Laser Physics II

PH582-VT (Mirov)

Optics of Anisotropic Media
Class lecture and ch.18

Lectures 17-18

Spring 2014
C. Davis, “Lasers and Electro-optics”
Wave Propagation in Anisotropic Media.

Optics of Anisotropic Media

The Dielectric Tensor

- In an isotropic medium, the propagation characteristics of e.m. waves are independent of their propagation direction.
- Ex: Gases, liquids in the absence of external fields.

- $\mathbf{D} = \varepsilon_\infty \mathbf{E}$ for isotropic media
- Electric displacement vector $\mathbf{D}$ and its associated electric field $\mathbf{E}$ are parallel.
- $\varepsilon_\infty$ - scalar dielectric constant, which in the general case is a function of frequency.

- $\mathbf{P} = \varepsilon_0 \mathbf{E} + \mathbf{P}$
  - $\mathbf{P}$ - polarization vector
  - $\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$ - polarization induced by the field and the field are $\mathbf{E}$.
  - $\chi$ - scalar susceptibility.

- Assumption: Materials that we consider are transparent for excitation light.
- $\varepsilon_\infty$, $\chi$, $\chi$ are real.
In an anisotropic medium $\mathbf{B}$ and $\mathbf{E}$ are no longer necessarily parallel.

\[
\mathbf{B} = \varepsilon_r \varepsilon_0 \mathbf{E}
\]

\(\varepsilon_r\) is the dielectric tensor, which in matrix form referred to 3 arbitrary orthogonal axes is

\[
\varepsilon_r = \begin{pmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\
\varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\
\varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}
\end{pmatrix}
\]

\[\rho_x = \varepsilon_0 (\varepsilon_{xx} E_x + \varepsilon_{xy} E_y + \varepsilon_{xz} E_z)\]
\[\rho_y = \varepsilon_0 (\varepsilon_{yx} E_x + \varepsilon_{yy} E_y + \varepsilon_{yz} E_z)\]
\[\rho_z = \varepsilon_0 (\varepsilon_{zx} E_x + \varepsilon_{zy} E_y + \varepsilon_{zz} E_z)\]

By making the appropriate choice of axes the dielectric tensor can be diagonalized. This axes are called principal axes of material.

\[
\begin{pmatrix}
\rho_x \\
\rho_y \\
\rho_z
\end{pmatrix} = \varepsilon_0 \begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix} \begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
\]

\(\varepsilon_x, \varepsilon_y, \varepsilon_z\) - principal dielectric constants.
With the aid of susceptibility tensor \( \mathbf{\overline{P}} = \sum \mathbf{\overline{x}} \mathbf{E} \)

where \( \mathbf{\overline{x}} \) has the matrix form when referred to 3 arbitrary orthogonal axes

\[
\mathbf{\overline{x}} = \begin{pmatrix}
X_{xx} & X_{xy} & X_{xz} \\
X_{yx} & X_{yy} & X_{yz} \\
X_{zx} & X_{zy} & X_{zz}
\end{pmatrix}
\]

In the principal coordinate system

\[
\mathbf{\overline{x}} = \begin{pmatrix}
X_x & 0 & 0 \\
0 & X_y & 0 \\
0 & 0 & X_z
\end{pmatrix}
\]

Since \( \mathbf{\overline{D}} = \sum \mathbf{\overline{E}} + \mathbf{\overline{P}} \)

it is clear that in a principal coordinate system

\[
\Sigma_x = 1 + X_x \\
\Sigma_y = 1 + X_y \\
\Sigma_z = 1 + X_z
\]
- Why susceptibilities depend on the direction of applied field.

- When a field \( E \) is applied to a crystal it displaces both \( E \) nuclei from their equilibrium positions in the lattice and induces a net dipole moment per unit volume (polarization)

\[
\vec{P} = \sum_j \vec{e}_j \Delta \vec{r}_j
\]

- \( \vec{e}_j \) - density of species \( j \) with charge \( e_j \) in the crystal
- \( \Delta \vec{r}_j \) - displacement of this charged species \( j \) from its equilibrium position.

- If the applied electric field has components \( E_x, E_y, E_z \) then in equilibrium

\[
\begin{align*}
E_x &= -K_{jx} (\Delta \vec{r}_j)_x \\
E_y &= -K_{jy} (\Delta \vec{r}_j)_y \\
E_z &= -K_{jz} (\Delta \vec{r}_j)_z
\end{align*}
\]

- \( k_{jx} \) - restoring force constant appropriate to the x component of the charge \( j \) from the displacement of the charge \( j \) from equilibrium position.

- \( \Delta \vec{r}_j = \left( \frac{E_x}{k_{jx}} \hat{i} + \frac{E_y}{k_{jy}} \hat{j} + \frac{E_z}{k_{jz}} \hat{k} \right) \)

- \( \Delta \vec{r}_j \) is \( \perp \) to \( \vec{E} \) only if \( k_{jx} = k_{jy} = k_{jz} \)
- It works only for electric crystals.
Stored Electromagnetic Energy in Anisotropic Media

- the electrical energy density in the crystal

\[ U_E = \frac{1}{2} \varepsilon_0 \mathbf{E} \cdot \mathbf{D} = \frac{1}{2 \varepsilon_0} \left( \frac{D_x^2}{\varepsilon_x} + \frac{D_y^2}{\varepsilon_y} + \frac{D_z^2}{\varepsilon_z} \right) \]

- Eq. shows that the electric displacement vectors from a given point that correspond to a constant stored electric energy describe an ellipsoid.
Propagation of Monochromatic Plane Waves in Anisotropic Media

1. Assume that a monochromatic plane wave of the form \( \mathbf{E} = \mathbf{E}_0 \, e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} = \mathbf{E}_0 \, e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \) can propagate through an anisotropic medium.

2. \( \mathbf{k} \) direction specifies the direction of polarization of this wave.

3. \( \mathbf{E} \) - wave vector is \( \perp \) to the wavefront.

4. \( K = \frac{2\pi}{\lambda} = \frac{2\pi v}{c} \)

5. \( C = \frac{\omega}{k} \)

6. \( \frac{k_x^2}{c^2 - c_x^2} + \frac{k_y^2}{c^2 - c_y^2} + \frac{k_z^2}{c^2 - c_z^2} = 0 \)

7. \( \frac{k_x^2 h_x^2}{h_x^2 - n_x^2} + \frac{k_y^2 n_y^2}{h_y^2 - n_y^2} + \frac{k_z^2 n_z^2}{h_z^2 - n_z^2} = 0 \)

Fresnel's equation

- \( n_x, n_y, n_z \) - principal refractive indices of the crystal.

- In general there are 2 possible solutions for the phase velocity of refractive index, \( c_1, c_2 \), for a monochromatic wave propagating through a medium with wave vector \( \mathbf{k} \).

- When \( \mathbf{k} \) lies in certain specific directions both roots become equal. These special directions are called optical axes of the crystal.
The Indicatrix

- The indicatrix, wave-normal, or index ellipsoid is an ellipsoid with the equation
  \[ \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \]
- it allows to determine \( n \) for the monochro. plane waves as a function of their direction of polarization.
- it is equivalent to the surface mapped out by the \( \mathbf{B} \) vectors corresponding to a constant energy density at a given frequency.

\[ U_e = \frac{1}{2} |\mathbf{E}|^2 = \frac{1}{2E_0} \left( \frac{D_x^2}{\varepsilon_x} + \frac{D_y^2}{\varepsilon_y} + \frac{D_z^2}{\varepsilon_z} \right) \]

- in any crystal with \( L \) symmetry axes, the axes of ellipsoid, which are the principal axes of the crystal, are \( \perp \) to the three axes of symmetry of the crystal.
We use the geometric properties of indicatrix to determine the refractive indices and polarization of the two monochromatic waves that can propagate through the crystal with a given wave vector.

- The plane surface $\mathbf{K}$ that passes through the center of indicatrix intersects this ellipsoid in an ellipse called the intersection ellipse.
- The two permitted linear polarizations directions $\mathbf{B}_1$ and $\mathbf{B}_2$ lie along the semi-axes of the intersection ellipse.
- The lengths of these semi-axes give the refractive indices experienced by these two polarizations.
Angular relationships between \( \vec{B}, \vec{E}, \vec{H}, \vec{\kappa} \) and the
poynting vector \( \vec{S} \) in an anisotropic medium.

- \( \vec{B}, \vec{E}, \vec{H} \) are coplanar
- \( \vec{B} \) & \( \vec{E} \) make an angle

\[
\theta = \arccos \left( \frac{\vec{E} \cdot (\vec{E} \times \vec{B})}{|\vec{E}| \cdot |\vec{E} \times \vec{B}|} \right)
\]

- \( \vec{B} \perp \vec{\kappa} \)
- \( \vec{S} = \vec{E} \times \vec{H} \Rightarrow \exists \vec{\Sigma}, \vec{\Sigma} \perp \vec{H} \)

- \( \vec{S} \) defines the direction of energy flow within the
medium - direction of the ray in geometro optics
- In isotropic media the ray is \( \parallel \) to the wave vector
and \( \perp \) to the wavefront.
- In anisotropic media this is no longer so, except
for propagation along one of the principal axes.

- Transverse electromagnetic waves can propagate through
anisotropic media, for propagation in a general direction two
- distinct allowed linear polarizations specified by
the direction of \( \vec{B} \) can exist for the wave.
- 2 allowed polarizations are \( \parallel \) and \( \perp \) to the wave propagation.
- 2 allowed polarizations with a phase velocity (the velocity of the surface of
constant phase - wavefront) which depends on which
of these 2 polarizations it has.
- A wave of arbitrary polarization entering an anisotropic medium
will be resolved into 2 linearly polarized components
with a different phase velocity.
- To characterize these allowed polarization directions we
have to specify their orientation with respect
to the principal axes of the medium.
- It is done with the help of geometric figure
index ellipsoid or indicatrix.
• in the general case there are two \( E \) vector directions through the center of the indicatrix for which the intersection ellipse is a circle. These two directions are called the principal optic axes.

• waves can propagate along these optic axes with any arbitrary polarization.

• in a cubic crystal the indicatrix is a sphere.

• in crystals belonging to the tetragonal, hexagonal and trigonal systems the crystal symmetry requires \( n_x = n_y \) and the indicatrix reduces to an ellipsoid of revolution. There is only one optic axis, oriented along the axis of highest symmetry of the crystal. These crystal classes are said to be uniaxial.

Uniaxial crystals

• The equation of the uniaxial indicatrix is

\[
\frac{x^2 + y^2}{n_e^2} + \frac{z^2}{n_o^2} = 1
\]

\( n_o \) - index of refraction experienced by waves polarized \( \perp \) to the optic axis - Ordinary or O-rays

\( n_e \) - polarized \( \parallel \) to the optic axis - Extraordinary or E-rays.
Fig. 18.3. (a) Indicatrix in a positive crystal, (b) indicatrix in a negative crystal.

\[ n_e > n_o \quad \text{- positive uniaxial} \]
\[ n_o > n_e \quad \text{- negative uniaxial} \]

- Because uniaxial crystals have indicatrices which are circularly symmetric about the optic axis, their optical properties depend only on the polar angle \( \theta \) that the vector \( \mathbf{R} \) makes with the optic axis and not on the azimuthal orientation of \( \mathbf{R} \) relative to the \( x \) and \( y \) axes.

- We can illustrate all their optical character by considering propagation in any plane containing the optic axis.
Fig. 18.5. Section of positive uniaxial indicatrix containing optic axis showing wave-normal and ray directions.

3 & E are II for the O-waves.
0 ray is II
0 wave vector.
E-ray is HT E-vector.

Ray is in direction of E-ray, independent of direction and is equal OP.

- The tangent to the ellipse at T is II to the ray direction OP.
- The tangent to the ellipse at point P is II to the wave vector.
Index Surfaces

Calculate the refractive index \( n_E(\theta) \) of an \( E \) wave propagating at angle \( \theta \) to the optic axis in a uniaxial crystal.

- \( n_E(\theta) = 0 \times \) major semi-axis of the intersection ellipse.
- The Cartesian coordinates of point \( P \) relative to the origin 0 are:
  \[ x = n_E(\theta) \cos \theta \]
  \[ z = n_E(\theta) \sin \theta \]
  \[ y = 0 \]
- This point lies on the indicatrix, so:
  \[ n_E^2(\theta) \cos^2 \theta + n_E^2(\theta) \sin^2 \theta = 1 \]

\[
\frac{n_E(\theta)}{n_0} = \sqrt{\frac{1}{n_E^2 \cos^2 \theta} + \frac{1}{n_E^2 \sin^2 \theta}}
\]

- We can use this relationship to specify a surface which collects the extraordinary index surface which shows geometrically the index of refraction of extraordinary waves in a uniaxial crystal as a function of their direction of propagation.
- The Cartesian coordinates of this surface must satisfy:
  \[ n_E^2(\theta) = x^2 + y^2 \]
  Also:
  \[ \sin^2 \theta = \frac{x^2 + y^2}{x^2 + y^2 + \frac{z^2}{x^2 + y^2}} \]
  \[ \cos^2 \theta = \frac{z^2}{x^2 + y^2} \]
Fig. 18.7. Ordinary and extraordinary index surfaces: (a) positive crystal; (b) negative crystal.

\[
\frac{x^2}{n_e^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1
\]

- for \( E \) wave parallel to the optic axis, \( z = 0 \)
  \[\sqrt{x^2 + y^2} = n_o(0) = n_o\]
- for an \( E \) wave along \( z \) axis
  \[x^2 + y^2 = 0\]
  \[z = n_e(90) = n_e\]

- the index surface for \( O \)-waves is a sphere since the index of refraction of such waves is independent of their propagation direction.
  \[x^2 + y^2 + z^2 = n_e^2\]

- Sections of the ordinary and extraordinary index surfaces for both positive and negative uniaxial crystals which contain the optical axis are called principal sections.
Other Surfaces related to the Uniaxial Indicators

Fig. 18.8. Ray-velocity surfaces for positive and negative uniaxial crystals.

The wave-velocity surface describes the velocity of waves in their direction of propagation; it is a two-shelled surface.

**O-waves wave velocity surf.**

\[ n_o^2 \left( x^2 + y^2 + z^2 \right) = c_o^2 \]

**E-waves**

\[ n_e \theta \left( x^2 + y^2 + z^2 \right) = c_e^2 \]

\[ \sin \theta = \frac{x^2 + y^2}{x^2 + y^2 + z^2}; \quad \cos \theta = \frac{z}{x^2 + y^2 + z^2} \]

\[ = \frac{x^2 + y^2}{n_e^2} + \frac{z^2}{n_o^2} = \left( \frac{x^2 + y^2 + z^2}{c_o^2} \right)^2 \]

(from \( n_e \theta = \frac{n_o n_e}{\sqrt{n_e^2 \sin^2 \theta}} \))

This surface is not an ellipsoid but an ovaloid of revolution.

- The ray-velocity surface describes the velocity of rays in their direction of propagation.
- The ray-velocity surface for O-rays is a sphere with equation \( x^2 + y^2 + z^2 = \frac{c_o^2}{n_o^2} \).
- The ray-velocity surface for E-rays is an ellipsoid of revolution with semi-axes \( c/o_0 \) and \( c/o_+ \) satisfying the equation \( \frac{x^2}{n_+^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_+^2} = c_o^2 \).
Huygenian Constructions

Fig. 18.9. Huygenian construction for light entering a negative uniaxial crystal normally and travelling along the optic axis \([18.1]\). The number on the crystal face is an index that gives the orientation of the face \([18.2]\).

\[D - \text{wave} \quad \sin \theta_D = \frac{n_0}{n_e} \sin \theta \]

\[E - \text{wave} \quad \sin \theta_E = \frac{n_e}{n_0} \sin \theta \]

Fig. 18.10. Huygenian construction for light entering a positive uniaxial crystal normally and travelling perpendicular to the optic axis \([18.1]\).

Fig. 18.11. Huygenian construction for light that does not enter the crystal normally and does not propagate along a principal axis direction \([18.1]\).

On leaving the crystal the \(O\)- and \(E\)-rays become \(\pi\) but \(E\)-ray has been displaced. Double refraction.
Fig. 18.12. Huygenian construction for light travelling perpendicular to optic axis that refracts on entering the crystal. O- and E-waves (and rays) refract at different angles. The indices on the crystal faces specify their orientation.

Fig. 18.13. Huygenian construction for a rather general case.
Canada balsam, refractive index $n_e$

Ahrens polarizer

Black coating

Optic-axis direction

O-ray

E-rays

$n_e < n_o < n_r$

1.486 1.55 1.658 at 598.3 nm

Length = 1.9

Width = 1.9

Glan–Foucault polarizer

Optic-axis direction

$\theta = 51.5^\circ$

Glan–Taylor polarizer

O-ray

E-ray

Rochon polarizer

O-ray

E-ray

Wollaston polarizer

O-ray

E-ray

Fig. 18.14. Construction of various kinds of polarizer. (For (d) – (f) the separation of the O- and E-rays is much exaggerated.)
Retardation

- unless a light beam is travelling in the direction of a principal axis, and is polarised to a principal axis, when it is incident on a planar uniaxial crystal slab, it will be resolved within this material into O- and E waves.
- if E has angle $\theta$ to the optic axis then wave velocities of O and E waves will be
  \[ v_O = \frac{c}{n_o} \text{ and } v_E = \frac{c}{n_E(\theta)} \]
  in a positive crystal $n_o > n_E$
  \[ v_o > v_E \]
  in a negative crystal $n_o < n_E$
  \[ v_o > v_E \]

- for ordinary wave
  \[ |k_o| = k_o = \frac{\omega n_o}{c_o} = \frac{2\pi n_o}{\lambda_0} \]
- for extraordinary wave
  \[ |k_E| = k_E = \frac{\omega n_E(\theta)}{c_o} = \frac{2\pi n_E(\theta)}{\lambda_0} \]

- on passing through a crystal of thickness $L$
  the phase changes for the O- and E waves are
  \[ \phi_o = k_o L = \frac{2\pi n_o L}{\lambda_0} \]
  \[ \phi_e = k_E L = \frac{2\pi n_E(\theta) L}{\lambda_0} \]

- the phase difference (retardation) introduced by the crystal is
  \[ \Delta \phi = \phi_e - \phi_o = \frac{2\pi L}{\lambda_0} (n_E(\theta) - n_o) \]

- for an incident wave of the form
  \[ D = A \cos (\omega t - kr) \text{ linearly polarized at an angle } \beta \text{ to the ordinary polarisation direction, } \]
  the ordinary and extraordinary waves
\[ D_e = A \cos \beta \cos (\omega t - \Phi_e) \]
\[ D_o = A \sin \beta \cos (\omega t - \Phi_o) \]

\( \Phi_o = \phi_o \pm \frac{2\pi m}{L} \)
\( \Phi_e = \phi_e \pm \frac{2\pi n \lambda}{c} \)

- at the exit face of the crystal
  \( D_o = A \cos \beta \cos (\omega t - \Phi_o) \)
  \( D_e = A \sin \beta \cos (\omega t - \Phi_e) \)

- if the input to the crystal is a narrow beam of light - these two beams will be displaced from one another.
- if the input light to the optical axes these two electric vectors recombine to form a resultant single displacement vector with magnitude
  \[ D_{\text{out}} = \sqrt{D_e^2 + D_o^2} \]
which makes an angle \( \alpha \) with the ordinary polarization direction

- simplest case
  \[ \Phi_e - \Phi_o = 2\pi m \]
  \[ m = 0 \pm 1, \pm 2, \ldots \]
  \[ \tan \alpha = \tan \beta \]
  \[ \tan \alpha = \frac{D_e}{D_o} = \tan \beta \frac{\cos (\omega t - \Phi_e)}{\cos (\omega t - \Phi_o)} \]

- if \( \Phi_e - \Phi_o = (2m+1)\pi \Rightarrow \tan \alpha = -\tan \beta; \alpha = \frac{\pi}{2} \)

- output wave is linearly polarized but rotated by \( \alpha \) from its original polarized direction.

- since a retardation \( \Phi = (2m+1)\pi \) it is equivalent to a path difference of \( (2m+1)\frac{\lambda}{2} \) a crystal that rotates the plane of linearly \( \Phi \) polarized by \( \alpha \) is called a half wave retardation plate.
it is most usual to cut such a crystal so its faces
are perpendicular to the optic axis and to polarize the
input at 45° to the optic axis so the output is
linearly polarized and rotated 90° from
input.

for such an input and \( \phi = \phi - \phi_0 = (2m+1) \frac{\pi}{2} \)
then \( \tan \theta = \frac{\pi}{2} \tan (\omega t - \phi) \)
(i) For \( m \) even \( \tan \theta = \omega t - \phi_0 \)
(ii) For \( m \) odd \( \tan \theta = - (\omega t - \phi_0) \)

For both cases

\[
P_{out} = \frac{A}{\sqrt{2}} \sqrt{\cos^2(\omega t - \phi_0) + \cos^2(\omega t - \phi_0 - \phi)} = \frac{A}{\sqrt{2}}
\]

\[
P_0 = A \cos B \cos (\omega t - \phi_0)
\]
\[
P_\phi = A \sin \omega \cos (\omega t - \phi_0)
\]
\[
P_{out} = \sqrt{P_0^2 + P_\phi^2}
\]
\[
\phi = 45°
\]

The output displacement vector has a
constant magnitude but rotates about the
direction of propagation with constant velocity.
(Circular polarized light).

if the electric vector rotates in a clockwise
direction when viewed in the direction of propogation when \( m \) is even, the light is said to be
left-hand circular polarized.

when \( m \) is odd, the rotation is in counterclockwise direction.
right-hand circular polarized.

a crystal which introduces a retardation
\( \phi = (2m+1) \frac{\pi}{2} \) is called a quarter-wavelength plate.
\((2m+1)\)th order quarter-wave plate.
Fig. 18.16. Left-hand circularly polarized light (a) and (b) left hand elliptically polarized light[18.1].

In the general case when \( \theta \) is not an integer # of \( \frac{\pi}{2} \) or if the input to a quarter-wave plate is not polarized at 45° to the optical axis, the resultant output will be elliptically polarized.

In this case the displacement vector and the electric vector trace out an ellipse as they rotate in time as the wave propagates.
Intensity Transmission Through Polarizer/Waveplate Combinations.

A wave is passed through a linear polarizer (P) whose preferred direction is at angle $\beta$ to the O-dir. of a succeeding waveplate, and then transmitted through a second linear polarizer (A) whose preferred axis makes direction of the wave plate. A vector transmitted through the analyzer has

$$D_0 = A \cos \beta \cos \psi \cos \Delta \phi + \sin \beta \sin \psi \frac{1}{\cos \beta}$$

$$\tan \chi = \frac{\cos \beta \cos \psi \sin \Delta \phi}{\cos \beta \cos \psi \cos \Delta \phi + \sin \beta \sin \psi}$$

$$\Delta \phi = \phi_0 - \phi_1$$ - retardation produced by the wave plate.
Specific Examples.

1) The input & output polarizers are ||: \( \psi = 0 \)

\[ D_0 = A \sqrt{1 - \frac{1}{2} \sin^2 \frac{\Delta \Phi}{2}} (1 - \cos \Delta \Phi) \]

- if \( \Delta \Phi = 0 \) or \( \pm \frac{\pi}{4} \)
  \[ D_0 = A \]
- if \( \beta = 45^\circ \)
  \[ D_0 = A \sqrt{1 - \frac{1}{2} (1 - \cos \Delta \Phi)} \]
  - if \( \Delta \Phi = \pi, 3\pi, 5\pi \)
    \[ D_0 = 0 \]
  - if \( \Delta \Phi = \frac{2\pi}{3}; \frac{4\pi}{3}; \ldots \)
    \[ D_0 = \frac{A}{\sqrt{2}} \]

2) Input & output polarizers are crossed: \( \psi = \frac{\pi}{2} \)

\[ D_0 = \frac{1}{\sqrt{2}} \sin \frac{\pi}{3} \sqrt{1 - \cos \Delta \Phi} \]

- if \( \beta = 45^\circ \)
  \[ D_0 = \frac{1}{\sqrt{2}} \sqrt{1 - \cos \Delta \Phi} \]
- if \( \Delta \Phi = 0 \Rightarrow D_0 = 0 \)
  \[ \Delta \Phi = (2m+1)\pi \]
  \[ D_0 = A \left( \frac{\alpha}{2} \right) \]
  \[ \Delta \Phi = (2m+1)\frac{2\pi}{3} \]
  \[ D_0 = \frac{A}{\sqrt{2}} \left( \frac{\alpha}{2} \right) \]

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