}}
\]

\( S \) - fraction of photons surviving a round trip in the passive cavity.

\[
\ln \left( \frac{I_2}{I_1} \right) + \frac{I_2}{I_1} (1 - S) = \frac{\partial m}{\partial p} \frac{n_a}{I_{sp}} \frac{I_0 (c)}{I_{sp}}
\]

\[
I_2 = \frac{I_{left} - I_{u}}{I_1} \frac{I_{u}}{(1 - S)} ; \text{ threshold is defined by}
\]

round trip gain = 1 or \( S \cdot \exp \left[ \Delta m \Delta n \right] = 1 \Rightarrow \frac{I_2}{I_1} = I_{th}
\]

\[ \Rightarrow \frac{I_2}{I_1} = 0 \text{ for threshold.} \]
\[ \frac{\frac{\Delta m}{2^*}}{I_0} \cdot \eta_a \frac{I_p(\text{th})}{I_{sp}} = \ln \frac{1}{5} \]

or \[ I_p(\text{th}) = \frac{2^*}{\Delta m} \cdot \frac{I_{sp}}{2^*} \cdot \ln \frac{1}{5} \]

- Thus the intensity impinging on mirror M2 is given by \[ I_2 = \frac{\Delta m}{2^*} \cdot \eta_a \cdot \frac{I_s}{I_{sp}} \left[ \frac{I_p - I_p(\text{th})}{1-5} \right] \]

- The output intensity is just \((1-R_2)I_2\).

- The remaining task is to use the definition of the saturation intensity of the laser to find that most of the cross sections disappear from the expression.

\[ I_{out} = 2^* \cdot \eta_a \cdot 2\eta_c \left[ \frac{1-R_2}{1-5} \right] \int (I_p - I_p(\text{th}))^2 \]

\[ = 2^* \cdot \eta_a \cdot 2\eta_c \cdot 2e \cdot \int (I_p - I_p(\text{th}))^2 \]

where \( \eta_c = \frac{1-R_2}{1-5} \) = coupling loss

\( 2\eta_c = \frac{\eta_c}{\eta_p} = \frac{\Delta P}{\lambda_e} = 0.762 \) = quantum efficiency.

- \[ I_p(\text{th}) = \frac{2^*}{\Delta m} \cdot \eta_a \frac{I_s}{2^*} \cdot \ln \frac{1}{5} = \frac{I_s}{2\eta_c \cdot 2^* \cdot 2^*} \cdot \ln \left( \frac{1}{5} \right) \]
if we assume a pumping effic. $\eta_p = 90\%$, a pump utilization efficiency $\eta = 100\%$, $I_s = 4.4 \text{ kW/cm}^2$ and if the survival factor for the passive cavity $S = 0.9$ ($\ln S = 0.1515$) then the threshold pump intensity would be $6.76 \text{ W/cm}^2$.
For a mode size of 500 $\mu\text{m}$ diameter, a threshold pump power is 1.33 W.

High gain laser. Some lasers have very high gains. For example, some lasers have very high gains. For example, semiconductor lasers or color center lasers. For a laser with large output transmission complex, for a laser with large output transmission complex, the fraction of photons surviving a round trip is small and the denominator in $\frac{I_2}{I_s} = \frac{\ln S}{1 - S}$ can be replaced by 1.

\[
S = \exp \left( -\text{lnt} \frac{I_2}{I_s} \right)
\]

\[
\ln S = \text{lnt} \frac{I_2}{I_s} + \ln \frac{1}{1 - T_2}
\]

\[
I_{out} \approx \left( \frac{T_2}{I_s} I_s \right) \left[ \text{log} \frac{I_2}{I_s} - \ln \left( \frac{1}{1 - T_2} \right) \right]^2
\]

After differentiating to find an optimum we obtain a transcendental equation for $T_2$.

\[
\frac{T_2}{1 - T_2} + \ln \frac{1}{1 - T_2} = (\text{log} I_2 - \text{lnt} I_s)
\]
The layout of the Coherent, Inc. "Verdi" laser, wherein intracavity frequency doubling with a lithium triborate (LiB$_3$O$_5$, LBO) crystal is used to generate 530 nm from 1.06 µm. The initial laser oscillation is at 1.064 µm using a neodymium-doped yttrium vanadate (Nd:YVO$_4$) crystal pumped with an 808-nm diode laser. HR, high-reflectance mirror; OC, output-coupling mirror.
Three important methodological problems related to diode pumped solid state lasers

1. Internal loss estimation

Assume ring cavity & clockwise traveling wave

Exper. 1: $R_1<1$; Exper. 2: $R_2<1$;
2. A four level scheme solid state laser operates in a continuous wave regime at the wavelength 850 nm being pumped by the radiation of 514 nm. Assume a survival factor of the cavity 90%, an output mirror reflectivity $R_2=95\%$, branching ratio $\eta_B=1$, and a pump utilization efficiency $\eta_a=30\%$. Calculate the laser slope efficiency.

\[
P_{\text{out}} = \frac{2p \cdot 2a \cdot 2_{\text{ge}} \cdot 2_c}{2s} (P_P - P_{\text{th}})
\]

\[
2_p = \frac{R_2}{1 - R_2} - \text{branching ratio}
\]

\[
2_a = \frac{P_{\text{in}} - P_{\text{out}}}{P_{\text{in}}} - \text{pump through the crystal} = 0.3 - \text{pump utilization efficiency}
\]

\[
2_{\text{ge}} = \frac{h \nu_c}{h \nu_P} = \frac{\lambda_P}{\lambda_c} = \frac{514}{850} = 0.604
\]

\[
2_c = \frac{1 - R_2}{1 - S} = \frac{1 - 0.95}{1 - 0.9} = \frac{0.05}{0.1} = 0.5
\]

\[
2_s = 2_p \cdot 2_a \cdot 2_{\text{ge}} \cdot 2_c = 1 \cdot 0.3 \cdot 0.604 \cdot 0.5 = 0.0906
\]

\[
\eta_s = 9.1
\]

**Laser slope efficiency**
3. A continuous wave Ho:YAG (2090 nm) laser is longitudinally pumped at 1908 nm. The laser mode has a spot size of 1 mm and the length of the cavity is 10 cm; the stimulated emission cross-section is \( \sigma_e = 12.9 \times 10^{-21} \text{ cm}^2 \) and the upper level lifetime is \( \tau = 8.5 \text{ ms} \). Assume that an output coupler with a transmission \( T = 30\% \) is used, passive cavity losses are 2\% per pass, branching ratio \( \eta_p = 1 \), and a pump utilization efficiency \( \eta_a = 80\% \). Calculate the threshold pump power, slope efficiency, as well as the pump power required to obtain an output power \( P_{\text{out}} = 30 \text{ W} \) from this laser.

1) **Threshold gain** 
\[
\gamma_{\text{th}} = \alpha - \frac{1}{l} \ln \sqrt{R_1 \cdot R_2} = 0.02 - \frac{1}{10} \ln \sqrt{1 \cdot 0.70} = 0.0198 \text{ cm}^{-1}
\]

2) **Population inversion** 
\[
N_{\text{th}} = \frac{\gamma_{\text{th}}}{\sigma} = 1.54 \times 10^{18} \text{ cm}^{-3}
\]

3) **The absorbed threshold pump power per unit volume** 
\[
P_{\text{th}}^v = \frac{N_{\text{th}} \cdot h \nu_p}{\pi \eta(v) \eta_{qe} \eta_a} = 25.9 \text{ W/cm}^3
\]

4) **The absorbed pump power** 
\[
P_{\text{th}} = P_{\text{th}}^v \cdot V = 25.9 \cdot \left( \pi \cdot 0.1^2 / 4 \right) \cdot 10 = 2.0 \text{ W}
\]

5) **Output power** 
\[
P_{\text{out}} = \eta_p \eta_a \eta_{qe} \eta_c \left( P_p - P_{\text{th}} \right) = 1 \cdot 0.8 \cdot 0.913 \cdot \frac{(1-0.7)}{[1-(e^{-20.02 \times 0.7})]}(P_p - 2.0) ; \ P_p = \frac{30}{0.669} + 2.0 = 47 \text{ W} ; \ \eta_{\text{slope}} = 66.9\%; \ \text{Alternatively:}
\]

6) **Slope length** 
\[
I_s = \frac{h \nu}{\sigma \tau} = \frac{6.6 \times 10^{-34} \cdot 3 \times 10^8}{12.9 \times 10^{-21} \cdot 8.5 \times 10^{-3}} = 864 \text{ W/cm}^2 ; \ P_s = I_s A = 864 \cdot \left( \pi \cdot 0.1^2 / 4 \right) = 6.8 \text{ W}
\]

7) **Threshold pump power** 
\[
P_{\text{th}} = \frac{P_s}{\eta_p \eta_{qe} \eta_a} \ln \frac{1}{S} = \frac{P_s}{\eta_p \eta_{qe} \eta_a} (1-S) = \frac{6.8}{1 \cdot \frac{1908}{2090} \cdot 0.8} [1-(e^{-20.02 \times 0.7})] = 3.0 \text{ W}
\]

8) **Output power** 
\[
P_{\text{out}} = \eta_p \eta_a \eta_{qe} \eta_c \left( P_p - P_{\text{th}} \right) = 1 \cdot 0.8 \cdot 0.913 \cdot \frac{(1-0.7)}{[1-(e^{-20.02 \times 0.7})]}(P_p - 3.0) ; \ P_p = \frac{30}{0.669} + 3.0 = 46 \text{ W}
\]
Fiber lasers

- Cylindrical optical fibers will be discussed later in Chapter 16 and are the most important in optical communications.
- In the simplest form they have a central core region of a higher index than that of a surrounding medium – the cladding.
- Light in the core region traveling at a small enough angle to the axis is trapped because of total internal reflection at the core cladding boundary.

For the meridional ray in Fig. 16.1, total internal reflection (TIR) occurs within the core if $\theta_1 > \theta_c$, or $\sin \theta_1 > \frac{n_2}{n_1}$. From Snell’s law, applied to the ray entering the fiber,

$$\sin \theta = \sin \theta_0 / n_1.$$

(16.1)

Since $\theta + \theta_1 = 90^\circ$, the condition for TIR is

$$\sin \theta_0 < \sqrt{n_1^2 - n_2^2}.$$

(16.2)
For most optical fibers the relative difference between the index of the core and that of the cladding, \( \Delta = (n_1 - n_2)/n_1 \), is small, so Eq. (16.2) can be written

\[
\sin \theta_0 < \sqrt{(n_1 - n_2)(n_1 + n_2)},
\]

(16.3)

which, since \( n_1 \approx n_2 \), can be written

\[
\sin \theta_0 < n_1 \sqrt{2\Delta}.
\]

(16.4)
If a lens is used to focus light from a point source into a fiber, as shown in Fig. 16.3, then there is a maximum aperture size $D$ that can be used. When the end of the fiber is a distance $d$ from the lens, light rays outside the cross-hatched region enter the fiber at angles too great to allow TIR. The quantity $\sin \theta_0 \approx D/(2d)$ is called the numerical aperture (NA) of the system. So, from Eq. (16.4),

$$NA = \sin \theta_0 = \sqrt{n_1^2 - n_2^2} = n_1 \sqrt{2\Delta}.$$  \hspace{1cm} (16.5)

If the point source in Fig. 16.3 is placed a great distance from the lens, then $d = f$. In this case

$$NA = \frac{D}{2f}.$$ \hspace{1cm} (16.6)

If the lens is chosen to be no larger than necessary, then the lens diameter will be $D$. The ratio $f/D$ is a measure of the focusing/light-collecting properties of the lens, called the $f$/number. So to match a distant source to the fiber $2NA = 1/(f$/number).
Fiber lasers

- Fiber lasers use a gain medium that is an optical fiber doped with rare-earth ions such as erbium $\text{Er}^{3+}$, neodymium $\text{Nd}^{3+}$, ytterbium $\text{Yb}^{3+}$, thulium $\text{Tm}^{3+}$, or praseodymium $\text{Pr}^{3+}$.
- Research on fiber lasers based on ytterbium $\text{Yb}^{3+}$ ions has been particularly intense because these lasers have a simple energy-level structure.
A fiber laser using a double-clad fiber. HR-S, high reflectance at fiber laser signal wavelength.

The double-clad fiber design used in fiber lasers.
Very-high-power fiber lasers with powers above 10 kW are increasingly used in industrial applications because they have high electrical efficiency and good output beam quality.

Fiber lasers can be mode-locked to generate very short pulses (<1 ps) in the infrared.

An important application of Erbium doped fiber is erbium doped optical fiber amplifiers (EDFAs) which are widely used in fiber –optic communication networks to amplify 1.55 um semiconductor laser radiation.
Thin disk lasers

- The thin-disk laser (sometimes called thin-disc laser or active-mirror laser) is a special kind of diode-pumped high-power solid-state laser, which was introduced in the 1990s by the group of Adolf Giesen at the University of Stuttgart, Germany.
- The main difference from conventional rod lasers or slab lasers is the geometry of the gain medium: the laser crystal is a thin disk, where the thickness is considerably smaller than the laser beam diameter. The heat generated is extracted dominantly through one end face, i.e., in the longitudinal rather than in the transverse direction. The cooled end face has a dielectric coating which reflects both the laser radiation and the pump radiation.
- The thin disk is also often called an active mirror, because it acts as a mirror with laser gain. Within the laser resonator, it can act as an end mirror or as a folding mirror. In the latter case, there are two double passes of the laser radiation per resonator round trip, so that the gain per round trip is doubled and the threshold pump power is reduced.
Competition with Fiber Lasers

- Thin-disk lasers are currently facing fierce competition from high-power fiber lasers and amplifiers.
- In the domain of ultrashort pulse generation, fiber amplifier systems based on chirped-pulse amplification allow one to reach even higher average powers and shorter pulse durations than thin-disk lasers can generate without amplification.
- Within the next few years, both thin-disk lasers and fiber lasers are expected to show significant further progress, and it is currently not clear which technology will acquire the larger market share.