Tentative Schedule:

	Date	Module	Topics
1	Aug. 25 (Mo)	Module 1. Spontaneous and Stimulated Transitions	Introduction, Spontaneous and Stimulated Transitions (Ch. 1)
2	Aug. 27 (We)		Spontaneous and Stimulated Transitions (Ch. 1) Homework 1: PH481 Ch.1 problems 1.4 &1.6. PH581 Ch.1 problems 1.4, 1.6 & 1.8 due Sep.3 before class
	Sep. 1 (Mo) No classes		**Labor Day Holiday**
3	Sep. 3 (We)	Module 2. Optical Frequency Amplifiers	Optical Frequency Amplifiers (Ch. 2.1-2.4) Problem solving for Ch.1
4	Sep. 8 (Mo)		Optical Frequency Amplifiers (Ch. 2.5-2.10)
5	Sep. 10 (We)		Optical Frequency Amplifiers (Ch. 2.5-2.10) Homework 2: PH481 Ch.2 problems 2.2 (a,b), 2.4 & 2.5 (a,b). PH581 Ch.2 problems 2.2 (a,b), 2.4 & 2.5 (a,b,c,d) due Sep.22 before class
6	Sep. 15 (Mo)	Module 3. Introduction to two practical Laser Systems	Problem solving for Ch.2 Introduction to two Practical Laser Systems (The Ruby Laser, The Helium Neon Laser) (Ch. 3)
7	Sep. 17 (We)		Review Chapters 1 & 2
8	Sep. 22 (Mo)		**Exam 1** Over Chapters 1-3; Grades for exam 1
9	Sep. 24 (We)	Module 4. Passive Optical Resonators	Exam 1 problem solving. Passive Optical Resonators – Lecture Notes
10	Sep. 29 (Mo)		Passive Optical Resonators – Lecture Notes.
11	Oct. 1 (We)		Passive Optical Resonators – Lecture Notes. Physical significance of χ' and χ'' (Ch.2.8-2.9). Homework 3: read Ch.2 & notes. Work out problems (see Canvas). Due Oct. 8
12	Oct. 6 (Mo)	Module 5. Optical Resonators Containing Amplifying Media	Optical Resonators Containing Amplifying Media (4.1-2).
13	Oct. 8 (We)		Optical Resonators Containing Amplifying Media (Ch.4.3-4.7) Homework 4: Ch. 4 problems 4.7 and 4.9. Due Oct 15.
14	Oct. 13 (Mo)	Module 6. Laser Radiation	Laser Radiation (Ch. 5.1-5.4)
15	Oct. 15 (We)	Module 7. Control of Laser Oscillations	Control of Laser Oscillators (6.1-6.3) Homework 5: Ch. 5 problems 5.1 and 5.5. Due Oct 29.
16	Oct. 20 (Mo)		Control of Laser Oscillators (6.4-6.5) and exam 2 review
17	Oct. 22 (We)	Module 8. Optically Pumped Solid State Lasers	Optically Pumped Solid State Lasers (7.1-7.11)
18	Oct. 27 (Mo)		Optically Pumped Solid State Lasers (7.1-7.11)
19	Oct. 29 (We)		**Exam 2 Over Chapters 4-6** Grades for exam 2 Exam 2 correct solution; Homework 6 Due Nov.5; see Canvas including article on Cr:CdSe
20	Nov. 3 (Mo)	Module 8. Optically Pumped Solid State Lasers	Optically Pumped Solid State Lasers (7.14-7.15)
21	Nov. 5 (We)		Optically Pumped Solid State Lasers (7.16-7.17) Homework 7 (see Canvas) Due Nov. 17
22	Nov. 10 (Mo)	Module 9. Spectroscopy of Common Lasers and Gas Lasers	Spectroscopy of Common Lasers and Gas Lasers (Ch. 8.1-8.10)
23	Nov. 12 (We)	Module10. Molecular Gas Lasers I	Molecular Gas lasers I (Ch. 9.1-9.5)
24	Nov. 17 (Mo)		Molecular Gas lasers I (Ch. 9.1-9.5) Homework 8 (see Canvas) Due Dec. 1
25	Nov. 19 (We)	Module 11. Molecular Gas Lasers II	Molecular Gas Lasers II (Ch. 10.1-10.8) and review for exam 3 (Ch. 10.1-0.8) Homework 9 (see Canvas) Due Dec. 1
	Nov. 24 (Mo) No classes		**Thanksgiving - no classes held**
	Nov.26 (We) No classes		**Thanksgiving - no classes held**
26	Dec. 1 (Mo)		**Exam 3** Over Chapters 7-10 Grades; Exam 3 Correct solution
27	Dec. 3 (We)		Review for Final
28	**Dec. 8 (We) in ESH 3160**		**FINAL EXAM Over Chapters 1-10 (4:15-6:45pm) in ESH 3160 Final Grades**

LASER PHYSICS I PH 481/581-VT1 (MIROV)
Exam I (09/22/25)

STUDENT NAME: ____________________________STUDENT id #: ________________________________

Opened textbook and notes

PH 581 ALL QUESTIONS ARE WORTH 50 POINTS. WORK OUT ANY 3 PROBLEMS OUT OF 6.
PH481 ALL QUESTIONS ARE WORTH 75 POINTS. WORK OUT ANY 2 PROBLEMS OUT OF 6

NOTE: Clearly write out solutions and answers (circle the answers) by section for each part (a., b., c., etc.)

1. The Nd:YAG 1.06 μm laser transition has, to a good approximation, a Lorentzian shape of width (FWHM) ~ 195 GHz at room temperature. The measured upper state lifetime is τ=230 μs, the fluorescence quantum yield η of the laser transition is~ 0.42, and the YAG refractive index is 1.82. Calculate the peak transition cross-section. (Hint: the spontaneous, measured lifetimes and quantum yield of the transition are related as follows: $\tau_{spont} = \frac{\tau}{\eta}$).

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For Lorentzian shape transition the cross section of emission is described by the following equation:

$$\sigma = \frac{c_o^2 A_{21}}{n^2 8\pi\nu^2} \cdot \frac{\frac{2}{\pi\Delta\nu}}{1+\left[\frac{2(\nu-\nu_o)}{\Delta\nu}\right]^2},$$

where $A_{21} = \frac{1}{\tau_{spont}} = \frac{\eta}{\tau}$, where τ is measured lifetime of the upper laser level and

η is fluorescence quantum yield.

c_o is speed of light in vacuum; n is index of refraction of YAG crystal

ν is frequency of Nd:YAG transition; $\nu = \frac{c_o}{\lambda}$, where λ is wavelength of Nd:YAG transition

$\Delta\nu$ is FWHM of the Nd:YAG transition

Since Nd:YAG operates at central frequency, $\nu = \nu_o$ the value of Lorentzian lineshape function will be $g(\nu_o,\nu) = \frac{2}{\pi\Delta\nu}$

Hence, the cross-section of emission for central frequency will be described by the following equa

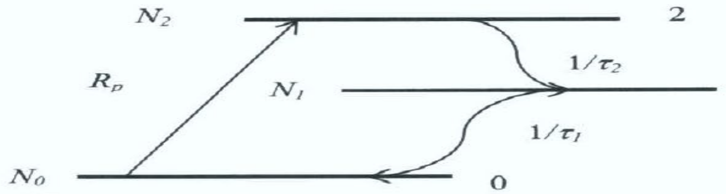
$$\sigma_o = \frac{\lambda^2\eta}{4\pi^2 n^2 \tau\Delta\nu} = \frac{\left(1.06\times10^{-4}\,cm^2\right)\cdot 0.42}{4\pi^2 1.82^2\left(230\times10^{-6}\,s\right)\left(190\times10^9\,Hz\right)} = 8\times10^{-19}\,cm^2$$

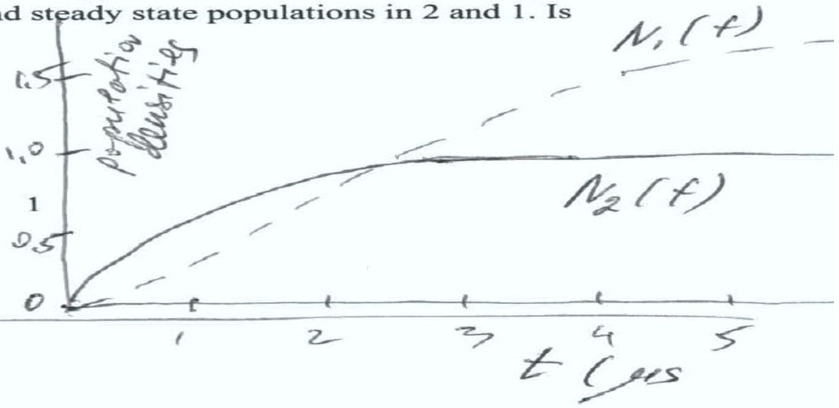
2. Consider the energy level scheme shown below. Atoms are raised from level 0 to level 2 at a pump rate R_p. The lifetime of levels 1 and 2 are τ_1 and τ_2, respectively. Assuming that the ground state is not depleted to any significant extent and neglecting stimulated emission:

(i) Write the rate equations for the population densities, N_1 and N_2 of levels 1 and 2, respectively.

(ii) Assuming that levels 1 and 2 have the same degeneracies, position of level 1 is much higher than kT, $\tau_1 = 2$ μs and $\tau_2 = 1$ μs, and $R_p = 10^{20}$ cm^{-3} s^{-1} find steady state populations in 2 and 1. Is steady state population inversion feasible?

(iii) Calculate N_2 as a function of time if $N_2=0$ at $t=0$.





(i) $$\frac{dN_1}{dt} = \frac{N_2}{\tau_2} - \frac{N_1}{\tau_1} \quad (1a)$$
$$\frac{dN_2}{dt} = R_p - \frac{N_2}{\tau_2} \quad (1b)$$

(ii) $$\left.\begin{aligned} 0 &= \frac{N_2}{\tau_2} - \frac{N_1}{\tau_1} \\ 0 &= R_p - \frac{N_2}{\tau_2} \end{aligned}\right\} \quad N_1 = \frac{N_2 \cdot \tau_1}{\tau_2} = R_p \tau_1 = 2\times10^{14}\ \text{cm}^{-3}$$
$$N_2 = R_p \tau_2 = 10^{14}\ \text{cm}^{-3}$$
$\Rightarrow$ population inversion is not feasible.

(iii) Eq 1b can be solved using $N_2 = 0$ at $t=0$
$$N_2(t) = R_p \tau_2 \left[1 - \exp\left(-\frac{t}{\tau_2}\right)\right] \quad (2)$$

Eq (1a) can be written as $\frac{dN_1}{dt} + \frac{N_1}{\tau_1} = \frac{N_2}{\tau_2}$ (3)
To solve we multiply both sides by $\exp(t/\tau_1)$ and left-hand side of Eq (3) becomes a perfect differential:
$$\frac{d}{dt}\left[N_1(t)\exp(t/\tau_1)\right] = \frac{N_2(t)}{\tau_2}\exp(t/\tau_1) \quad (4)$$
using $N_1=0$ at $t=0$ from Eq (4)
$$N_1(t) = \frac{\exp(-t/\tau_1)}{\tau_2}\int_0^t N_2(t')\exp(-t'/\tau_1)\,dt' \quad (5)$$
using $N_2(t)$ from Eq (2)
$$N_1(t) = R_p \tau_1\left[1 + \frac{\tau_1}{\tau_2-\tau_1}\exp\left(-\frac{t}{\tau_1}\right) - \frac{\tau_2}{\tau_2-\tau_1}\exp\left(-\frac{t}{\tau_2}\right)\right]$$
As $t\to\infty$ a steady state is reached $N_1(\infty) = R_p \tau_1$; $N_2(\infty) = R_p \tau_2$

3. Calculate the total stored energy in a 100 cm^3 box that lies within a bandwidth $\Delta\lambda$=10 nm centered at λ=10600 nm at a temperature of 6000K.

1) The energy density of the radiation within the cavity per volume, per frequency interval is

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{\frac{h\nu}{kT}}-1}\right)$$

2) The total energy stored is

$$E = V\int_{\nu_1}^{\nu_2}\rho(\nu)d\nu, \text{ since } V = 100\,cm^3 \cdot \frac{10^{-6}\,m^3}{cm^3} = 10^{-4}\,m^3$$

$$E = 10^{-4}\int_{\nu_1}^{\nu_2}\rho(\nu)d\nu \quad [J]$$

where $\nu_1 = \frac{c}{\lambda_1} = \frac{3\times10^8\,m/s}{10605\times10^{-9}\,m} = 2.829\times10^{13}\,Hz$

$\nu_2 = \frac{c}{\lambda_2} = \frac{3\times10^8\,m/s}{10595\times10^{-9}\,m} = 2.832\times10^{13}\,Hz$

3) Let us estimate $\frac{h\nu}{kT} = \frac{6.62\times10^{-34}\,J.s \times 2.83\times10^{13}\,Hz}{1.38\times10^{-23}\,J/k \times 6000k} = 0.23$

$e^{+0.23} - 1 = 0.25; \Rightarrow$ approximation $e^{a\nu}-1 \approx a\nu$ is valid, where $a = \frac{h}{kT}$.

4) $$E = 10^{-4}\int_{\nu_1}^{\nu_2}\frac{8\pi \cancel{h}\nu^{\cancel{3}\,2}}{c^3}\cdot\frac{kT}{\cancel{h}\cancel{\nu}} = 10^{-\cancel{4}}\frac{8\pi kT}{c^3}\int_{\nu_1}^{\nu_2}\nu^2 d\nu =$$

$$= 10^{-4}\frac{8\pi kT}{c^3}\left.\frac{\nu^3}{3}\right|_{2.829\times10^{13}}^{2.832\times10^{13}} =$$

$$= \underbrace{10^{-4}m^3 \cdot \frac{8\pi\cdot1.38\times10^{-23}\,J/K \times 6000\,K}{(3\times10^8\,m/s)^3}}_{7.71\times10^{-48}} \times \left[\underbrace{\frac{(2.832\times10^{13})^3}{3}}_{7.571\times10^{39}} - \underbrace{\frac{(2.829\times10^{13})^3}{3}}_{7.547\times10^{39}}\right] =$$

$= 1.85\times10^{-10}$ J

4. A krypton ion laser emits 0.5W of power at 647.1 nm in a 2-mm diameter beam. What would be the effective blackbody temperature of the output beam of that laser radiating over the frequency width of the laser transition, given that the laser linewidth is approximately one fifth of the Doppler linewidth (the atomic mass of Krypton is 36)? Assume that the laser is operating at a krypton gas temperature of 1500 K and that the laser output is uniform over the width of the beam.

1) Find $\Delta\nu_D$ for krypton gas at 1500 K

$$\Delta\nu_D = 2\nu_0\sqrt{\frac{2KT\ln 2}{MC^2}} = 2\times 4.63\times10^{14}\sqrt{\frac{2\cdot 1.38\times10^{-23}\cdot 1500\ln 2}{(6\times10^{-26})(3\times10^{8})^2}} = 2.13\times10^{9}\,Hz$$

$$M = \frac{m_{KR}}{N_A} = \frac{36\times10^{-3}\,kg/mole}{6.02\times10^{23}\,1/mole} = 6\times10^{-26}\,kg$$

$$\nu_0 = \frac{c}{\lambda_0} = \frac{3\times10^{8}\,m/s}{647.1\times10^{-9}\,m} = 4.63\times10^{14}\,Hz$$

$$\Delta\nu_L = \frac{\Delta\nu_D}{5} = 0.43\,GHz$$

2) $$A_{beam} = \frac{\pi D^2}{4} = \frac{\pi(2\times10^{-3})^2}{4} = 3\times10^{-6}\,m^2$$

3) $$Power = I(\nu)\cdot area\cdot\Delta\nu_L = \rho(\nu)\cdot c\cdot area\cdot\Delta\nu_L$$

$$\Rightarrow \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{\frac{h\nu}{KT}}-1}\right]\cdot c\cdot A\cdot\Delta\nu_L = 0.5$$

$$e^{\frac{h\nu}{KT}}-1 = \frac{8\pi(6.6\times10^{-34})(4.63\times10^{14})^3\cdot(3\times10^{-6})(0.43\times10^{9})}{(3\times10^{8})^2\cdot 0.5} = 0.047$$

$$e^{\frac{h\nu}{KT}} = 1.047;\quad \frac{h\nu}{KT} = 0.046$$

$$T = \frac{h\nu}{0.046K} = \frac{6.6\times10^{-34}\cdot 4.63\times10^{14}}{0.046\cdot 1.38\times10^{-23}} =$$

$$= 4.8\times10^{5}\,K$$

5. When a signal of intensity 3.0 kW/m^2 is applied at the input of an amplifier of a length of 10 m with a homogeneously broadened transition, the output signal intensity is 36 kW/m^2. When the output signal is reduced to 1.0 kW/m^2, the output is 20 kW/m^2. Calculate the value of the saturation intensity of the amplifier and the small signal gain coefficient.

$$I = I_0 e^{\gamma_0 \ell - \frac{I - I_0}{I_s}}$$

$$\left.\begin{aligned} 36 &= 3 e^{10\gamma_0 - \frac{36-3}{I_s}} \quad (1) \\ 20 &= e^{10\gamma_0 - \frac{20-1}{I_s}} \quad (2) \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} 12 &= e^{10\gamma_0 - \frac{33}{I_s}} \quad (3) \\ 20 &= e^{10\gamma_0 - \frac{19}{I_s}} \quad (4) \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 2.4849 &= 10\gamma_0 - \frac{33}{I_s} \quad (5) \\ 2.9957 &= 10\gamma_0 - \frac{19}{I_s} \quad (6) \end{aligned}\right\} \quad (6) - (5)$$

$$0.5108 = -\frac{19}{I_s} + \frac{33}{I_s} = \frac{14}{I_s}$$

$$\Rightarrow I_s = 27.4 \frac{kW}{m^2}$$

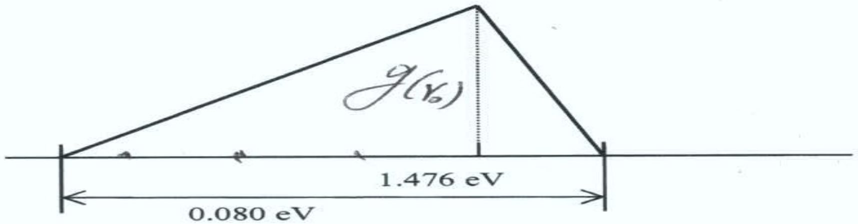
Substitute in (6)

$$2.9957 = (10m)(\gamma_0) - \frac{19 \cancel{kW/m^2}}{27.4 \cancel{kW/m^2}}$$

$$2.9957 \times 27.4 = (274m)(\gamma_0) - 19$$

$$\Rightarrow \gamma_0 = 0.37 m^{-1}$$

6. The spontaneous emission profile from a certain transition can be approximated by the shape shown below.



If the spontaneous lifetime were 5 ns and the gain coefficient were 10 cm^{-1}, find

a) The value of the line shape function (in sec) at $h\nu/e$=1.476 eV.

b) The inversion necessary to obtain that gain coefficient.

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a) $$\nu_0 = \frac{1.476\,eV \cdot 1.6\times10^{-19}\,J/eV}{6.6\times10^{-34}\,J\cdot s} = 3.58\times10^{14}\,Hz$$

$$\int_{-\infty}^{+\infty} g(\nu_0,\nu)\,d\nu = \frac{1}{2}\text{ base} \times \text{height} = 1$$

$$\Rightarrow \frac{1}{2}\,\frac{0.080\,eV\cdot 1.6\times10^{-19}\,J/eV}{6.6\times10^{-34}\,J\cdot s}\cdot g(\nu_0) = 1$$

$$\Rightarrow g(\nu_0) = \frac{1}{9.7\times10^{12}\,Hz} = 1.03\times10^{-13}\,s$$

b) $$\sigma(\nu_0) = \frac{c^2 A_{21}}{8\pi\nu_0^2}\,g(\nu_0) = \frac{(3\times10^8)^2\cdot\frac{1}{5\times10^{-9}}}{8\pi(3.58\times10^{14})^2}\cdot 1.03\times10^{-13} =$$

$$= 5.8\times10^{-19}\,m^2 = 5.8\times10^{-15}\,cm^2$$

$$\gamma(\nu_0) = \left(N_2 - \frac{g_2}{g_1}N_1\right)\sigma(\nu_0)$$

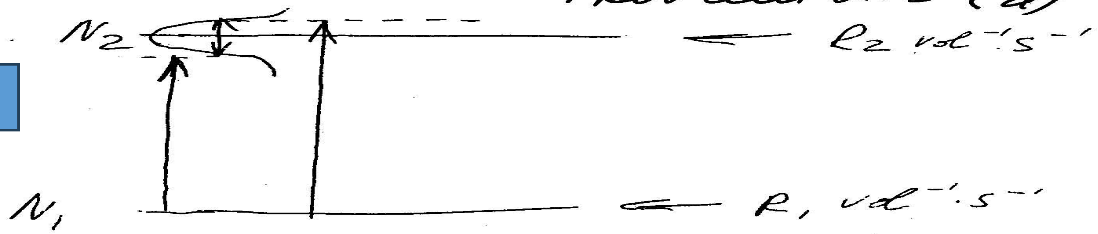
$$\Rightarrow \left(N_2 - \frac{g_2}{g_1}N_1\right) = \frac{\gamma(\nu_0)}{\sigma(\nu_0)} = \frac{10\,cm^{-1}}{5.8\times10^{-15}\,cm^2} = 1.7\times10^{15} \approx 2\times10^{15}\,cm^{-3}$$

Problem 2.5 (d)

$I_i\left(\nu_o + 500Hz\right) = 2 W/m^2$

$I_i\left(\nu_o - 500Hz\right) = 1 W/m^2$

$\gamma_o = 1m^{-1}$; $I_s = 1 W/m^2$ at ν_o



Find the output intensity 500 MHz above line center if an additional $1 \frac{W}{m^2}$ signal is injected 500 MHz below line center

$$(1) \quad 0 = \frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - N_2 B_{21} g(\nu_o, \nu_o+500)\rho(\nu_o+500) - N_2 B_{21} g(\nu_o, \nu_o-500)\rho(\nu_o-500) + N_1 B_{12} g(\nu_o ; \nu_o+500)\rho(\nu_o+500) + N_1 B_{12} g(\nu_o, \nu_o-500)\rho(\nu_o-500)$$

$$(2) \quad 0 = \frac{dN_1}{dt} = R_1 + N_2 A_{21} - \frac{N_1}{\tau_1} + N_2 B_{21} g(\nu_o, \nu_o+500)\rho(\nu_o+500) + N_2 B_{21} g(\nu_o, \nu_o-500)\rho(\nu_o-500) - N_1 B_{12} g(\nu_o, \nu_o+500)\rho(\nu_o+500) - N_1 B_{12} g(\nu_o, \nu_o-500)\rho(\nu_o-500)$$

$$\begin{cases} B_{12} g(\nu_o, \nu_o+500)\rho(\nu_o+500) = W_{12} \\ \left.\begin{array}{l} g(\nu_o, \nu_o+500) = g(\nu_o, \nu_o-500) \\ \rho(\nu_o-500) = \dfrac{\rho(\nu_o+500)}{2} \end{array}\right\} \text{given} \\ B_{12} g(\nu_o, \nu_o-500)\rho(\nu_o-500) = \dfrac{W_{12}}{2} \end{cases}$$

; assume $g_1 = g_2 \Rightarrow W_{12} = W_{21}$

$\Rightarrow$

$$\left.\begin{array}{l} (3) \quad R_2 - \dfrac{N_2}{\tau_2} - N_2 W - N_2 \dfrac{W}{2} + N_1 W + N_1 \dfrac{W}{2} = 0 \\ (4) \quad R_1 + N_2 A_{21} - \dfrac{N_1}{\tau_1} + N_2 W + N_2 \dfrac{W}{2} - N_1 W - N_1 \dfrac{W}{2} = 0 \end{array}\right\}$$

$$\begin{cases} (5) \quad R_2 - \dfrac{N_2}{\tau_2} - \dfrac{3}{2} N_2 W + \dfrac{3}{2} N_1 W = 0 \\ (6) \quad R_1 + N_2 A_{21} - \dfrac{N_1}{\tau_1} + \dfrac{3}{2} N_2 W - \dfrac{3}{2} N_1 W = 0 \end{cases}$$

From ⑤

$$R_2 - \frac{N_2}{\tau_2} - \frac{3}{2} N_2 W + \frac{3}{2} N_1 W = 0$$

$$\boxed{N_2 = \frac{R_2 + \frac{3}{2} N_1 W}{\frac{1}{\tau_2} + \frac{3}{2} W}}$$

Compare

from Paris

$$\boxed{N_2 = \frac{R_2 + N_1 W}{\frac{1}{\tau_2} + W}}$$

From ⑥

$$R_1 + N_2 A_{21} - \frac{N_1}{\tau_1} + \frac{3}{2} N_2 W - \frac{3}{2} N_1 W = 0$$

$$\cancel{N_1 = R_1 \tau_2 + R_2 A_{21} + \frac{3}{2} W(}$$

$$\boxed{N_2 = \frac{N_1/\tau_1 + \frac{3}{2} N_1 W - R_1}{A_{21} + \frac{3}{2} W}}$$

Compare

$$\boxed{N_2 = \frac{N_1/\tau_1 + N_1 W - R_1}{A_{21} + W}}$$

$$\Rightarrow \quad N_2 - N_1 = \frac{R_2 \tau_2 - R_1 \tau_1 - R_2 \tau_1 \tau_2 A_{21}}{1 + \frac{3}{2} W \tau_2 (1 + \tau_1/\tau_2 - A_{21} \tau_1)} \qquad R_2 \tau_2 - R_1 \tau_1 - R_2 \tau_1 \tau_2 A_{21} = \Delta N_0$$

$$N_2 - N_1 = \frac{\Delta N_0}{1 + \frac{3}{2} W \tau_2 (1 + \tau_1/\tau_2 - A_{21} \tau_1)}$$

$$\Phi = A_{21} \tau_2 \left[1 + (1 - A_{21} \tau_2) \frac{\tau_1}{\tau_2} \right]$$

$$N_2 - N_1 = \frac{\Delta N_0}{1 + \Phi \frac{3}{2} W / A_{21}}$$

$$W = \frac{c^2 A_{21}}{8\pi h \nu^3} I_\nu g(\nu_0, \nu)$$

$$I_s(\nu) = \frac{8\pi h \nu^3}{c^2 \phi g(\nu_0, \nu)} \quad \text{then} \quad N_2 - N_1 = \frac{\Delta N_0}{1 + \frac{3}{2} I_\nu / I_s(\nu)}$$

$$\gamma(\nu) = (N_2 - N_1)\frac{c^2 A_{21}}{8\pi\nu^2} g(\nu_0,\nu) =$$

$$= \frac{\Delta N^0}{\left[1 + {}^{3/2\,I(\nu)}/_{I_s(\nu)}\right]} \frac{c^2 A_{21}}{8\pi\nu^2} g(\nu_0,\nu)$$

$$I = I_0(0)\, e^{\gamma(\nu_0+500)\cdot z} - \frac{I - I_0}{I_s}$$ — wrong

$$I_s(\nu_0+500) = \frac{8\pi h\nu^3}{c^2 \phi\, g(\nu_0, \nu_0+500)} = \frac{8\pi h\nu^3}{c^2 \phi\, \frac{g(\nu_0,\nu_0)}{2}} =$$

$$= 2\, I_s(\nu_0) = 2\ {}^{W}/_{m^2}$$

We know that

$$I = I_0 e^{\gamma(\nu)\ell}$$

$$\gamma(\nu) = \frac{1}{I}\frac{dI}{dz}$$

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + {}^{3/2\,I(\nu)}/_{I_s(\nu)}} = \frac{1}{I}\frac{dI}{dz}$$

$$\frac{dI}{I} + \frac{dI}{\frac{2}{3}I_s(\nu)} = \gamma_0(\nu)\,dz$$

$$\gamma_0(\nu_0) = 1\,m^{-1}$$

$$\gamma_0(\nu_0+500) = 0.5\,m^{-1}$$

$$I = I_0\, e^{\gamma_0(\nu_0+500)\ell - \frac{I - I_0}{2/3\, I_s(\nu_0+500)}}$$

$$I = 2 \times e^{\left(0.5\times 1 - \frac{I-2}{2/3\cdot 2}\right)} =$$

$$= 2 \times \left[e^{0.5 - \frac{3(I-2)}{4}}\right] = 2 \times \left[e^{0.5\times 4 - 3I + 6}\right] =$$

$$= 2 \times \left[e^{8-3I}\right]$$

Guess	Cal
2	7.38
2.5	1.648
2.2	4.05
2.3	3.0
2.4	2.22
2.36	2.5
2.37	2.43
2.38	2.36

$$I \approx 2.375\ W/m^2$$

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4	Sep. 8 (Mo)		Optical Frequency Amplifiers (Ch. 2.5-2.10)
5	Sep. 10 (We)		Optical Frequency Amplifiers (Ch. 2.5-2.10) Homework 2: PH481 Ch.2 problems 2.2 (a,b), 2.4 & 2.5 (a,b). PH581 Ch.2 problems 2.2 (a,b), 2.4 & 2.5 (a,b,c,d) due Sep.22 before class
6	Sep. 15 (Mo)	Module 3. Introduction to two practical Laser Systems	Problem solving for Ch.2 Introduction to two Practical Laser Systems (The Ruby Laser, The Helium Neon Laser) (Ch. 3)
7	Sep. 17 (We)		Review Chapters 1 & 2
8	Sep. 22 (Mo)		**Exam 1** Over Chapters 1-3; Grades for exam 1
9	Sep. 24 (We)	Module 4. Passive Optical Resonators	Exam 1 problem solving. Passive Optical Resonators – Lecture Notes
10	Sep. 29 (Mo)		Passive Optical Resonators – Lecture Notes.
11	Oct. 1 (We)		Passive Optical Resonators – Lecture Notes. Physical significance of χ' and χ'' (Ch.2.8-2.9). Homework 3: read Ch.2 & notes. Work out problems (see Canvas). Due Oct. 8
12	Oct. 6 (Mo)	Module 5. Optical Resonators Containing Amplifying Media	Optical Resonators Containing Amplifying Media (4.1-2).
13	Oct. 8 (We)		Optical Resonators Containing Amplifying Media (Ch.4.3-4.7) Homework 4: Ch. 4 problems 4.7 and 4.9. Due Oct 15.
14	Oct. 13 (Mo)	Module 6. Laser Radiation	Laser Radiation (Ch. 5.1-5.4)
15	Oct. 15 (We)	Module 7. Control of Laser Oscillations	Control of Laser Oscillators (6.1-6.3) Homework 5: Ch. 5 problems 5.1 and 5.5. Due Oct 29.
16	Oct. 20 (Mo)		Control of Laser Oscillators (6.4-6.5) and exam 2 review
17	Oct. 22 (We)	Module 8. Optically Pumped Solid State Lasers	Optically Pumped Solid State Lasers (7.1-7.11)
18	Oct. 27 (Mo)		Optically Pumped Solid State Lasers (7.1-7.11)
19	Oct. 29 (We)		**Exam 2 Over Chapters 4-6** Grades for exam 2 Exam 2 correct solution; Homework 6 Due Nov.5; see Canvas including article on Cr:CdSe
20	Nov. 3 (Mo)	Module 8. Optically Pumped Solid State Lasers	Optically Pumped Solid State Lasers (7.14-7.15)
21	Nov. 5 (We)		Optically Pumped Solid State Lasers (7.16-7.17) Homework 7 (see Canvas) Due Nov. 17
22	Nov. 10 (Mo)	Module 9. Spectroscopy of Common Lasers and Gas Lasers	Spectroscopy of Common Lasers and Gas Lasers (Ch. 8.1-8.10)
23	Nov. 12 (We)	Module10. Molecular Gas Lasers I	Molecular Gas lasers I (Ch. 9.1-9.5)
24	Nov. 17 (Mo)		Molecular Gas lasers I (Ch. 9.1-9.5) Homework 8 (see Canvas) Due Dec. 1
25	Nov. 19 (We)	Module 11. Molecular Gas Lasers II	Molecular Gas Lasers II (Ch. 10.1-10.8) and review for exam 3 (Ch. 10.1-0.8) Homework 9 (see Canvas) Due Dec. 1
	Nov. 24 (Mo) No classes		**Thanksgiving - no classes held**
	Nov.26 (We) No classes		**Thanksgiving - no classes held**
26	Dec. 1 (Mo)		**Exam 3** Over Chapters 7-10 Grades; Exam 3 Correct solution
27	Dec. 3 (We)		Review for Final
28	**Dec. 8 (We) in ESH 3160**		**FINAL EXAM Over Chapters 1-10 (4:15-6:45pm) in ESH 3160 Final Grades**

Laser Physics I

PH481/581-VT1 (Mirov)

Lectures 9-11. Passive Optical Resonators
chapter 4 and notes

Fall 2025

C. Davis, “Lasers and Electro-optics”

Passive Optical Resonators

We shall examine the passive properties of optical resonators consisting of two plane-parallel, flat mirroes placed a distance apart.

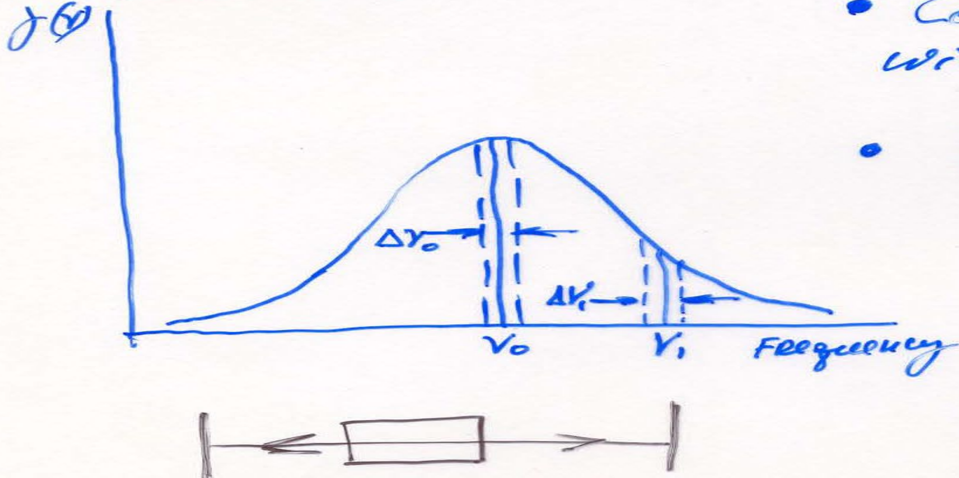
- properties of standing electromagnetic waves in a system
- the way in which stored energy is lost if the mirrors are not totally reflecting
- Fabry-Perot etalon analysis - passive resonant structure widely used in a laser. It has a series of equally spaced resonant frequencies and in transmission acts as a comb filter.

Preliminary Consideration of Optical Resonators

Oscillation in laser system occurs because:

- the amplifying medium is placed between suitable aligned mirrors that provide positive feedback
- the passive properties of this pair of mirrors (optical resonator, or cavity of the laser) affect the way in which the oscillation occurs.
- the resonator has resonance frequencies of its own that interact with line center frequency of the amplifying medium and control the output oscillation frequency of the laser.

Consider at what frequency a laser would oscillate if the resonator has no interaction with the gain profile of the amplifying medium.



- Consider amplifying medium with a Gaussian gain profile $\Delta\nu_D >> \Delta\nu_H$ (gaseous medium)
- γ_0 – maximum gain (line center) logical to expect that oscill. will build up at this frequency
    - view the build-up of oscillation as a progress triggered by spontaneous emission

- a photon travelling in a direction that keeps it bouncing back and forth within the resonator is more likely to be emitted in a narrow band of frequencies $\Delta\nu_0$ near ν_0 than in some other band $\Delta\nu_1$ at frequency ν_1.
- as oscillation builds up, photons spontaneously emitted at all points of the lineshape will be amplified to some extent, but oscillation a ν_0 builds up fastest.
- as intensity at ν_0 grows it depletes the atomic population by causing sufficient stimulated emission that the medium ceases to be amplifying at frequencies near ν_0 within few $\Delta\nu_H$.
- if the medium is homogeneously broadened, since photons at ν_0 can stimulate emission from all the atoms in the medium – oscillation at ν_0 can suppress oscillation at any other frequencies.

Monochromatic character of the oscillation

- In the early stages of oscillation, photons with a frequency distribution $g(\nu_0, \nu)$ are being amplified in a material whose gain/ν response is $\gamma(\nu) \sim$ to $g(\nu_0, \nu)$.
- the amplification process changes the lineshape of the emitted photons circulating in the cavity by a process that is dependent on $g(\nu_0,\nu) \times \gamma(\nu_0\nu)$ that is on $[g(\nu_0,\nu)]^2$.
- the resulting profile of the laser radiation is dependent on higher powers of $[g(\nu_0,\nu)]^2$ as the oscillation is dependent on many passes of photons back & forth through the amplifying medium.

For gaussian lineshape

$$e^{-[2(\nu-\nu_0)/\Delta\nu_D]^2 \ln 2} = e^{-x^2/\sigma^2}$$

the product of two lineshapes produces a narrower profile

$$e^{-(x^2/\sigma^2)^2} = e^{-2x^2/\sigma^2} = e^{-x^2/(\sigma/\sqrt{2})^2}$$

a function that has a width $\frac{1}{\sqrt{2}}$ less than original

In a noninteractive laser cavity (cavity doesn't interact with a gain profile) the laser oscillation will be highly monochromatic – gain of the medium causes a narrowing of the original spontaneously emitted lineshape.

Passive aspects of the simple plane-parallel resonator

- purpose of the cavity – to provide positive feedback necessary to cause oscillation
- the oscillation occurs as a result of spontaneous emission into those modes that keep radiation within the resonator after multiple reflections between the cavity mirrors.
- how many such modes do we have at $\lambda = 1\mu m$? the amount of modes at $\lambda = 1\mu m$, in $1\,m^3$ that lie in a a frequency range of the order of a typical spontaneous emission linewidth, 10^9 Hz is

$$\frac{8\pi\nu^2}{c^3}\cdot\Delta\nu = \frac{8\pi\times(3\times10^{14})^2\times10^9}{(3\times10^8)^3} \approx 8\times10^{13} \text{ modes}$$

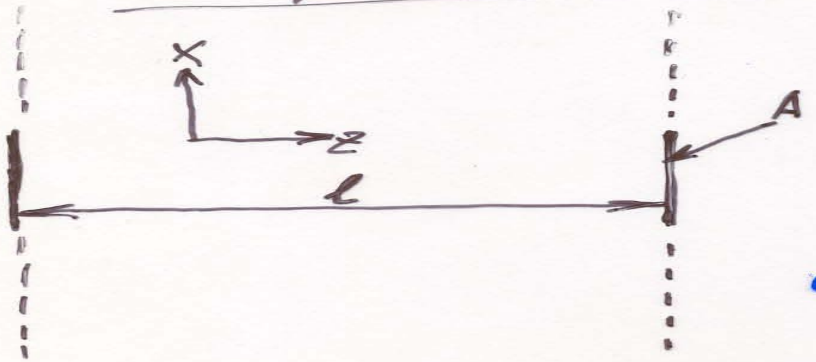
most of these modes will not lie on, or near, the normals to the resonator mirrors and will not undergo feedback.

- The optical resonator has a quality factor Q that varies from one spontaneous emission mode to another.
- those modes that lie perpendicular to the parallel resonator mirror surfaces have the highest Q.

$$\boxed{Q = \frac{2\pi\nu_0 U}{P}}$$

ν_0 – resonant frequency
U – field energy stored in the resonator
P – power dissipated by the resonator

Calculation of the Energy Stored in an Optical Resonator



- Consider the case of a standing e-m wave between two perfectly conducting infinite planes of separation l.
- Such a wave corresponds to a mode of the resonator that lies $\perp$ to its end planes.
- Any energy stored in such a mode suffers no losses and remains stored indefinitely.
- Calculate e-m. energy stored between area A of these plates in a volume $V = A \times l$
- $E(z,t) = E_x \sin \omega t \sin Kz$ – electric field of the standing wave inside the resonator
- on each reflector $E(z,t) = 0$

$$\Rightarrow K = \frac{n\pi}{l} \quad ; \quad n\text{ - integer}$$

- the total averaged stored energy per unit volume

$$\frac{U}{V} = \frac{1}{2} \overline{(\varepsilon E_x^2 + \mu H_y^2)}$$

ε, μ are permittivity & permeability of the medium filling the cavity. —— indicates averaging over the whole resonator.

- Since $H_y = \frac{E_x}{Z} = E_x\sqrt{\frac{\varepsilon}{\mu}}$

  where Z - impedance of the medium between the plates

  => the total average stored energy per unit volume

$$\frac{U}{V} = \overline{\varepsilon E_x^2}$$

- the total energy stored is

$$U = \frac{A}{T}\,\varepsilon \int_0^{\ell}\int_0^{T} E^2(z,t)\,dz\,dt = \frac{1}{4}\varepsilon E^2 V$$

  where T - oscillation period of the field.

- if the power input to the resonator is P,

$$\boxed{Q = \frac{2\pi\nu_0 U}{P} = \frac{2\pi\nu_0\,\varepsilon E^2 V}{4P}}$$ and

$$\boxed{E = \sqrt{\frac{4QP}{2\pi\nu_0\,\varepsilon V}}}$$

if $Q\uparrow$ and a given P input in a resonator E stored is high.

Quality factor of a resonator in terms of the Transmission of its End Reflectors

- Consider quality factor "Q" of an open resonator that has two reflectors with equal transmittance $T << 1$.

- consider the decay of stored energy in a resonator of length "ℓ"
- the initial stored energy "U_0"

- at a later time this has been reduced to $U(t)$ because of transmission through the end reflectors.
- the resonator is symmetrical — equal amounts of energy $\frac{U(t)}{2}$ are propagating towards each mirror.
- in a short time dt, the energy lost from the resonator is

$$-dU = U(t)\left(\frac{T\cdot c}{\ell}\right)dt$$

$$-\frac{dU}{U} = \left(\frac{c}{\ell}T\right)dt$$

with boundary conditiones at $t=0$ $U(0)=U_0$

$$\boxed{U = U_0\, e^{-\left(\frac{cT}{\ell}\right)t}}$$

time constant of decay of energy stored in reson.

$$\tau_0 = \frac{\ell}{cT}$$

$$U = U_0 e^{-\left(\frac{cT}{\ell}\right)t}$$

the rate at which energy is dissipated is

$$-\frac{dU}{dt} = \left(\frac{cT}{\ell}\right) U_0 e^{-\left(\frac{cT}{\ell}\right)t}$$

- The Q of the resonator is

$$Q = \frac{2\pi\nu_0 \times \text{stored energy}}{\text{rate at which energy is dissipated}} =$$

$$= \frac{2\pi\nu_0 \cdot U_0 e^{-\left(\frac{cT}{\ell}\right)t}}{\left(\frac{cT}{\ell}\right) U_0 e^{-\left(\frac{cT}{\ell}\right)t}}$$

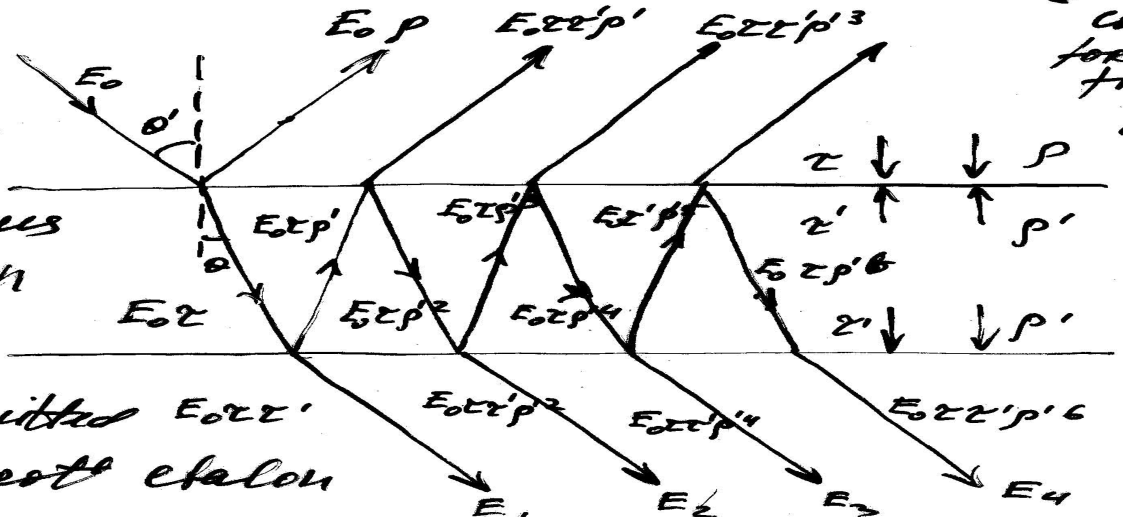
$$\boxed{Q = \frac{2\pi\nu_0 \ell}{cT} = 2\pi\nu_0\tau_0}$$

- if τ_0 is high $\rightarrow$ Q is high
- if the resonator contains an amplifying medium, and the gain of the medium is high, it prevents the decay of energy and sustains an oscillation.

Fabry-Perot Etalons and Interferometers

Fabry-Perot structures serve as the resonator in most laser systems.

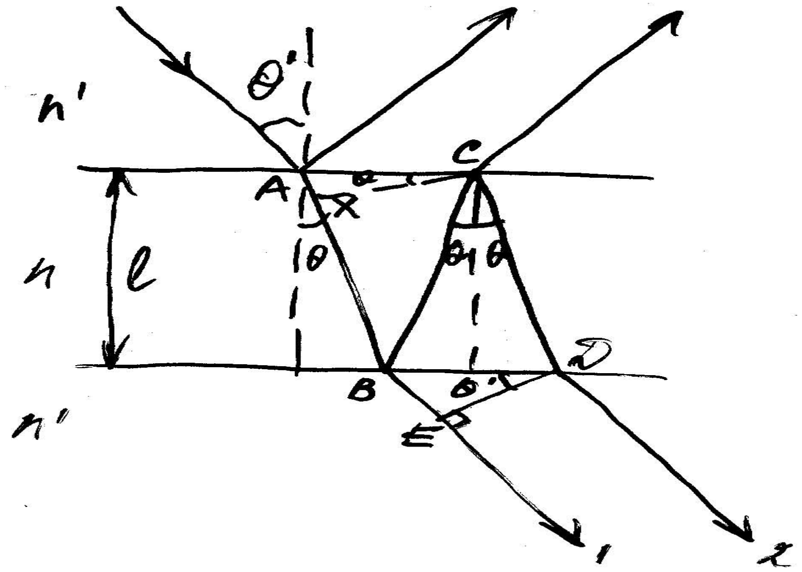
- In Fabry-Perot **Etalon**, which consists of a pair of plane, parallel, optical interfaces or reflectors of constant separation, interference occurs between the beams of light that are multiply reflected between the two interfaces or reflectors.
- If the optical spacing of these interfaces or reflectors can be changed the device is called a Fabry-Perot interferometer.
- The simplest kind of etalon consists of a piece of plane, parallel-sided material of refr. index "n", immersed in a medium refr. ind. "n'". Parallel faces should be flat to two of λ of operation.
- Consider what happens when a plane wave "r" is incident at etalon at angle of incidence θ'



Electric field amplitudes of the various singly and multiply reflected and transmitted waves in Fabry-Perot etalon

- the reflection coefficient ("ρ") for waves travel. from n' to n and (ρ') - for waves travel. from n to n'.
- similarly τ and τ' - transmission coefficients.

Consider the phase difference between two successive transmitted waves



path difference between two successive transmitted waves 1 and 2 is

$$n(BC+CD) - n'(BE)$$

$$CD = BC = \frac{l}{\cos\theta}$$

$$BD = 2l\tan\theta$$

$$BE = BD\cdot\sin\theta' = BD\cdot\sin\theta\frac{n}{n'}$$

$$\Rightarrow \quad n(BC+CD) - n'(BE) = \frac{2nl}{\cos\theta} - 2nl\tan\theta\sin\theta =$$

$$= \frac{2nl}{\cos\theta} - \frac{2nl\sin^2\theta}{\cos\theta} = \underline{2ln\cos\theta}$$

the phase difference $\delta = k\times PD = \frac{2\pi}{\lambda_0}\times PD =$

$$\boxed{\delta = \frac{4\pi n l\cos\theta}{\lambda_0} + 2\varepsilon}$$

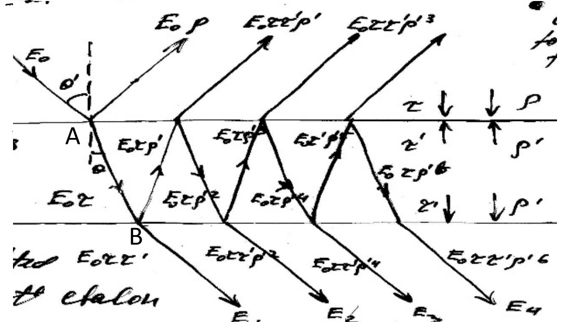
ε is the phase change (if any) that occurs on reflection and λ_0 is the wavelength of the wave in vacuum.

If the incident wave is of the form

$$E = |E_0| e^{i(\omega t - kz)}$$

or in terms of its complex amplitude,

$$E = E_0 e^{i\omega t} \quad (\text{where } E_0 = |E_0| e^{-ikz}) \text{ then}$$



$$E_1 = E_0 e^{-i\delta_0} \tau \tau'$$

$$E_2 = E_0 e^{-i\delta_0} \tau \tau' \rho'^2 e^{-i\delta}$$

$$E_3 = E_0 e^{-i\delta_0} \tau \tau' \rho'^4 e^{-2i\delta}$$

where δ_0 is the phase difference introduced by the optical path AB.

- The total complex amplitude of the transmitted beam is

$$E_t = E_0 e^{-i\delta_0} \tau \tau' (1 + \rho'^2 e^{-i\delta} + \rho'^4 e^{-2i\delta} + \ldots)$$

This is a geometrical series with ratio $\rho'^2 e^{-i\delta}$, and first term $= 1$.

Its sum to n terms is

$$(E_t)_n = E_0 e^{-i\delta_0} \tau \tau' \frac{[1 - (\rho'^2)^n e^{-in\delta}]}{(1 - \rho'^2 e^{-i\delta})}$$

since $|\rho'| < 1$ the sum to infinity is

$$\boxed{E_t = \frac{E_0 e^{-i\delta_0} \tau \tau'}{1 - \rho'^2 e^{-i\delta}}}$$

if the interfaces between the media with indices n and n' are not made specially reflecting then in the case of normal incidence

$$\rho = \frac{n'-n}{n'+n}$$

$$\rho' = \frac{n-n'}{n+n'}$$

So $\rho = -\rho'$ and there is a phase change on reflection from the interface if $n' > n$

$$\tau = \frac{2n'}{n'+n}$$

$$\tau' = \frac{2n}{n'+n}$$

$|\rho^2| = (\rho')^2 = R$ - is called reflectance of the interface

$$R = \left| \frac{E_{reflected}}{E_{incident}} \right|^2$$

By a similar procedure as for transmitted wave

$$E_r = E_0 \rho + E_0 \tau \tau' \rho' e^{-i\delta} + E_0 \tau \tau' \rho'^3 e^{-2i\delta} + \ldots =$$

$$= E_0 [\rho + \tau \tau' \rho' e^{-i\delta} (1 + \rho'^2 e^{-i\delta} + \rho'^4 e^{-2i\delta} + \ldots)]$$

Summing to infinity gives

$$\boxed{E_r = \frac{E_0 (\rho - \rho \rho'^2 e^{-i\delta} + \tau \tau' \rho' e^{-i\delta})}{1 - \rho'^2 e^{-i\delta}}}$$

As far as the transmitted intensity through one interface is concerned we can define (next)

$$T = \frac{I_{transm}}{I_{incident}} = \left(\frac{|E_{trans}|^2}{Z \text{ in transmitted medium}}\right)\left(\frac{Z \text{ in incid. m.}}{|E_{incident}|^2}\right) =$$

$$= \left|\text{transmission coeff.}\right|^2 \left(\frac{Z \text{ in incident med.}}{Z \text{ in transmitted med.}}\right)$$

for the $n' \to n$ interface

$$Z' = \sqrt{\frac{\mu' \mu_0}{\varepsilon' \varepsilon_0}}$$

ε', μ' - dielectric constant and relative magnetic permeability, respectively.

if $\mu' = 1 \Rightarrow \sqrt{\varepsilon'} = n'$, so

$$Z' = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{1}{n'}$$

In normal incidence

$$T = \frac{(\tau')^2 Z'}{Z} = \left(\frac{2n'}{n'+n}\right)^2 \frac{n}{n'} = \frac{4nn'}{(n'+n)^2}$$

For the $n \to n'$ interface at normal incidence

$$T = \frac{(\tau)^2 Z}{Z'} = \frac{4nn'}{(n'+n)^2}$$

So the T coefficient is independent of which way the wave travels through the interface and

$$\boxed{R + T = \frac{(n-n')^2}{(n+n')^2} + \frac{4nn'}{(n'+n)^2} = 1}$$

- the ratio of the mutually orthogonal $\bar{E}$ and $\bar{H}$ components is called the impedance Z of the medium

$$\frac{E_x}{H_y} = \frac{-E_y}{H_x} = Z = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}}$$

μ_r - relative magnetic permeability
ε_r - dielectric constant of the medium
ε_0 - permittivity of free space 8.85×10^{-12} F/m
μ_0 - permeability of free space $4\pi \times 10^{-7}$ H/m

$$C_0 = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = 2.9979 \times 10^8 \text{ m/s}$$

- The Poynting vector $\bar{S} = \bar{E} \times \bar{H}$ is a vector which points in the direction of energy propagation of the wave.

- The average magnitude of the Poynting vector is called the intensity I

$$I = \langle |S| \rangle_{AV} = \frac{|E|^2}{2Z}$$

Now $\tau\tau' = \dfrac{4nn'}{(n'+n)^2} = T$

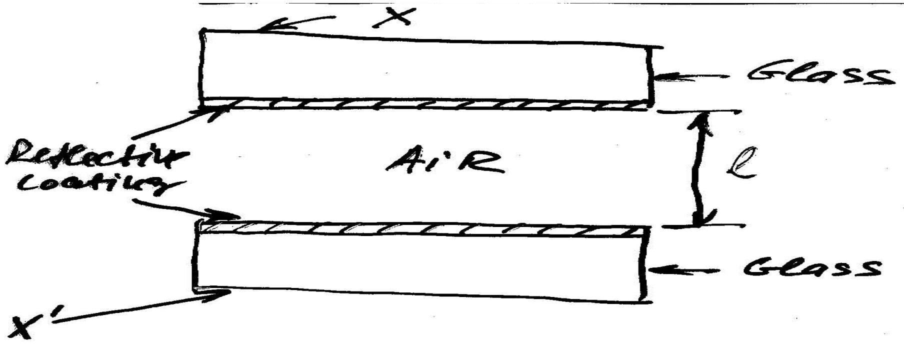
So $\boxed{E_t = \dfrac{E_0 e^{-i\delta_0}\tau\tau'}{1-\rho'^2 e^{-i\delta}} = \dfrac{E_0 T e^{-i\delta_0}}{1-Re^{-i\delta}}}$

$$\frac{E_0(\rho-\rho\rho'^2 e^{-i\delta}+\tau\tau'\rho' e^{-i\delta})}{1-\rho'^2 e^{-i\delta}} = \boxed{E_r = \frac{E_0 r(1-e^{-i\delta})}{1-Re^{-i\delta}} = \frac{E_0\sqrt{R}\,(1-e^{-i\delta})}{1-Re^{-i\delta}}}$$

- these formulae apply in general when R and T are the reflectance and transmittance at the appropriate angle of incidence of the plane-parallel reflecting faces that constitute the etalon.

- the relations $R=\rho^2=\rho'^2$, $T=\tau\tau'$ are correct results independent of the angle of incidence.

A practical air-spaced Fabry-Perot Etalon



- The outer faces X, X' are generally antireflection coated and make a small wedge angle with the inner reflecting faces. So, it can be treated as a single device.

- if the reflecting faces of an air-spaced etalon are metal coated – the absorption of light in passing through the reflecting film should be taken into account.

- if fractional energy absorption in passing through the reflective film is A' then

$$R + T + A' = 1$$

if A' - small $\quad R + T = 1$

- The overall intensity reflection characteristics

conjugate

$$\frac{I_r}{I_i} = \frac{E_r E_r^*}{E_0 E_0^*} = \frac{E_0 E_0^* R(1-e^{-i\delta})(1-e^{i\delta})}{E_0 E_0^* (1-Re^{-i\delta})(1-Re^{i\delta})} =$$

I_i - incident ; I_r - reflected intensity

$$= \frac{R[2-(e^{i\delta}+e^{-i\delta})]}{1+R^2-R(e^{i\delta}+e^{-i\delta})} = \frac{R(2-2\cos\delta)}{1+R^2-R(2\cos\delta)}$$

since $\cos\delta = \cos(\delta/2+\delta/2) = \cos^2(\delta/2) - \sin^2(\delta/2) = 1-2\sin^2\frac{\delta}{2}$

$$\boxed{\frac{I_r}{I_i} = \frac{4R\sin^2(\delta/2)}{(1-R)^2+4R\sin^2(\delta/2)}}$$

$$\frac{I_t}{I_i} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(\delta/2)} = \frac{1}{1+[4R/(1-R)^2]\sin^2(\delta/2)}$$

- the transmittance is unity whenever

$$\sin(\delta/2) = 0$$

$$\delta/2 = m\pi \qquad m \text{ - integer}$$

$$\frac{4\pi n\, l\cos\theta}{\lambda_0} + 2\varepsilon = 2m\pi$$

- if the phase change on reflection is 0 or π then for max transmittance

$$l = \frac{m\lambda_0}{2n\cos\theta} = \frac{m\lambda}{2\cos\theta};$$

λ - wavelength in the material between the reflectors

In normal incidence

$$\boxed{l = \frac{m\lambda}{2}}$$

- the frequencies of max transmission

$$\boxed{\nu_m = \frac{mc_0}{2nl\cos\theta}}$$

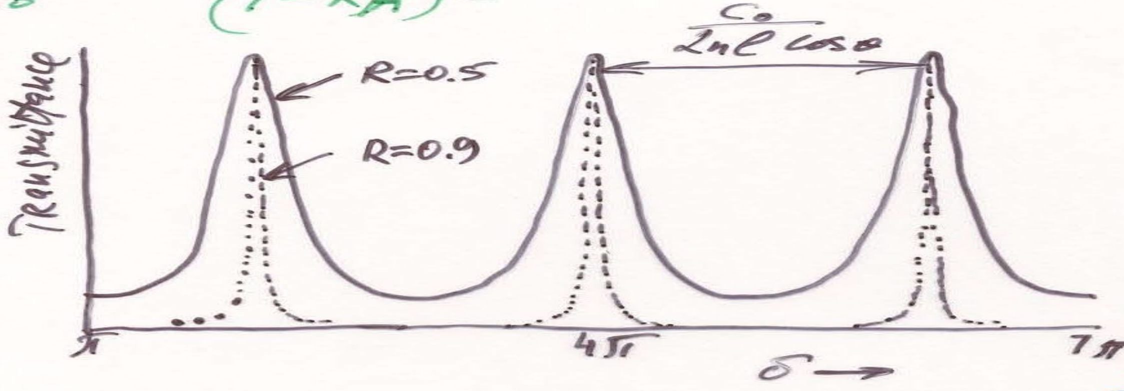
- adjacent frequencies at which etalon shows max transmission

$$\boxed{\Delta\nu = \frac{c_0}{2nl\cos\theta}}$$ — free spectral range of the etalon.

frequencies are equally spaced - comb filter.

- if there are losses in the ethalon

$$\frac{I_t}{I_0} = \frac{(1-R)^2 A}{(1-RA)^2} \text{ — peak transmission falls}$$

- 

coefficient of finesse $F = \frac{4R}{(1-R)^2}$

$\mathcal{F} = \frac{\pi}{2}\sqrt{F}$ finesse

As $R \uparrow$ the sharpness $\uparrow$

$$\frac{I_t}{I_i} = \frac{1}{1+4R(1-R)^2\sin^2(\delta/2)}$$

- $\boxed{\mathcal{F} = \frac{\pi\sqrt{R}}{1-R}}$ — finesse of the instrument

- near a transm. max $\delta = 2m\pi + \Delta$; Δ – small angle.

$$\frac{I_t}{I_i} = \frac{1}{1+\mathcal{F}^2\Delta^2/\pi^2}$$

$$\frac{(2/\pi\,\Delta\nu)}{1+[2(\nu-\nu_m)/\Delta\nu]^2}$$

this is Lorentzian function of Δ with FWHM $\frac{2\pi}{\mathcal{F}}$.

- since $\delta = \frac{4\pi n l \nu \cos\theta}{c_0}$

and $\nu_m = \frac{m c_0}{2nl\cos\theta} \Rightarrow m = \frac{2nl\nu_m\cos\theta}{c_0}$

$$\Delta = \delta - 2m\pi = \frac{4\pi n l \nu \cos\theta}{c_0} - \frac{4\pi n l \nu_m \cos\theta}{c_0} =$$

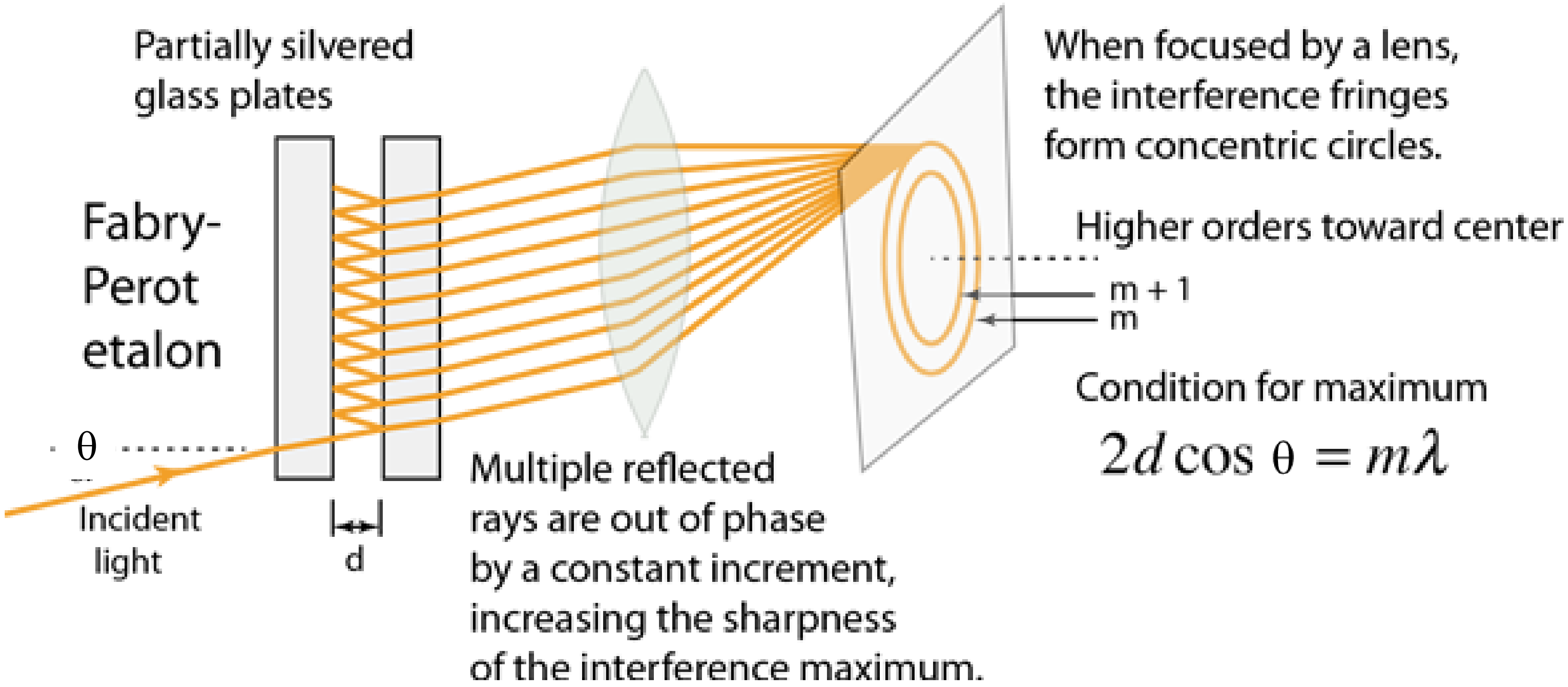
$$= \frac{4\pi n l}{c_0}(\nu - \nu_m)\cos\theta$$

=> transmission peaks are Lorentzian functions of the frequency spacing from the transmission maximum ν_m

$$\Delta\nu_{1/2} = \frac{c_0}{(2nl\cos\theta)\mathcal{F}} = \frac{\Delta\nu}{\mathcal{F}} = \frac{\text{free spectral range}}{\text{finesse}}$$

Fabry-Perot Interferometer

This interferometer makes use of multiple reflections between two closely spaced partially silvered/coated surfaces. Part of the light is transmitted each time the light reaches the second surface, resulting in multiple offset beams which can interfere with each other. The large number of interfering rays produces an interferometer with extremely high resolution.



Fringe Profiles: The Airy Function

- The variation of the irradiance in the fringe pattern of the Fabri-Perot as a function of the phase or path difference is called the <u>fringe profile</u>.
- The sharpness of the fringes is important to the ultimate resolving power of the instrument.
- The irradiance provided by the resultant of the transmitted beams

$$I_T = \left[\frac{(1-r^2)^2}{1+r^4-2r^2\cos\delta} \right] I_i$$

- using trigonometric identity
$\cos\delta = 1 - 2\sin^2\frac{\delta}{2}$ and simplifying $\Rightarrow$
the Transmittance T, or Airy function

$$T = \frac{I_T}{I_i} = \frac{1}{1+\left[\frac{4r^2}{(1-r^2)^2}\right]\sin^2\left(\frac{\delta}{2}\right)}$$

- the square-bracket factor which is a function of the reflection coefficient or reflectance was called the coefficient of <u>finesse</u>.

$$F = \frac{4r^2}{(1-r^2)^2} = \frac{4R}{(1-R)^2}$$

$$\Rightarrow \quad T = \frac{1}{1+F\sin^2\left(\frac{\delta}{2}\right)}$$

- the quantity F is a sensitive function of the reflection coefficient, since as r varies from 0 to 1, F varies from 0 to infinity.

It is important to realize that the **intensity** is defined as the amount of energy per unit time going through an area perpendicular to the beam, while **irradiance** refers to what amount of energy arrives on a certain surface with a given orientation.

The irradiance caused by a laser beam, for example, which hits a workpiece under some angle θ against normal direction, is the beam intensity times cos θ.

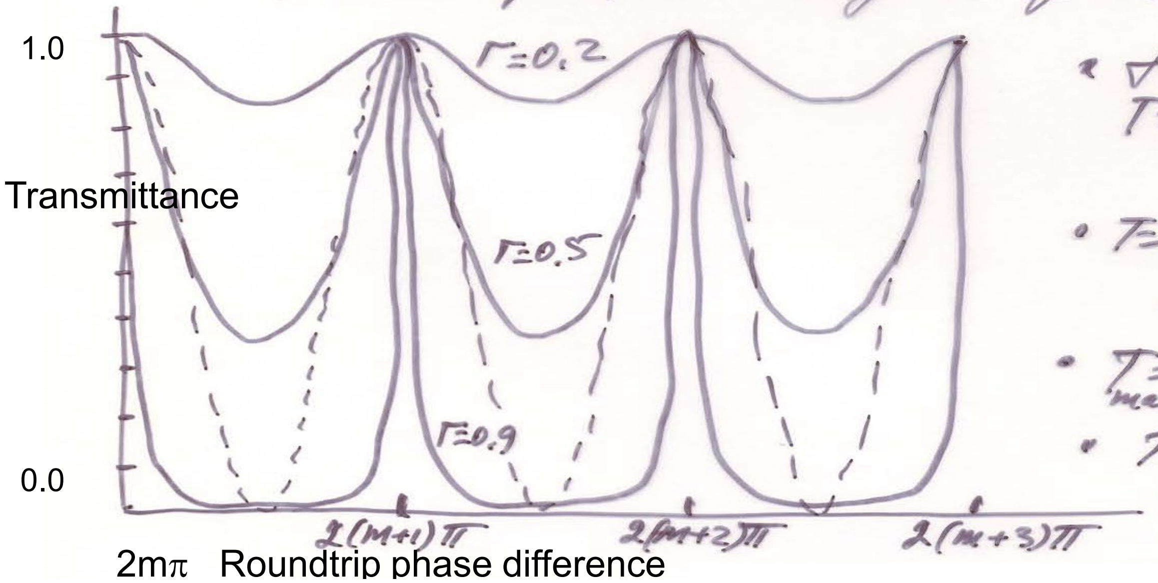
- F also represents a certain measure of fringe contrast,

$$\frac{(I_T)_{max} - (I_T)_{min}}{(I_T)_{min}} = \frac{T_{max} - T_{min}}{T_{min}}$$

$$\left.\begin{aligned} &T_{max} = 1 \text{ when } \sin\delta/2 = 0 \\ &T_{min} = \frac{1}{1+F} \quad \text{when } \sin\delta/2 = \pm 1 \end{aligned}\right\} \Leftarrow T = \frac{1}{1+F\sin^2\frac{\delta}{2}}$$

$$\Rightarrow \frac{1 - \frac{1}{1+F}}{\frac{1}{1+F}} = F$$

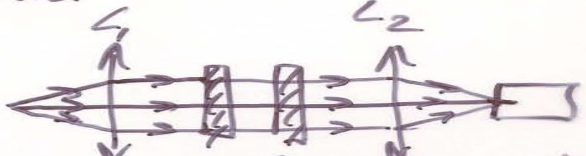
- the fringe profile may be plotted if r is chosen.



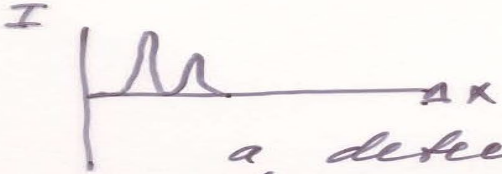
- for each curve $T = T_{max} = 1$ at $\delta = m(2\pi)$
- $T = T_{min} = \frac{1}{1+F}$ at $\delta = (m+\frac{1}{2})2\pi$
- $T_{max} = 1$ regardless of r
- T_{min} is never zero
- $T_{min} \to 0$ r → 1.

Resolving Power

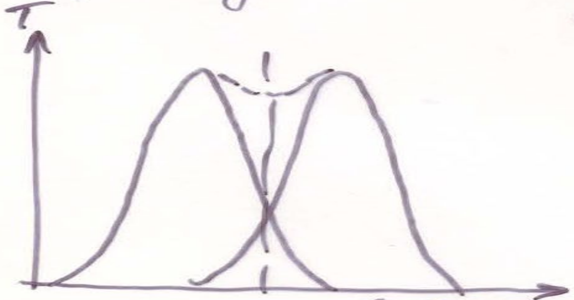
- when 2 wavelength components are present in the incident light the Fabry-Perot interferometer gives a double set of circular fringes, each set belonging to one of the wavelengths.
- a detector scanning across the width of two closely spaced rings in a particular order m of the interference, might give a plot like



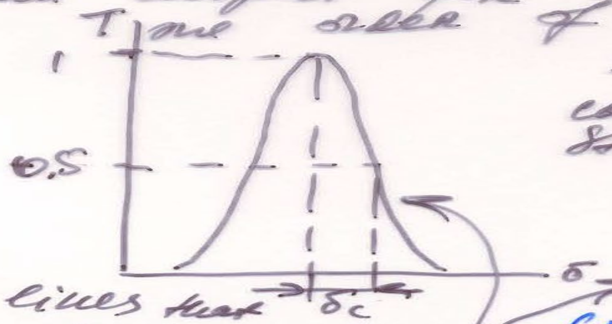
F-P interf with point source and variable spacing



a detector at the focal point of L_2 is connected to a plotter gives an output for a translation through one order of interference.



only sum of two lines that follows dashed line is measured.



Scan of 2 wavelength components of comparable strength in a F-P fringe pattern.

Resolution criterion: half-width of the peak at half-maximum corr. to phase interval $\left(\sin\frac{\delta_c}{2}\right) = \frac{1}{\sqrt{F}}$

Crossover point be not more than half the max. irradiance of each indiv. peaks — Rayleigh criterion. $T = \frac{1}{1 + F\sin^2\left(\frac{\delta}{2}\right)}$

$$\sin\frac{\delta_c}{2} = \frac{\delta_c}{2} = \frac{1}{\sqrt{F}}; \quad \to \delta_c = \frac{2}{\sqrt{F}}$$

Phase diff. between two fringe maxima is twice larger $(\Delta\delta)_{min} = \frac{4}{\sqrt{F}}$

- The corresponding min resolvable wavelength difference will be found as follows:

$$\delta = 4\pi t \cos\theta \left(\frac{1}{\lambda}\right)$$

- for small wavelength intervals

$$\Delta\delta = (4\pi t \cos\theta_t)\,\Delta\left(\frac{1}{\lambda}\right) = \left(\frac{4\pi t \cos\theta_t}{\lambda^2}\right)\Delta\lambda$$

- combining with $(\Delta\delta)_{min} = \frac{4}{\sqrt{F}}$

$$(\Delta\lambda)_{min} = \frac{\lambda^2}{\pi\sqrt{F}\, t\cos\theta_t}$$

- Since at the fringe maxima

$$2t\cos\theta_t = m\lambda$$

we write more simply.

$$(\Delta\lambda)_{min} = \frac{2\lambda}{m\pi\sqrt{F}}$$

- Resolving power $\boxed{R = \frac{\lambda}{(\Delta\lambda)_{min}} = \left(\frac{\pi}{2}\right) m\sqrt{F}}$ =m∗Finess

- large resolving powers are desirable.
  $R\uparrow$ when $m\uparrow$ — near the center of fringe pattern. and $F\uparrow$ or $r\uparrow$

- to maximize m at the center

$$m = \frac{2t}{\lambda}$$ — separation t is max.

Example

F-P. interf has a 1-cm spacing a $r = 0.95$. For $\lambda = 500\ nm$ determine its max. order of interference F, its min resolvable $(\Delta\lambda)_{min}$ and R.

1) Using $F = \frac{4r^2}{(1-r^2)^2}$; $(\Delta\lambda)_{min} = \frac{2\lambda}{m\pi\sqrt{F}}$

$R = \frac{\lambda}{(\Delta\lambda)_{min}}$; $R = \left(\frac{\pi}{2}\right) m\sqrt{F}$; $m_{max} = \frac{2t}{\lambda}$

Find

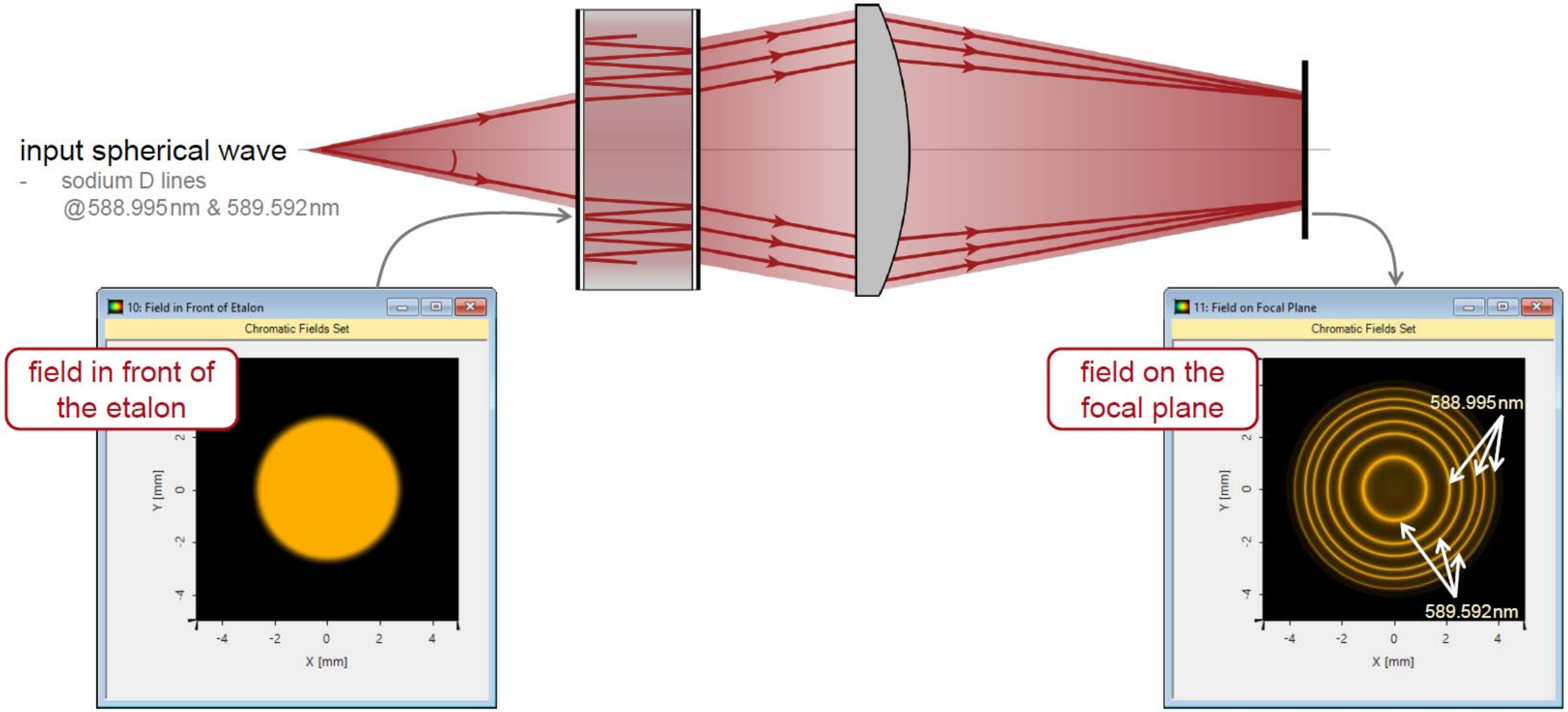
2) $m_{max} = \frac{2t}{\lambda} = \frac{2(1\times10^{-2})}{500\times10^{-9}} = 40{,}000$

3) $F = \frac{4r^2}{(1-r^2)^2} = \frac{4(0.95)^2}{(1-0.95^2)^2} = 380$

4) $\Delta\lambda_{min} = \frac{2\lambda}{m\pi\sqrt{F}} = \frac{2(5000\,Å)}{(40{,}000)\pi\sqrt{380}} = 0.004\,Å$

5) $R = \frac{\lambda}{(\Delta\lambda)_{min}} = \frac{5000}{0.004} = 1.2\times10^6$

Visualization of Both Spectrum Lines



Fabry-Perot Interferometers as Optical Spectrum Analyzers

Since F-P etalon or interferometer has $T = f(\nu)$, it can be used to analyze the spectral output of a source of light.

For a white light illumination, θ = const, for a given spacing ℓ, the frequency distribution of transmitted light only shows large intensities for frequencies

$$\nu_m = \frac{mc_o}{2n\ell\cos\theta}$$

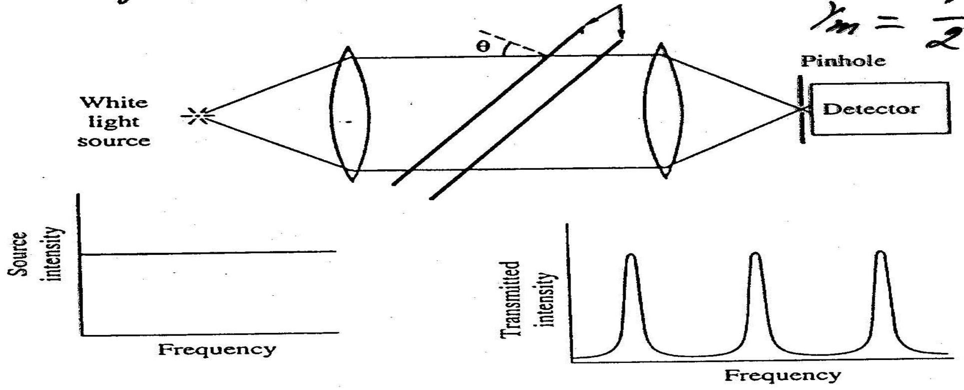


Fig. 4.9. Schematic arrangement in which a Fabry–Perot etalon is illuminated at angle θ with parallel light from a white light source.

$$2l\cos\theta = m\lambda_m;\ 2l\cos\theta = m\frac{c}{\nu_m};$$

$$2l\cos\theta = m\frac{c_o}{n\nu_m};\ \nu_m = \frac{mc_o}{2nl\cos\theta}$$

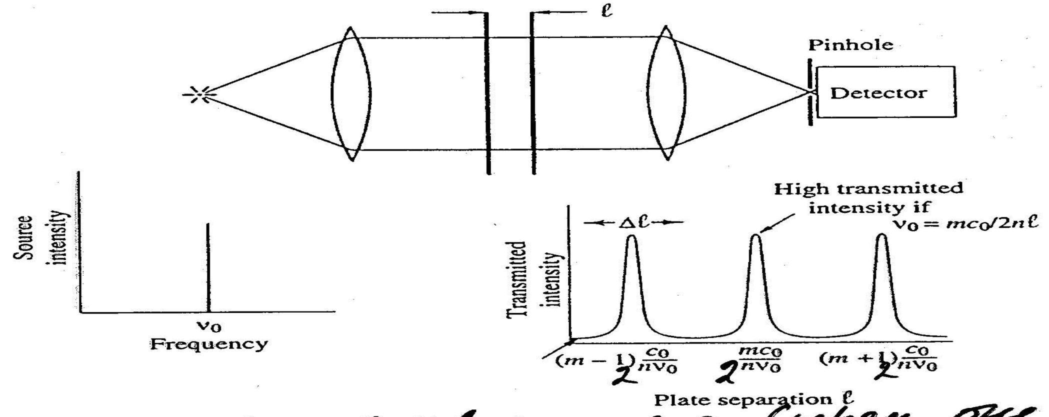


Fig. 4.10. Schematic arrangement in which a Fabry–Perot interferometer of adjustable spacing is illuminated normally with monochromatic parallel light.

signal will only be transmitted when frequency of monochromatic source

$$\nu_o = \frac{mc_o}{2n\ell}$$

$$l = \frac{mc_o}{2n\nu_o}$$

For F-P interferometer (when one plate is movable) transm. max occur for plate separations $\ell = \frac{mc_o}{2n\nu_o\cos\theta}$. The plate movement between successive max $\Delta\ell$, is equal $\frac{c_o}{2n\nu_o\cos\theta} = \frac{\lambda_o}{2n\cos\theta} = \frac{\lambda}{2}$ in normal incidence in the medium.

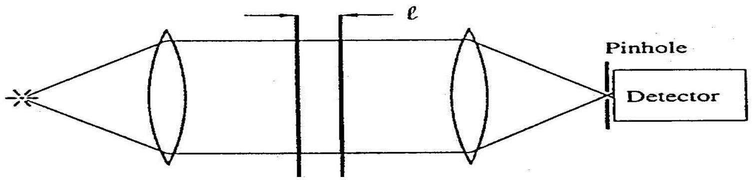
λ - wavelength of the signal in the medium.

In normal incidence plate separations for max. T are separated by a half wavelength interval.

White light is passed through a Fabry-Perot interferometer in the arrangement shown in Figure b, where the detector is a spectroscope. A series of bright bands appear. When mercury light is simultaneously admitted into the spectroscope slit 150 of the bright bands appear between the violet and green lines of mercury at 435.8 nm and 546.1 mm, respectively. What is the thickness of the etalon?

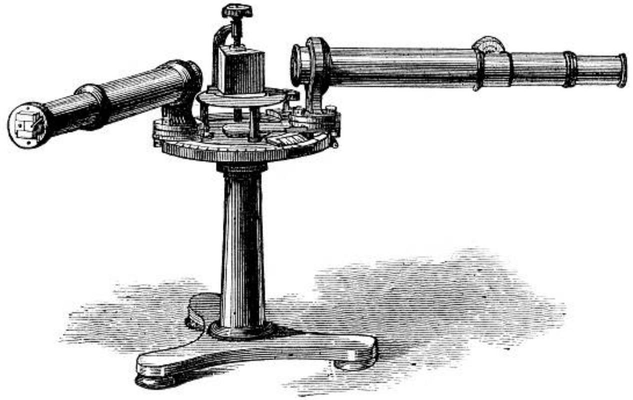
(1) $2L = m\lambda_1$

(2) $2L = (m+150)\lambda_2$



eqn (1) $\Rightarrow$ $m = \frac{2L}{\lambda_1}$ (3)

rewrite eqn. (2) interms of eqn. (3)



$\Rightarrow$ $2L = \left(\frac{2L}{\lambda_1} + 150\right)\lambda_2$

$\Rightarrow$ $2L - \frac{2L\lambda_2}{\lambda_1} = 150\lambda_2$

$\Rightarrow$ $L\left(1 - \frac{\lambda_2}{\lambda_1}\right) = 75\lambda_2$

$\Rightarrow$ $L = \dfrac{75\lambda_2}{1 - \frac{\lambda_2}{\lambda_1}}$, $\lambda_2 = 435.8$ nm, $\lambda_1 = 546.1$ nm

$\Rightarrow$ $L = \dfrac{75(435.8 \text{ nm})}{1 - \frac{435.8 \text{ nm}}{546.1 \text{ nm}}}$

$= 1.62 \times 10^5$ nm

$\therefore$ $\boxed{L = .162 \text{ mm}}$

It is possible to use a F-P. Etalon for angular discrimination between transmitted beams at different frequencies. For monochr. pointlike source illumination of etalon the angles at which transmission maxima occur

$\cos\Theta_m = \frac{mc_o}{2n\ell\nu_m}$

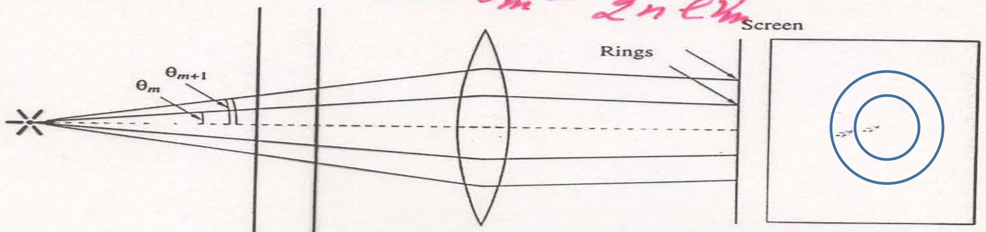


Fig. 4.11. Schematic arrangement in which a Fabry–Perot etalon is illuminated with a monochromatic point source and a ring pattern of transmitted light is observed.

If the source under study is diffuse the lens brings all the transmitted intensity max. at angle Θ_m into focus in a ring in the focal plane.

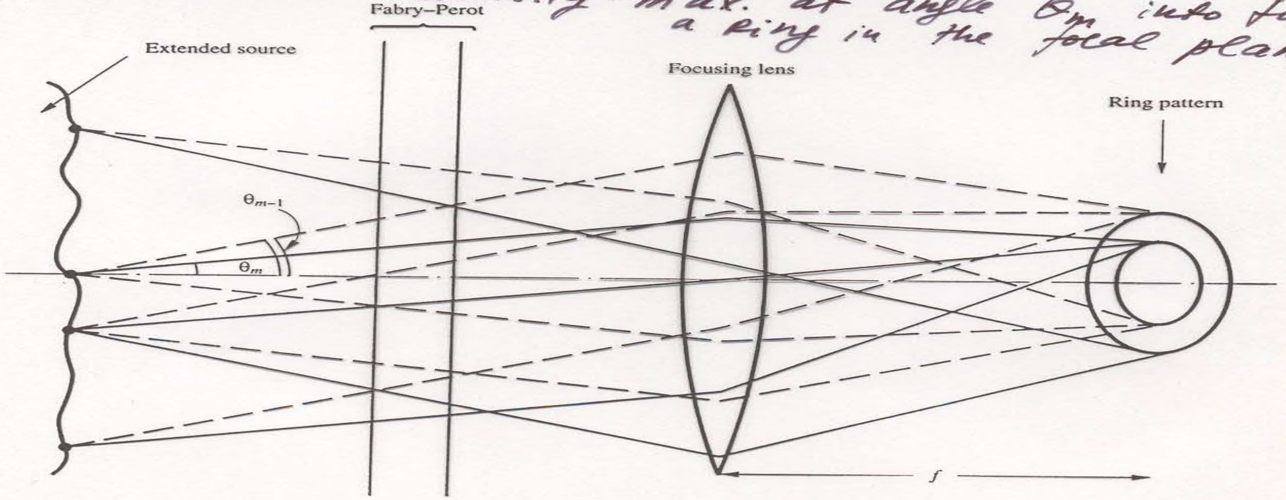


Fig. 4.12. Schematic arrangement used for viewing Fabry–Perot rings with a diffuse monochromatic source and an air-spaced etalon.

If there is a bright point at the center of the focused ring pattern this implies $\Theta_m = 0$ =>

$m\lambda_o = 2n\ell$

For the next order max $\cos\Theta_{m-1} = (m-1)\frac{\lambda_o}{2n\ell}$

If Θ_{m-1} is small

$$-\begin{cases} 2n\ell\left(1 - \frac{\Theta_{m-1}^2}{2}\right) = (m-1)\lambda_o \\ 2n\ell = m\lambda_o \end{cases}$$

$$2n\ell\,\frac{\Theta_{m-1}^2}{2} = \lambda_o \quad \Rightarrow \quad \Theta_{m-1} = \sqrt{\frac{\lambda_o}{n\ell}}$$

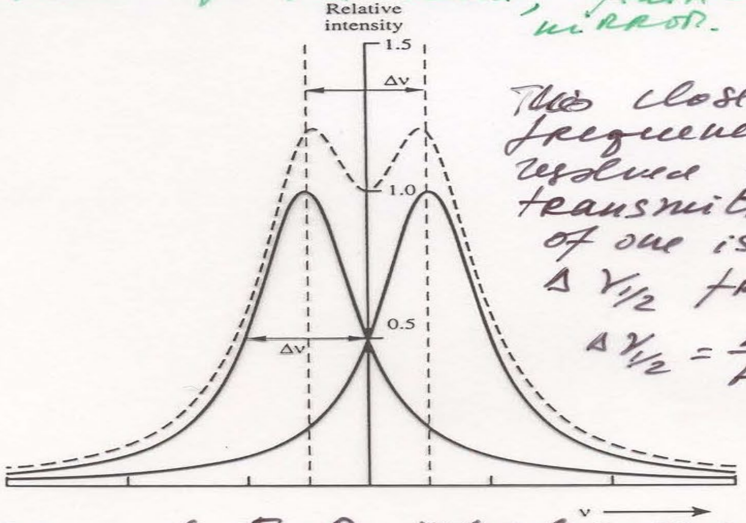
Example

- For $\lambda_0 = 500\,nm$, $n = 1$; $\ell = 100\,mm$, $\Theta \approx 2.24\times10^{-3}\,rad$.
- if the focal length of the lens is "f" the physical separation of adjacent rings near the center of the pattern is $\rho \simeq f\Theta = f\sqrt{\frac{\lambda_0}{n\ell}}$

For $f = 500\,mm$; $f\Theta \approx 1\,mm$

Although the finesse $\boxed{F = \frac{\pi\sqrt{R}}{1-R}}$ is the factor that determines the sharpness of transmission maxima, the true finesse is dependent on the surface flatness of the mirror, angular spread of the beam, finite-size of the mirror.

Fig. 4.13. Transmitted intensity (dashed line) as a function of plate separation when a Fabry–Perot interferometer is illuminated with two monochromatic lines (solid lines) closely spaced in frequency that the interferometer is *just* able to resolve.



The closely spaced frequencies can be resolved if the transmitted max. of one is a frequency $\Delta\nu_{1/2}$ from the other.

$\Delta\nu_{1/2} = \frac{\Delta\nu}{F}$ – resolution

Characteristics of F.-P. interferometers with different reflectances & spacings.
$\Theta = 0$; $\lambda = 500\,nm$; $n = 1$

ℓ, (mm)	$\Delta\nu = \frac{c_0}{2\ell}$ (Hz)	R	F	$\Delta\nu_{1/2}$ (Hz	$\frac{\nu}{\Delta\nu_{1/2}}$ = Resolving Power
10	1.5×10^{10}	0.9	30	5×10^{8}	1.2×10^{6}
100	1.5×10^{9}	0.9	30	5×10^{7}	1.2×10^{7}
10	1.5×10^{10}	0.95	61	2.46×10^{8}	2.44×10^{6}
100	1.5×10^{9}	0.95	61	2.46×10^{7}	2.44×10^{7}

Problem

A Fabry-Perot etalon is illuminated with monochromatic radiation at a wavelength of 488.79 nm (in vacuo). The etalon has $n = 1.55$ and is 7.4 mm thick.

(i) Calculate the minimum change in temperature necessary to produce a bright spot at the center of the ring pattern. Take the coefficient of expansion of the etalon as $3\times10^{-6}\,K^{-1}$.

(ii) What is the maximum divergence of the input light if only one ring is seen?

(i) $\Delta l = \alpha l \Delta T$; $\Delta T = \dfrac{\Delta l}{\alpha l}$ where $\alpha = 3\times10^{-6}\,K^{-1}$

For a bright spot at the center

$$m\lambda_0 = 2nl \implies m = \frac{2nl}{\lambda_0} = \frac{2\cdot(1.55)(7.4\times10^{-3})}{488.79\times10^{-5}} =$$

$$= 46932.220$$

m is not integer

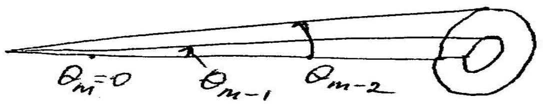
want m to be integer to have a bright spot at the center

$\therefore \Delta m = 0.220$

$$\implies \Delta l = \frac{\Delta m \lambda}{2n} = -36.69\,nm$$

$$\Delta T = \frac{\Delta l}{\alpha l} = \frac{(-34.69\times10^{-9}m)}{(3\times10^{-6}K^{-1})(7.4\times10^{-3}m)} = -1.56\,°C$$

(ii)



$$\cos\theta_{m-2} = \frac{(m-2)\lambda}{2nl} = \frac{(46932.0-2)(488.79\times10^{-9})}{2\cdot(1.55)(7.4\times10^{-3})} =$$

$$= 0.999957$$

$$\theta_{m-2} = 9.23\,mrad = 0.529°$$

(iii) A second monochromatic signal is present at 489.32 nm.

a) Do the transmitted rings have the same order within a spectral range

b) What min. equal reflectance of the plates of the etalon is needed to just resolve the two wavelengths

---

a) The free spectral range of the etalon is

$$\Delta\nu_{FSR} = \frac{c_0}{2n\ell\cos\theta}$$

take $\cos\theta = 1$ where resolution is the best

$$\Delta\nu_{FSR} = \frac{3\times10^8}{2\cdot1.55\cdot7.4\times10^{-3}} = 1.31\times10^{10}\ Hz$$

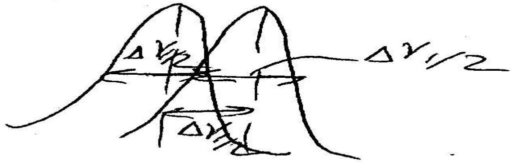
The frequency difference

$$\Delta\nu = \frac{c\,\Delta\lambda}{\lambda_1\lambda_2} = \frac{(3\times10^8)(489.32-488.79)}{(489.32)(489.79)} = 6.63\times10^{11}\ Hz$$

Since $\Delta\nu_{FSR} < \Delta\nu$ the transmitted rings do not have the same order.

---

b) $\Delta\nu_{1/2} = \dfrac{\Delta\nu_{FSR}}{F}$



$F = \dfrac{\pi\sqrt{R}}{1-R}$

assume $\Delta\nu_{1/2} = 1\,GHz$.

$$F = \frac{1.31\times10^{10}}{10^9} = 13.1$$

$F^2(1-R^2) = \pi R^2$; $\quad R^2 - \left(2+\frac{\pi^2}{F^2}\right)R + 1 = 0$

$$R = \frac{\left(2+\frac{\pi^2}{F^2}\right) \pm \sqrt{\left(2+\frac{\pi^2}{F^2}\right)^2 - 4}}{2}$$

$R = 1.27$; $R = 0.787$!

not physical

Laser Physics I

PH481/581-VT1 (Mirov)

Optical Resonators Containing Amplifying Media

Lectures 12-13 chapter 4

Fall 2025

C. Davis, “Lasers and Electro-optics”

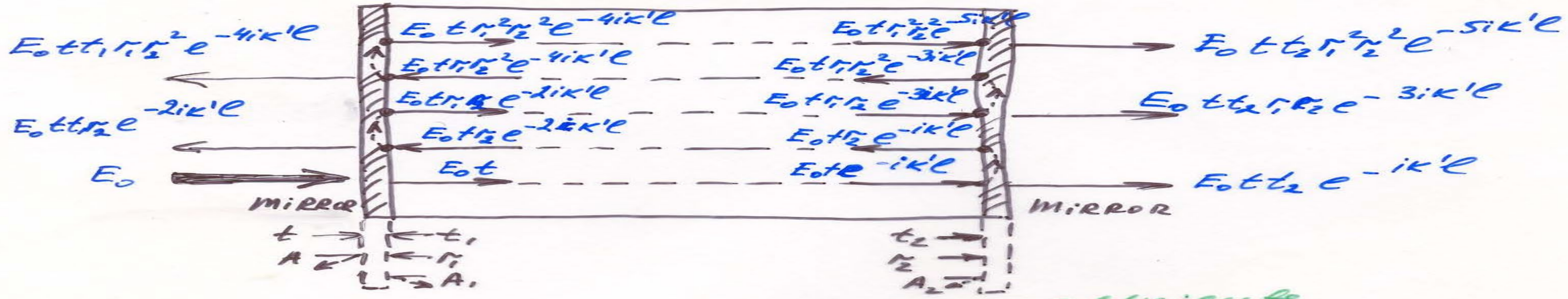
Optical resonators containing amplifying media

- Combination of our knowledge of optical frequency amplification and feedback characteristics of Fabry-Perot systems
- Laser oscillation will occur at specific frequencies if the gain of the medium is large enough to overcome the loss of energy through the mirrors and by other mechanisms within the laser medium.
- Once laser oscillation is established, it stabilizes at a level that depends on the saturation intensity of amplifying medium and the reflectance of the laser mirrors.
- How these factors (saturation intensity & reflectance of the laser mirrors) affect the output power that can be obtained from a laser and how this can be optimized.

Fabry-Perot Resonator Containing an Amplifying Medium

- Consider a F.-P. resonator with plane mirrors that is filled with an amplifying medium.
- Consider the complex amplitudes of the waves bouncing backwards and forwards normally between the resonator mirrors
- These waves result from an incident beam with electric vector E_0 at the first mirror.

The amplitudes of the electric field vectors of the successively transmitted amplified and reflected waves



t, t_1, and t_2 – transmission coefficients

r_1, r_2 – reflection coefficients

A, A_1, A_2 – absorption coefficients at the two mirrors that include any reflection losses or scattering that sends energy out of resonator.

- if there were nothing incide the resonator then a wave propagating between the mirrors would prop. as
  $E_0 e^{i(\omega t - kz)}$ – to the right
  $E_0 e^{i(\omega t + kz)}$ – to the left.
- the presence of a gain medium changes the passive propagation factor k
  $$k'(\omega) = k + \Delta k = k + \frac{k\chi'(\omega)}{2n^2}$$
- gain coefficient $\gamma(\omega) = -\frac{k\chi''(\omega)}{n^2}$ causes the complex amplitude of the wave to change with distance as
  $$e^{(\gamma/2)z}$$
- take into account existence of distributed loss per pass "α" due to scattering by crystals imperfections complex amplitude is modified by a factor $e^{-(\frac{i\alpha l}{2})}$ per pass
- full propagation constant of the wave
  $$k'(\omega) = k + k\frac{\chi'(\omega)}{2n^2} - \frac{ik\chi''(\omega)}{2n^2} - \frac{i\alpha}{2}$$
  the wave propagates as $e^{i(\omega t \pm k'z)}$

The output beam through the right mirror arises from the transmission of waves travelling to the right: its total electric field amplitude is:

$$E_t = E_0 t t_2 e^{-ik'l} + E_0 t t_2 r_1 r_2 e^{-3ik'l} + E_0 t t_2 r_1^2 r_2^2 e^{-5ik'l} =$$

$$= E_0 t t_2 e^{-ik'l}\left(1 + r_1 r_2 e^{-2ik'l} + r_1^2 r_2^2 e^{-4ik'l} + \ldots\right) =$$

$$= \frac{E_0 t t_2 e^{-ik'l}}{1 - r_1 r_2 e^{-2ik'l}} =$$

$$= \frac{E_0 t t_2 e^{-i(k+\Delta k)l} \cdot e^{(\gamma-\alpha)l/2}}{1 - r_1 r_2 e^{-2i(k+\Delta k)l} e^{(\gamma-\alpha)l}}$$

where $\gamma(\nu) = \left[N_2 - \left(\frac{g_2}{g_1}\right)N_1\right]\left(\frac{c^2 A_{21}}{8\pi\nu^2}\right) g(\nu_0, \nu)$

The ratio of output to input intensities is

conjugate

$$\frac{I_t}{I_0} = \frac{E_t \cdot E_t^*}{E_0 \cdot E_0^*} = \frac{\left[E_0 t t_2 e^{-i(k+\Delta k)l} \cdot e^{(\gamma-\alpha)\frac{l}{2}}\right]\left[E_0 t t_2 e^{i(k+\Delta k)l} \cdot e^{(\gamma-\alpha)\frac{l}{2}}\right]}{E_0^2\left[1 - r_1 r_2 e^{-2i(k+\Delta k)l} \cdot e^{(\gamma-\alpha)l}\right]\left[1 - r_1 r_2 e^{+2i(k+\Delta k)l} \cdot e^{(\gamma-\alpha)l}\right]} =$$

$$= \left\{ \begin{array}{l} e^{z_1+z_2} = \left(e^{z_1}\right)\left(e^{z_2}\right) \quad \text{for } z_1, z_2 \text{ - complex} \\ e^{-i(k+\Delta k)l} \times e^{i(k+\Delta k)l} = 1 \ ; \end{array} \right\} =$$

$$= \frac{t^2 t_2^2 e^{(\gamma-\alpha)l}}{\left[1 + r_1 r_2 e^{-2i(k+\Delta k)l} \cdot e^{(\gamma-\alpha)l}\right]\left[1 - r_1 r_2 e^{+2i(k+\Delta k)l} \cdot e^{(\gamma-\alpha)l}\right]} =$$

$$\frac{I_t}{I_0} = \frac{t_1^2 t_2^2 e^{(\gamma-\alpha)\ell}}{1 + r_1^2 r_2^2 e^{2(\gamma-\alpha)\ell} - 2 r_1 r_2 e^{(\gamma-\alpha)\ell}[\cos 2(k+\Delta k)\ell]}$$

As $\gamma - \alpha$ increases from 0 denominator approaches 0 and the whole expression blows up when

$$\boxed{r_1 r_2 e^{-2i(k+\Delta k)\ell} e^{(\gamma-\alpha)\ell} = 1}$$

we have an infinite amplitude transmitted wave for a finite amplitude incident wave.

or a finite amplitude transmitted wave for zero incident wave — oscillation.

- physically this condition must be satisfied for a wave to make a complete round trip inside the resonator and return to its starting point with the same amplitude and phase.
- it is an amplitude condition for oscillation that gives an expression for the threshold gain ~~constant~~ $\gamma_t(\nu)$

$$\boxed{r_1 r_2 e^{[\gamma_t(\omega) - \alpha]\ell} = 1}$$

- to satisfy $r_1 r_2 e^{-2i(k+\Delta k)\ell} e^{(\gamma-\alpha)\ell} = 1$
  $e^{-2i(k+\Delta k)\ell}$ must be real, so we have phase condition

  $e^{iy} = \cos y + i \sin y$
  $e^{-iy} = \cos y - i \sin y$

$$2[k + \Delta k(\nu)]\ell = 2\pi m \; ; \quad m = 1, 2, 3 \ldots$$

- threshold gain coefficient

$$\boxed{\gamma_t = \alpha - \frac{1}{\ell} \ln r_1 r_2}$$

- population inversion needed for oscillation

$$\boxed{\left(N_2 - \frac{g_2}{g_1} N_1\right) = \frac{8\pi}{g(\nu_0 \nu) A_{21} \lambda^2}\left(\alpha - \frac{1}{\ell} \ln r_1 r_2\right)}$$

For a homogeneously broadened transition

$$\left(N_2 - \frac{g_2}{g_1} N_1\right)_t \sim \frac{\Delta\nu}{A_{21}\lambda^2}$$

For an inhomogeneously broadened transition since $\Delta\nu_D \sim \frac{1}{\lambda}$

$$\left(N_2 - \frac{g_2}{g_1} N_1\right)_t \sim \frac{1}{A_{21}\lambda^3}$$

- lower inversions are needed to achieve laser oscillation at longer wavelengths

---

- Let us find population inversion at threshold using time constant of the cavity.
- Consider a passive F.-P. resonator having small distributed losses α.
- A wave starting with intensity I inside the resonator will after one complete round trip have intensity $I R_1 R_2 e^{-2\alpha l}$

if $R_1 = r_1^2 \simeq 1$; $R_2 = r_2^2 \simeq 1$

$$dI = I R_1 R_2 e^{-2\alpha l} - I = I\left(R_1 R_2 e^{-2\alpha l} - 1\right)$$

this loss occurs in a time $dt = \frac{2l}{c}$

$$\Rightarrow \frac{dI}{dt} = \frac{c\, I\left[R_1 R_2 e^{-2\alpha l} - 1\right]}{2l}$$

solution $\quad I = I_0 \exp\left\{-\frac{\left[1 - R_1 R_2 e^{-2\alpha l}\right]ct}{2l}\right\}$

I_0 - intensity at $t=0$

The time constant of the cavity for intensity loss

$$\tau_0 = \frac{2l}{c\left(1 - R_1 R_2 e^{-2\alpha l}\right)}$$

If $R_1 R_2 e^{-2\alpha l} \simeq 1 \Rightarrow$

$$(1 - R_1 R_2 e^{-2\alpha l}) \simeq -\ln(R_1 R_2 e^{-2\alpha l}) = -\ln(R_1 R_2) + 2\alpha l$$

use the relation $1 - x \simeq -\ln x$; when $x \approx 1$

$$\tau_0 = \frac{2l}{c(1 - R_1 R_2 e^{-2\alpha l})} = \frac{2l}{c(2\alpha l - \ln R_1 R_2)} = \frac{1}{c\left[\alpha - \left(\frac{1}{l}\right)\ln r_1 r_2\right]}$$

The Threshold population inversion

$$\boxed{N_t = \frac{8\pi}{A_{21}\lambda^2 g(\nu)\, c\tau_0}}$$

$$N_t = \frac{\gamma}{\sigma} = \frac{\left(\alpha - \frac{1}{l}\ln r_1 r_2\right)}{\frac{A_{21}\lambda^2 \cdot g(\nu_0, \nu)}{8\pi}}$$

Threshold Population Inversion- Numerical Example

For the 632.8 nm transition of He-Ne laser

$\lambda = 632.8\ nm$; $t_{spont} = 10^{-7}\ s$; $l = 12\ cm$

$$\frac{1}{g(\nu_0)} \approx \Delta\nu_0 \approx 10^9\ Hz \qquad \left(g(\nu_0, \nu_0) = \frac{2}{\Delta\nu_0}\sqrt{\frac{\ln 2}{\pi}} = \frac{0.94}{\Delta\nu_0} \approx \frac{1}{\Delta\nu_0}\right)$$

Assume $\alpha = 0$ and $R_1 = R_2 = 0.98$.

Since $R_1 = R_2 \approx 1$ we use approximation $-\ln x = 1 - x$, $x \approx 1$ to write

$$\tau_0 = \frac{2l}{c(2\alpha l - \ln R_1 R_2)} = \frac{2l}{c(-\ln R_1 R_2)} = \frac{2l}{c(1 - R_1 R_2)} = \frac{2 \cdot 0.12\, m}{(3\times10^8\, m/s)\cdot(1 - 0.96)} = \boxed{2\times10^{-8}\ s}$$

$$N_t = \frac{8\pi \times 10^9}{10^7 \times (632.8\times10^{-9})^2 \cdot (3\times10^8)\cdot 2\times10^{-8}} \approx 1\times10^{15}\ m^{-3} \approx \boxed{10^9\ cm^{-3}}$$

The oscillation frequency

To determine the frequency at which laser oscillation can occur return to the phase condition of oscillation

$$(k+\Delta k)l = m\pi$$

$$(k+\Delta k)l = \left(k + \frac{k\chi'(\nu)}{2n^2}\right)l = kl\left[1 + \frac{\chi'(\nu)}{2n^2}\right] = m\pi$$

we remember that $\chi'(\nu) = \frac{2(\nu_0-\nu)}{\Delta\nu}\chi''(\nu)$

ν_0 – central frequency

$\Delta\nu$ – homogeneous FWHM

$\gamma(\nu) = -\frac{k\chi''(\nu)}{n^2}$

$$\Rightarrow kl\left[1 + \underbrace{\frac{2(\nu_0-\nu)}{\Delta\nu}\cdot\overbrace{\left(-\frac{n^2\gamma(\nu)}{k}\right)}^{\chi''(\nu)}}_{\chi'(\nu)}\cdot\frac{1}{2n^2}\right] =$$

$$= kl\left[1 - \frac{(\nu_0-\nu)}{\Delta\nu}\cdot\frac{\gamma(\nu)}{k}\right] = \overbrace{\frac{2\pi\nu}{c}}^{k}l\left[1 - \frac{(\nu_0-\nu)}{\Delta\nu}\cdot\frac{\gamma(\nu)}{k}\right] = m\pi$$

$$\nu\left[1 - \frac{(\nu_0-\nu)}{\Delta\nu}\cdot\frac{\gamma(\nu)}{k}\right] = \frac{mc}{2l} = \nu_m$$

where ν_m is the mth resonance of the passive laser resonator in normal incidence

$$\nu - \frac{(\nu_0-\nu)}{\Delta\nu}\cdot\frac{\gamma(\nu)c}{\underbrace{2\pi\nu}_{k}}\cdot\nu = \nu_m \Rightarrow \nu = \nu_m - (\nu-\nu_0)\frac{\gamma(\nu)c}{2\pi\Delta\nu}$$

Since ν close to ν_m

$(\nu-\nu_0) \sim (\nu_m-\nu_0)$ and $\gamma(\nu) \approx \gamma(\nu_m)$

$$\Rightarrow \boxed{\nu = \nu_m - (\nu_m-\nu_0)\,\frac{\gamma(\nu_m)\,c}{2\pi\,\Delta\nu}}$$

- At threshold $\gamma_t(\nu_m) = \alpha - \frac{1}{\ell}\ln r_1 r_2$

For $x \simeq 1$: $-\ln x = 1-x$

if $\alpha = 0$ and $r_1 = r_2 = \sqrt{R}$ $\Rightarrow$

$\gamma_t(\nu_m) = \frac{1-R}{\ell}$ $\Rightarrow \nu = \nu_m - (\nu_m-\nu_0)\frac{c(1-R)}{2\pi\ell\,\Delta\nu}$

- FWHM of The transmission intensity maxima of F.-P.

$$\Delta\nu_{1/2} = \frac{\Delta\nu_{FSR}}{F} = \underbrace{\frac{c}{2\ell}}_{\Delta\nu_{FSR}} \cdot \underbrace{\frac{1-R}{\pi\sqrt{R}}}_{1/F} = \frac{c(1-R)}{2\pi\ell} = \Delta\nu_{1/2}$$

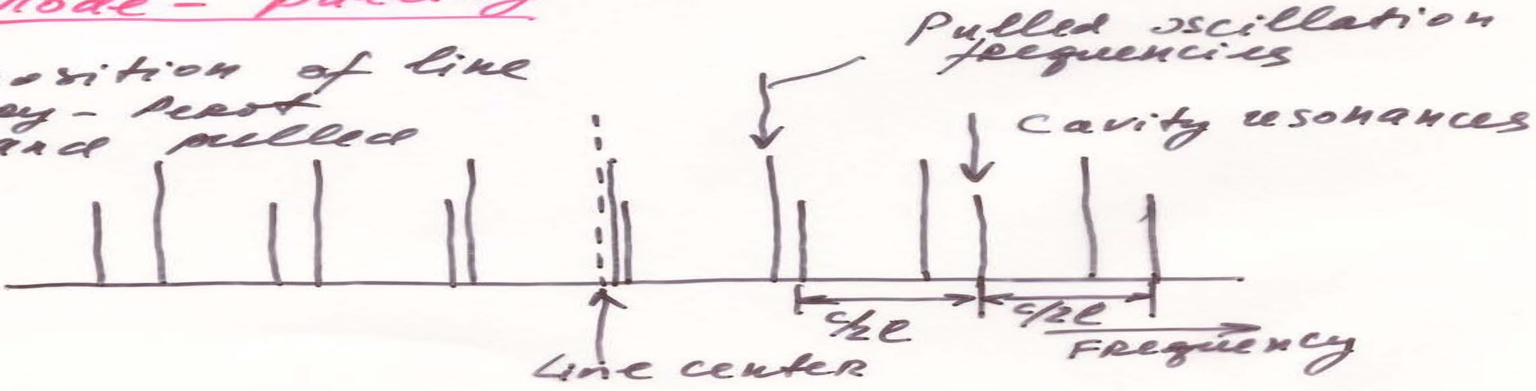
for $R \sim 1$

- $$\boxed{\nu = \nu_m - (\nu_m-\nu_0)\,\frac{\Delta\nu_{1/2}}{\Delta\nu}}$$

if $\nu_m \neq \nu_0$ – oscillation takes place near ν_m but is shifted slightly towards ν_0.

it is called Mode - pulling

Relative position of line center, Fabry-Perot resonances and pulled oscillation frequencies that satisfy the phase condition



Multimode laser oscillation

- When gain reach a threshold value $\gamma_t(\nu) = \alpha - \frac{1}{\ell} \ln r_1 r_2$ – oscillation will occur in a laser system.
- For gain coefficients greater than this, oscillation can occur at, or near (because of mode-pulling effect), one or more of the passive resonance frequencies of the F-P. laser cavity.
- The resulting oscillations of the system are called longitudinal modes.
- As The resulting oscillations at a particular one of these mode frequencies builds up, the growing intracavity energy density depletes the inverted population and gain saturation sets in.

  The reduction of gain continues until

$$\gamma(\nu) = \gamma_t(\nu) = \alpha - \frac{1}{\ell} \ln r_1 r_2$$

- Further reduction of $\gamma(\nu)$ below $\gamma_t(\nu)$ does not occur, otherwise the oscillation would cease.

  $\longrightarrow$ gain stabilizes at the loss level

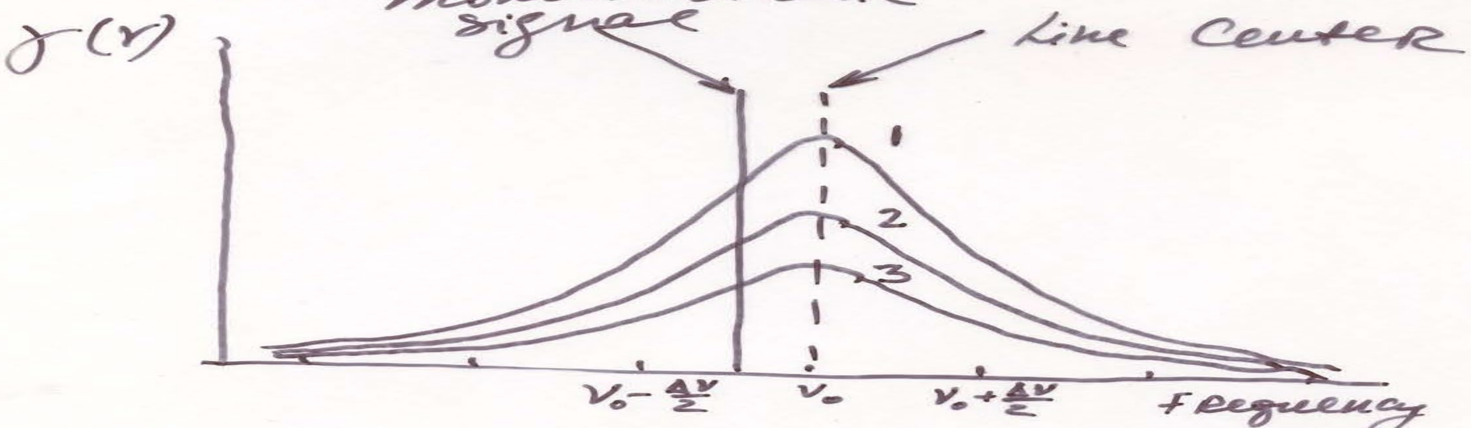
$$\alpha - \frac{1}{\ell} \ln r_1 r_2$$

- Usually α, r_1, and r_2 are ~ constant over the frequency range covered by typical transitions (10^{11} Hz). $\alpha - \frac{1}{\ell} \ln r_1 r_2$ as a function of ν is a straight line // to the frequency axis.

  Loss line

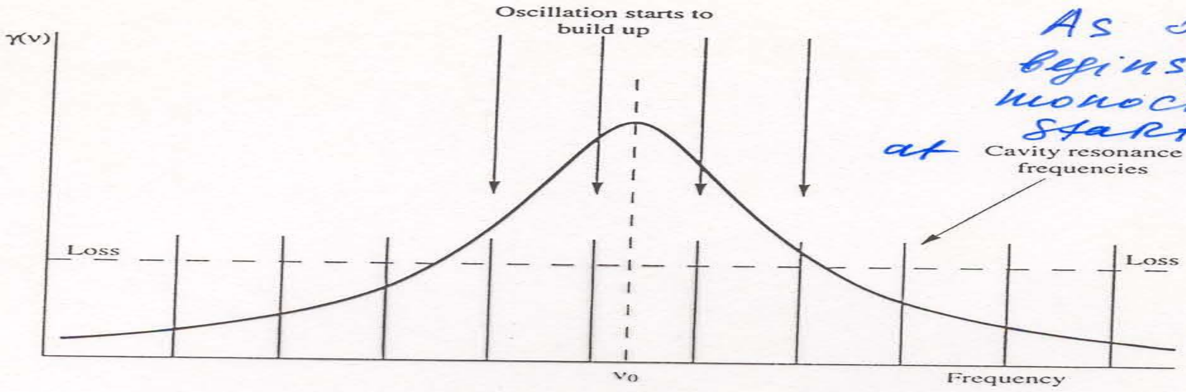
In a homogeneously broadened laser, because the reduction in gain caused by a monochromatic field is uniform across the whole gain profile, the clamping of the gain at $\gamma_t(\nu)$ leads to final oscillation at only one of the cavity resonance frequencies, the one where the original unsaturated gain was the highest.

- it can be shown by plotting $\gamma(\nu)$ at various stages ~~of~~ oscillation builds up.
- Saturation of gain of a homogeneously broadened transition by a monochromatic signal, whose intensity increases from 1→2→3



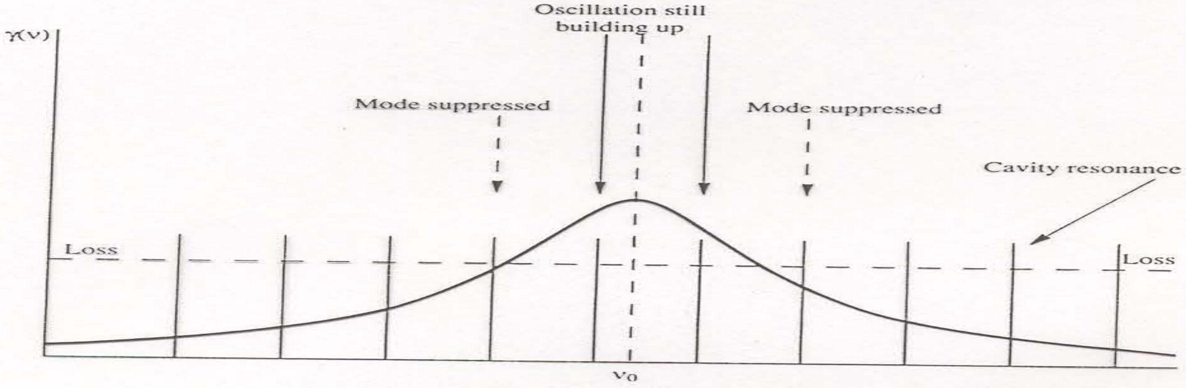
- the gain profile is suppressed uniformly even though the saturating signal is not at the line center

Fig. 5.4. Schematic illustration of the onset of oscillation at cavity resonances that lie above the loss line in a homogeneously broadened laser.



As oscillation begins, several such monochromatic fields start to build up at where gain exceeds loss.

Fig. 5.5. Oscillation building up in a homogeneously broadened laser. Gain saturation has already suppressed oscillation at two of the cavity modes that were above the loss line in Fig. (5.4).



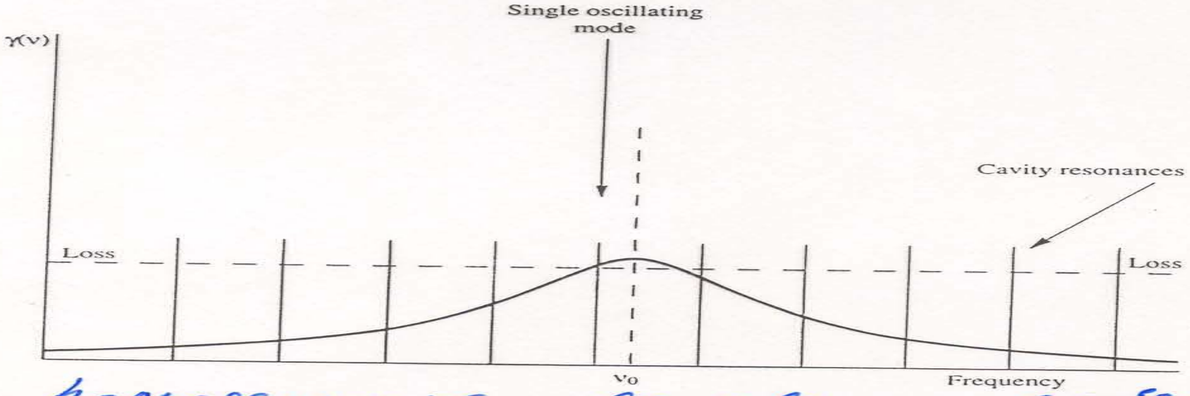


Fig. 5.6. Oscillation stabilized in a homogeneously broadened laser. The gain has been uniformly saturated until only one mode remains at the loss line.

In a homogeneously broadened laser, oscillation occurs at one longitudinal mode frequency.

In an inhomogeneously broadened laser, the onset of gain saturation due to a monochromatic signal only reduces the gain locally over a region which is of the order of homogeneous width. Only particles whose center emission frequencies lie within a homogeneous width of the monochrom. field can interact strongly with it.

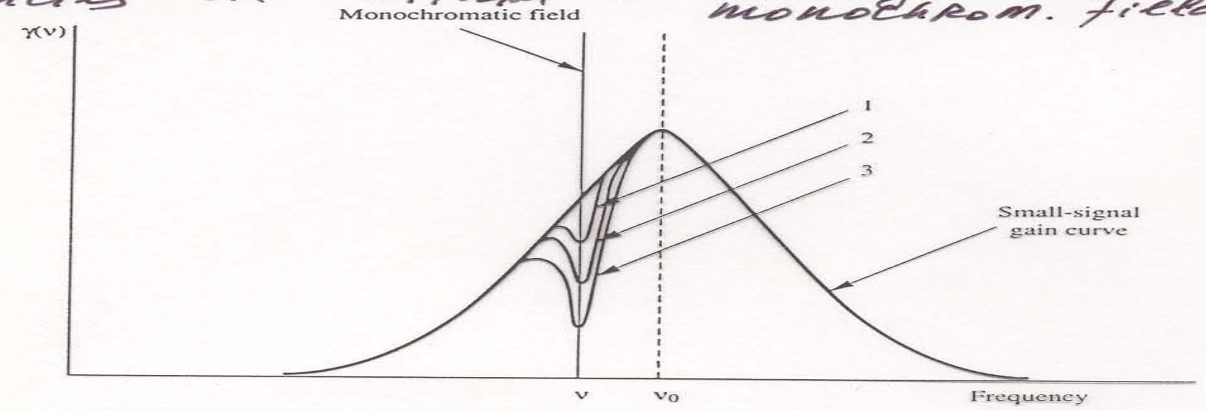


Fig. 5.7. Localized gain saturation in an inhomogeneously broadened amplifier produced by a monochromatic signal whose intensity increases from 1→2→3.

A localized hole in the gain profile occurs.

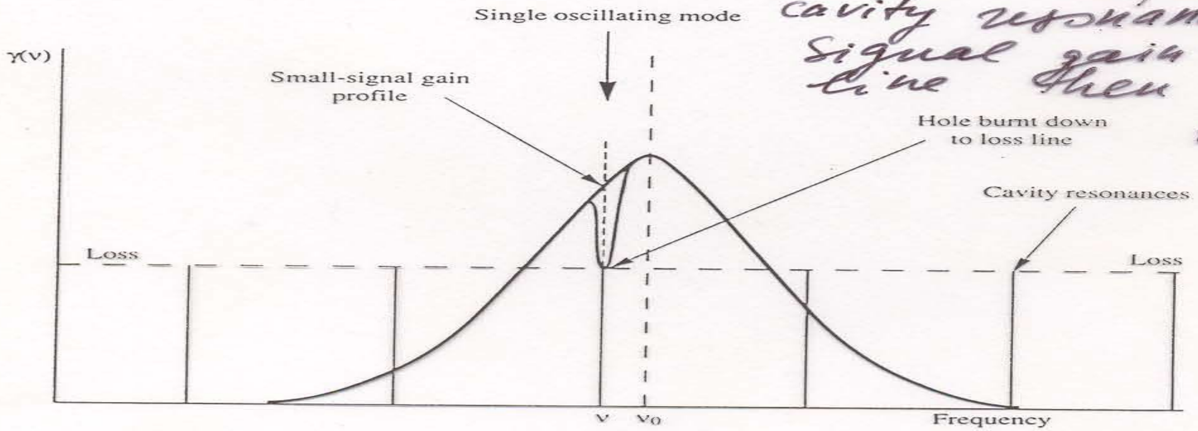


Fig. 5.8. Simplified illustration of stablization of oscillation at a single longitudinal mode in an inhomogeneously broadened laser.

If only one cavity resonance has a small signal gain above the loss line then only this longitudinal mode oscillates.

The situation is not quite as simple.

- Oscillation at single longitudinal mode frequency implies waves travelling in both directions inside the laser cavity.
  - a) wave travelling to the right $\sim E_0 e^{i(\omega t - kz)}$
  - b) ——————— " ——— left $\sim E_0 e^{i(\omega t + kz)}$
- $\omega = 2\pi\nu < 2\pi\nu_0$ - for convinience.
- wave a) interacts with particles with center frequency near ν.
- these particles are moving away from the observer looking into laser from right to left.

$$\nu = \nu_0 - \frac{|v_z| \nu_0}{c}$$

- wave (b) which is travelling in the opposite direction (to the left) and is monitored at $\nu (< \nu_0)$ by a second observer looking from left to right cannot interact with the same velocity group of particles as wave (a).
- the particles which interacted with wave (a) were moving away from the first observer and were Doppler shifted to lower frequencies so as to satisfy

$$\nu = \nu_0 - \frac{|v_z|}{c} \nu_0$$

- the second observer sees these particles approaching and their center frequency as

$$\nu = \nu_0 + \frac{|v_z|}{c} \nu_0$$

so they cannot interact with wave (b)
- wave (b) interacts with particles moving away from the second observer. Their velocities - solution of $\nu = \nu_0 - \frac{|v_z|}{c} \nu_0$
- these particles would be monitored by 1st observer at $\nu = \nu_0 + \frac{|v_z|}{c} \nu_0$

⇒ the oscillating waves interact with two groups of particles.

- this leads to saturation of the gain by a single laser mode in an inhomog. broadened laser both at the frequency of the mode ν and at a frequency which is equally spaced on the opposite side of line center. $\nu_0 + (\nu_0 - \nu)$,

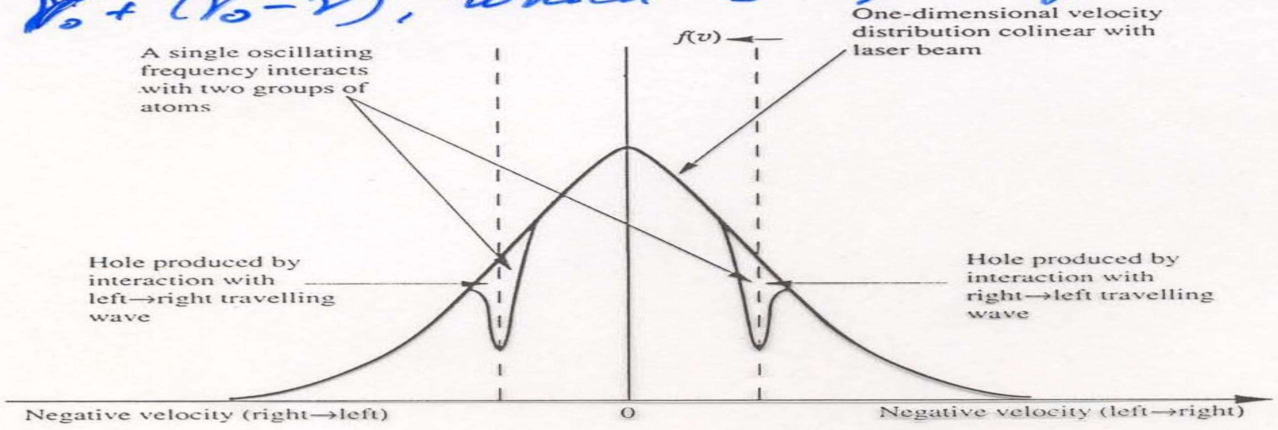


Fig. 5.9. Production of two holes in the velocity distribution of a collection of amplifying particles by a single cavity mode.

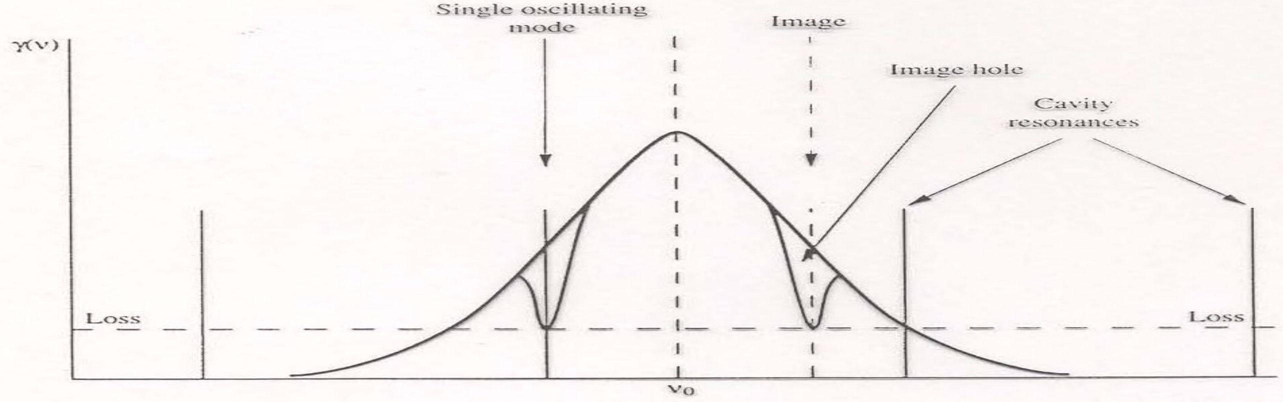
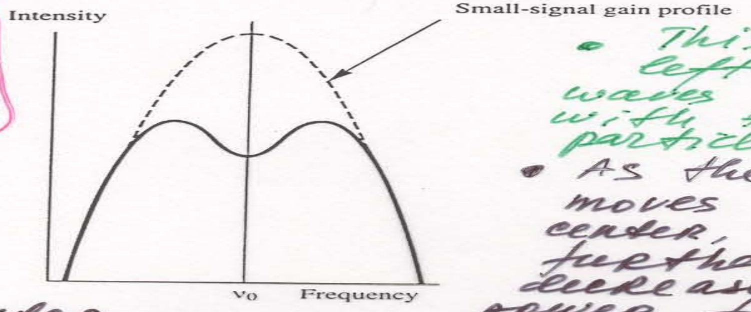


Fig. 5.10. Stabilization of a single longitudinal mode in an inhomogeneously broadened laser.

- the power output of the laser comes from those groups of particles that have gone into stimulated emission and left the two holes.
- The combined area of these two holes gives a measure of the laser power.

• If the frequency of the oscillating mode is moved in towards the line center, the main hole and image hole begin to overlap.

Fig. 5.11. The Lamb dip – a reduction in the intensity of a single oscillating longitudinal mode in an inhomogeneously broadened laser as its frequency is scanned through line center.



• This corresponds to the left and right travelling waves beginning to interact with the same group of particles.

• As the oscillating modes moves in towards the line center, the holes overlap further, the combined area decreases, the laser output power falls, reaching a minimum at the line center.

• because hole-burning in gain saturation in inhomogeneously broad. lasers is localized near the frequency of a cavity mode one oscillating mode doesn't reduce the gain at other modes, so simultaneous oscillation at several longitudinal modes is possible.

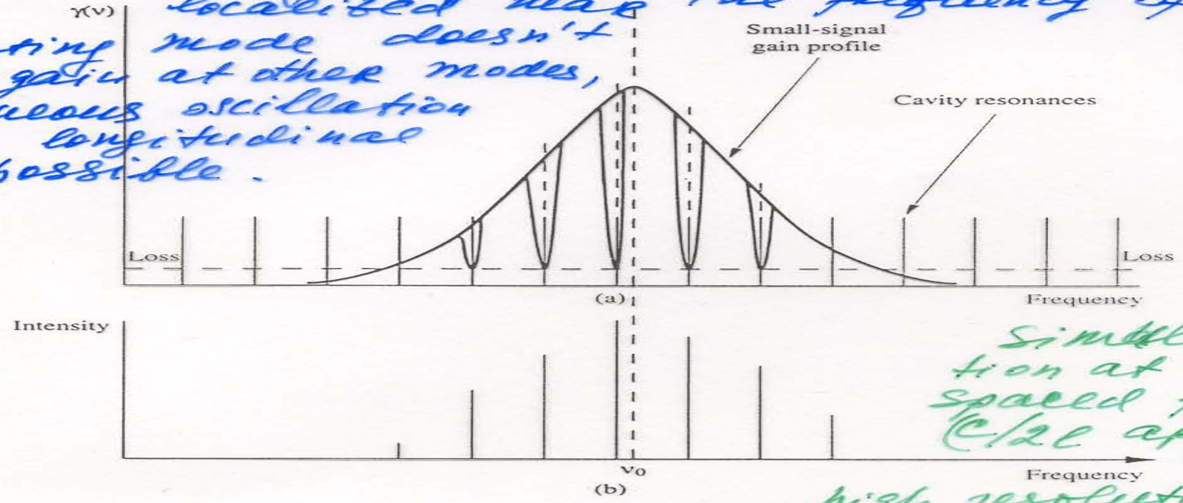


Fig. 5.12. Multi-longitudinal-mode oscillation in an inhomogeneously broadened laser. (a) Only the primary holes are shown burnt down to the loss line. The image holes are not shown. (b) Schematic laser output spectrum.

Simultaneous oscillation at several closed spaced frequencies ($c/2L$ apart) can be observed with a high resolution spectrometer.

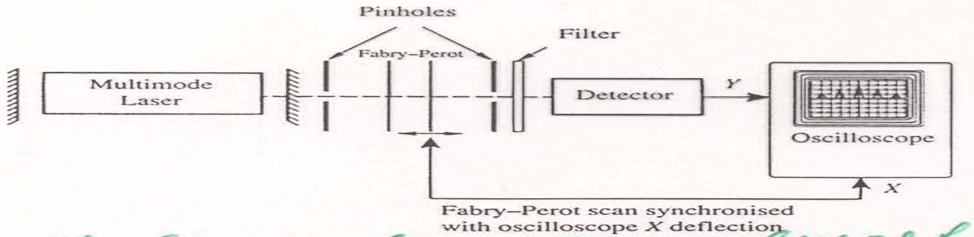
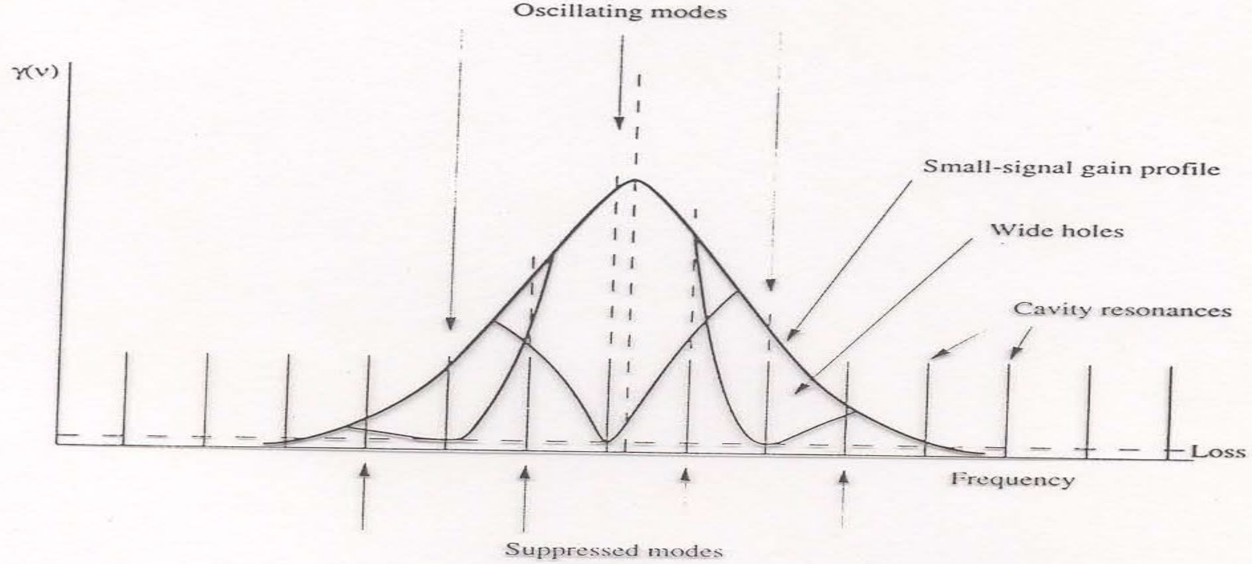


Fig. 5.13. Experimental arrangement with a scanning Fabry–Perot interferometer for observing multi-mode laser oscillation.

The multiple modes are almost exactly $\frac{c}{2L}$ in frequency apart, but are not exactly equally spaced because of mode-pulling.

• If a predominantly inhomogeneously broadened laser also has a significant amount of homogeneous broadening, the holes burnt in the gain curve start to overlap. (when $\Delta\nu \geq \frac{c}{2\ell}$)

Fig. 5.15. Schematic illustration of mode competition in an inhomogeneously broadened laser in which there is significant homogeneous broadening.



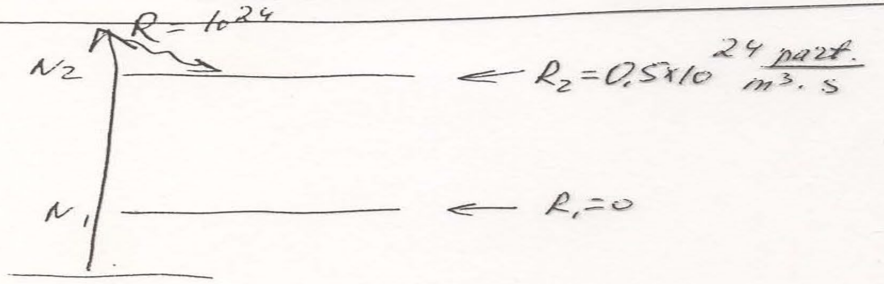
• If $\Delta\nu$ is large – neighboring oscillating modes compete and may lead to oscillation on a strong mode suppressing its weaker neighbors.

Problem 4.1

A four-level laser is pumped into its pump band at a rate $10^{24}\ m^{-3} s^{-1}$, the transfer efficiency to the upper level is 0.5. The lifetime of the upper laser level is 7×10^{-4} s. For the laser transition $A_{21} = 10^{3} s^{-1}$, $\lambda_0 = 1\,\mu m$. The laser is homogeneously broadened with $\Delta\nu = 1\,GHz$. The amplifying medium is 20 mm long. Assume $n=1.6$. Neglect lower laser level population.

a) What is the gain at line center?

b) What min value of R_2 is needed to get oscillation if mirror 1 has $R_1=1$? Assume $\alpha = 0$



$\tau_2 = 7\times10^{-4} s$
$A_{21} = 10^{3} s^{-1}$
$\lambda_0 = 1\,\mu m \Rightarrow \nu_0 = 3\times10^{14}\ Hz$
$\Delta\nu_H = 10^{9}\ Hz$
$n = 1.6$
$L = 2\,cm$
$N_1 = 0$
$\alpha = 0 \qquad R_1 = 1$

---

a)

$$\left.\begin{aligned}\frac{dN_2}{dt} &= R_2 - \frac{N_2}{\tau_2} = 0\\ \frac{dN_1}{dt} &= N_2 A_{21} = 0\end{aligned}\right\}\quad N_2 = R_2\tau_2$$

$$\gamma(\nu_0) = \left(N_2 - \frac{g_2}{g_1}N_1^{\,0}\right)\frac{c^2A_{21}}{8\pi\nu_0^2}\,g(\nu_0,\nu_0) = R_2\tau_2\cdot\frac{c^2A_{21}}{8\pi\nu_0^2}\,\frac{2/\pi\Delta\nu}{1+[2(\nu_0-\nu_0)/\Delta\nu]^2} =$$

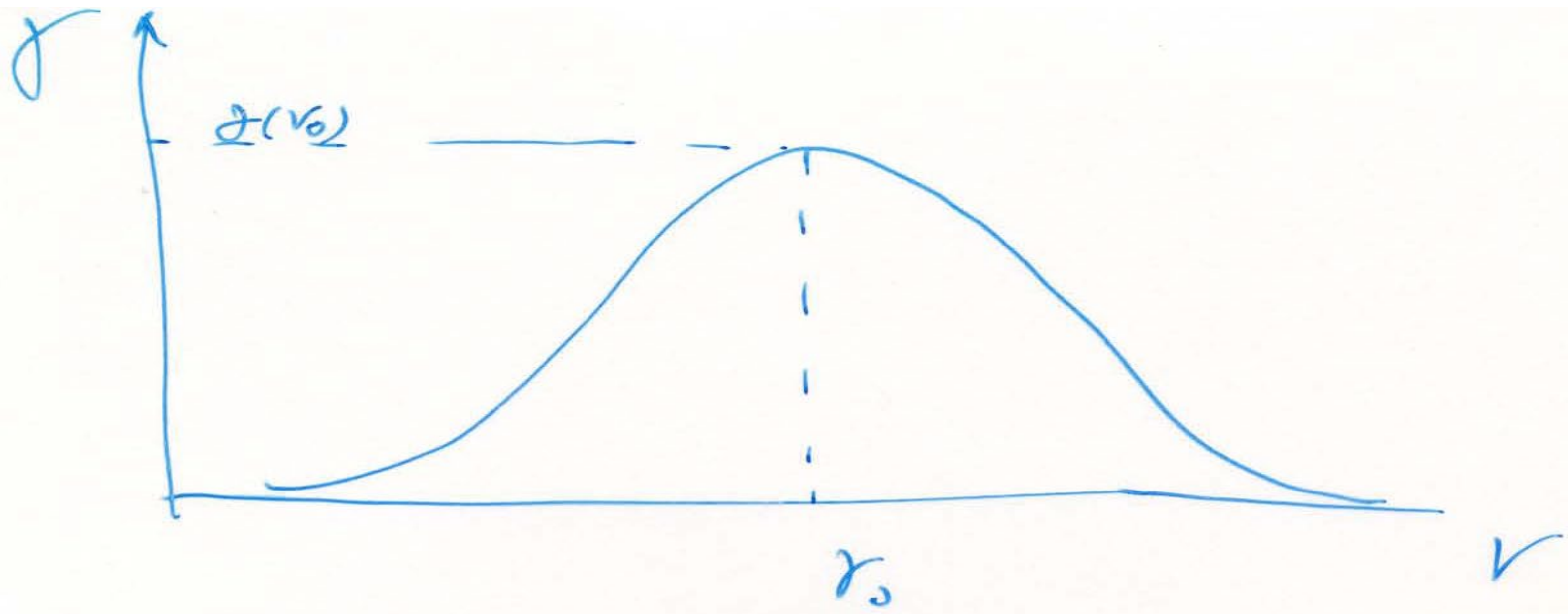
$$= \frac{\left(0.5\times10^{24}\frac{1}{m^3 s}\right)\cdot\left(7\times10^{-4}s\right)\cdot\left(\frac{3\times10^{8}\,m/s}{1.6}\right)^2\cdot\left(10^{3}s^{-1}\right)\cdot 2}{4\,8\pi\ \left(3\times10^{14}s^{-1}\right)^2\ 10^{9}} = 3.4\cdot10^{-5}\ m^{-1} =$$

$$= 3.4\times10^{-7}\ cm^{-1}$$

b)

$$\gamma(\nu) = \alpha^{0} - \frac{1}{l}\ln\sqrt{R_1R_2}$$

$$e^{\gamma L} = \frac{1}{\sqrt{R_1R_2}} \qquad R_2 = \frac{1}{R_1 e^{(2\gamma l)}} = \frac{1}{e^{(2\gamma l)}} \approx 99.99$$



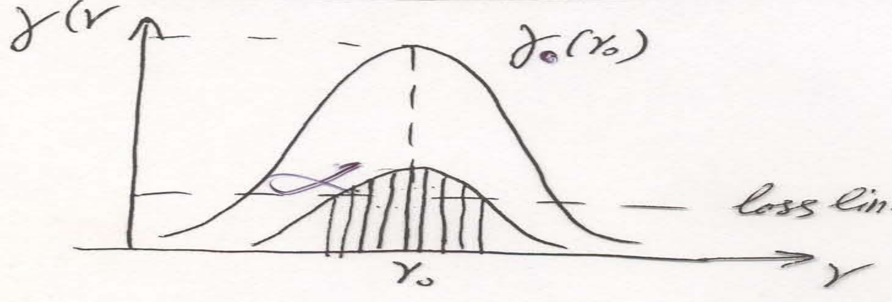
$$\gamma(\nu) = \Delta N \frac{c^2 A_{21}}{8\pi \nu^2} g(\nu_0, \nu) = \Delta N \frac{c^2 A_{21}}{8\pi \nu^2} \frac{2}{\Delta \nu_0} \sqrt{\frac{\ln 2}{\pi}} \times e^{-\left\{\left[\frac{2(\nu' - \nu_0)}{\Delta \nu_0}\right]^2 \ln 2\right\}}$$

$$\gamma(\nu_0) = \Delta N \frac{c^2 A_{21}}{8\pi \nu_0^2} g(\nu_0, \nu_0) = \Delta N \frac{c^2 A_{21}}{8\pi \nu_0^2} \frac{2}{\Delta \nu_0} \sqrt{\frac{\ln 2}{\pi}} \times 1 \Rightarrow$$

$$\Rightarrow \boxed{\gamma(\nu) = \gamma(\nu_0)\, e^{-\left\{\left[\frac{2(\nu' - \nu_0)}{\Delta \nu_0}\right]^2 \ln 2\right\}}}$$

Problem 4.2

How many longitudinal modes will oscillate in an inhomogeneously broadened gas laser with $\ell = 1\,m$; $\gamma(\nu_0) = 1\,m^{-1}$; $R_1 = R_2 = 99\%$; α distributed loss $= 0{,}001\,m^{-1}$; $\lambda_0 = 500\,nm$; $\Delta\nu_D = 3\,GHz$.



1) $\gamma_{th}(\nu) = \alpha - \frac{1}{\ell}\ln r_1 r_2 =$

$= (0.001\,m^{-1}) - \frac{1}{1}\ln\sqrt{0.99^2} =$

$= \left(1.105 \times 10^{-2}\,m^{-1}\right)$

2) Cavity resonances $\nu_m = \frac{m c_0}{2n\ell}$

Does ν_m coincide with line center for some m?

$$m = \nu_m \cdot \frac{2n\ell}{c_0} = \nu_m \cdot \frac{2\cdot 1\cdot 1}{3\times 10^8} = \frac{3\times 10^8 \cdot 2}{500\times 10^{-9} \times 3\cdot 10^8} = 4\times 10^6 \text{ exactly}$$

$\Rightarrow$ laser radiation will occur at the cavity resonances separated by $\frac{c_0}{2n\ell} = 150\,MHz$.

3) For an inhomogeneously broadened laser with $\gamma(\nu_0)$ =1 m^-1

$$\gamma(\nu) = \gamma(\nu_0)\, e^{-\left[\frac{2(\nu-\nu_0)}{\Delta\nu_0}\right]^2 \cdot \ln 2}$$

we are interested in determining the # of oscillating modes with a gain greater than the loss. What frequency corresponds to the situation where loss = gain

$$1.105\times 10^{-2} = 1 \cdot e^{-\left[\frac{2(\nu_{th}-\nu_0)}{\Delta\nu}\right]^2 \ln 2}$$

$$(\nu_{th}-\nu_0)^2 = \frac{-\ln(1.105\times 10^{-2})\,\Delta\nu_0^2}{4\ln 2} = 14.6\times 10^{18}\,Hz$$

$\nu_{th} = \nu_0 \pm \sqrt{14.6\times 10^{18}} = \nu_0 \pm 3.82\times 10^9\,Hz.$

The # of oscillating modes $= 1 + \frac{2(\nu_{th}-\nu_0)}{\Delta\nu_{FSR}} = \frac{2\cdot 3.82\times 10^3}{150\times 10^6} + 1 = 51$

Mode-Beating

- we shine the light from a two-mode laser on a square-law detector (responds to the intensity, not the electric field of an incident light signal)
- the incident electric field is

$$E_i = \Re\left(E_1 e^{i\omega t} + E_2 e^{i(\omega+\Delta\omega)t}\right)$$

E_1 and E_2 – complex amplitudes of the two modes and $\Delta\omega$ – frequency spacing between them.

- the output current "i" from the detector is

$$i \sim \left\{ |E_1| \cos[\omega t + \varphi_1) + |E_2| \cos[(\omega+\Delta\omega)t + \varphi_2] \right\}^2 \sim$$

$$\sim |E_1|^2 \cos^2(\omega t + \varphi_1) + |E_2|^2 \cos^2[(\omega+\Delta\omega)t + \varphi_2] +$$

$$+ 2|E_1||E_2| \cos[(\omega t + \varphi_1)] \cos[(\omega+\Delta\omega)t + \varphi_2] \sim$$

$$\sim |E_1|^2 \cos^2(\omega t + \varphi_1) + |E_2|^2 \cos^2[(\omega+\Delta\omega)t + \varphi_2] +$$

$$+ |E_1||E_2| \cos[(2\omega + \Delta\omega)t + \varphi_1 + \varphi_2] +$$

$$+ |E_1||E_2| \cos(\Delta\omega t + \varphi_2 - \varphi_1)$$

$$+\begin{cases} \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\sin\beta \end{cases}$$

$$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cdot\cos\beta$$

- Since $|E_1|^2 \cos^2(\omega t + \varphi_1) = \frac{1}{2}|E_1|^2 [1 + \cos 2(\omega t + \varphi_1)]$ the output frequency spectrum of the detector contains frequencies $\underbrace{2\omega;\ 2(\omega+\Delta\omega);\ 2\omega+\Delta\omega}_{\text{very high frequencies do not appear in the output of the detector.}}$ and $\Delta\omega$.

- $i \sim \frac{|E_1|^2}{2} + \frac{|E_2|^2}{2} + |E_1||E_2| \cos(\Delta\omega t + \varphi_2 - \varphi_1)$

only the difference frequency beat $\Delta\omega$ is observed.

If the output from the square-law detector is analyzed with a radio-frequency spectrum analyzer (freq. range where differ. frequency between longitudinal laser modes are observed) different displays are obtained according to how many longit. modes are oscillating.

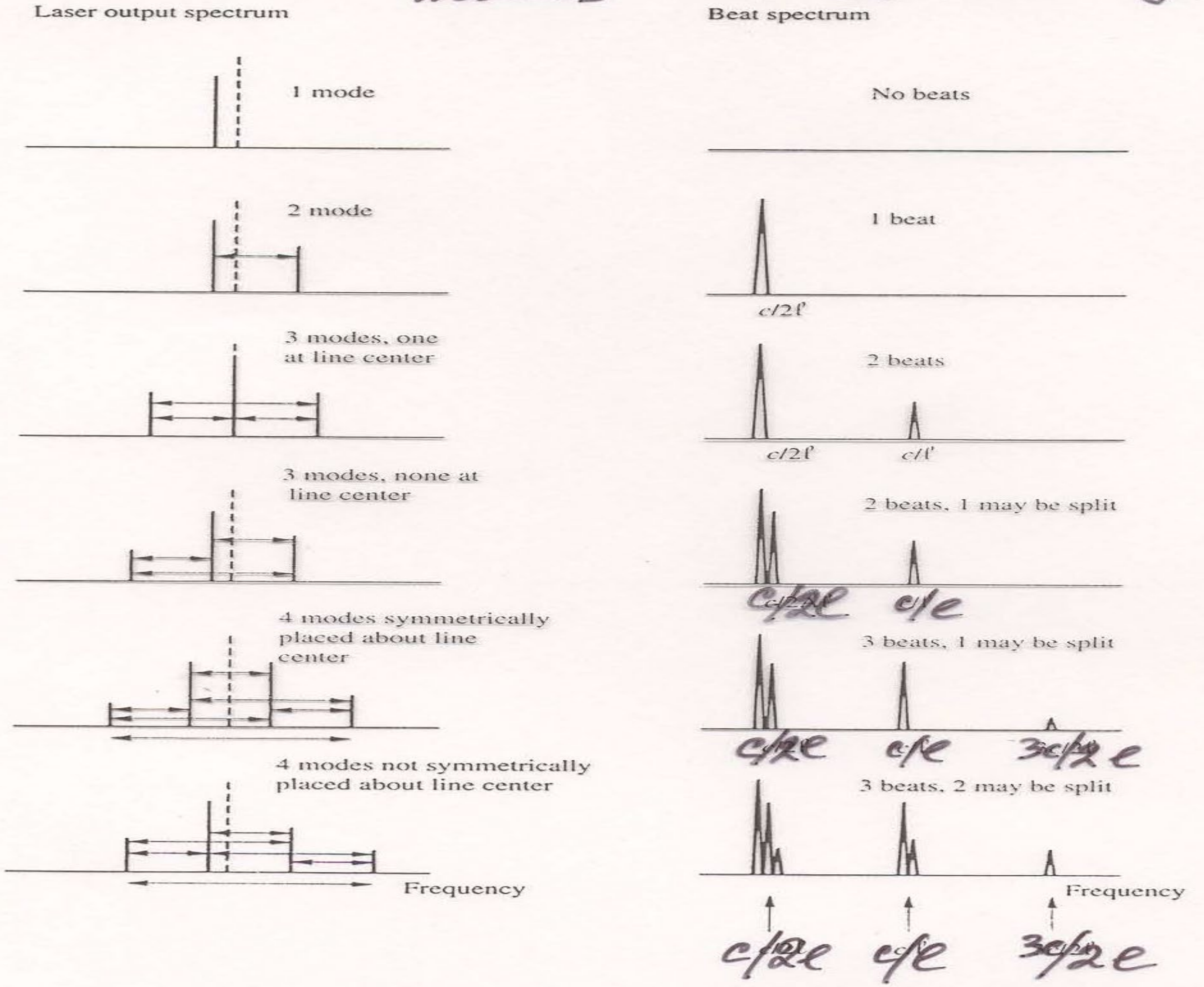
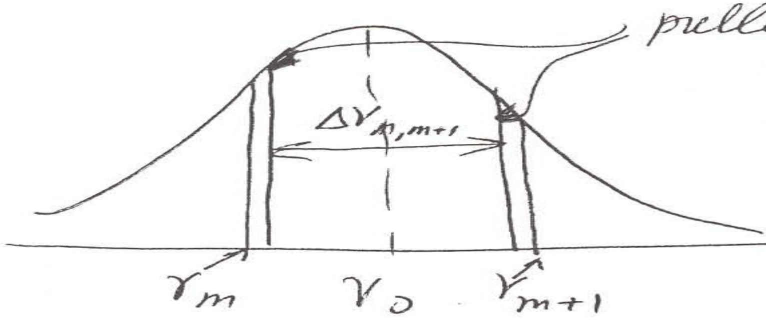


Fig. 5.14. Schematic mode-beating spectra observed with a square-law detector and a multimode laser.

Problem 4.6

A laser is exactly 1 m long and has a wavelength $\lambda_0 = 632.8$ nm. The mirrors of the laser have $R = 99\%$. The index of refraction of the amplifying medium is exactly 1.0001, $\Delta\nu = 100.000$ MHz. The laser operates on only the two modes nearest to the line center. The laser output illuminates a photodiode whose output is mixed with a 150 MHz local oscillator. What is the frequency of the lowest beat signal observed? Take $c_0 = 2.997 \times 10^8$ m/s.



1) $\nu = \nu_m - (\nu_m - \nu_0)\dfrac{\Delta\nu_{1/2}}{\Delta\nu}$

2) find

$$\Delta\nu_{m,m+1} = \nu_{m+1} - (\nu_{m+1} - \nu_0)\frac{\Delta\nu_{1/2}}{\Delta\nu} -$$

$$- \nu_m + (\nu_m - \nu_0)\frac{\Delta\nu_{1/2}}{\Delta\nu} =$$

$$= (\nu_{m+1} - \nu_m) + (\nu_m - \nu_{m+1})\frac{\Delta\nu_{1/2}}{\Delta\nu} =$$

$$= (\nu_{m+1} - \nu_m)\left(1 - \frac{\Delta\nu_{1/2}}{\Delta\nu}\right) =$$

$$= \left(\frac{m c_0}{2nl} + \frac{(m+1) c_0}{2nl}\right)\left(1 - \frac{c_0(1-R)}{2\pi l}\Big/\Delta\nu\right] = \frac{c_0}{2nl}\left[1 - \frac{c_0(1-R)}{2\pi l \Delta\nu}\right] =$$

$$= \frac{2.997\times10^8}{2\cdot 1.0001\cdot 1}\left[1 - \frac{2.997\times10^8(1-0.99)}{2\pi\times 1\cdot 100\times10^6}\right] = 149.1 \text{ MHz}$$

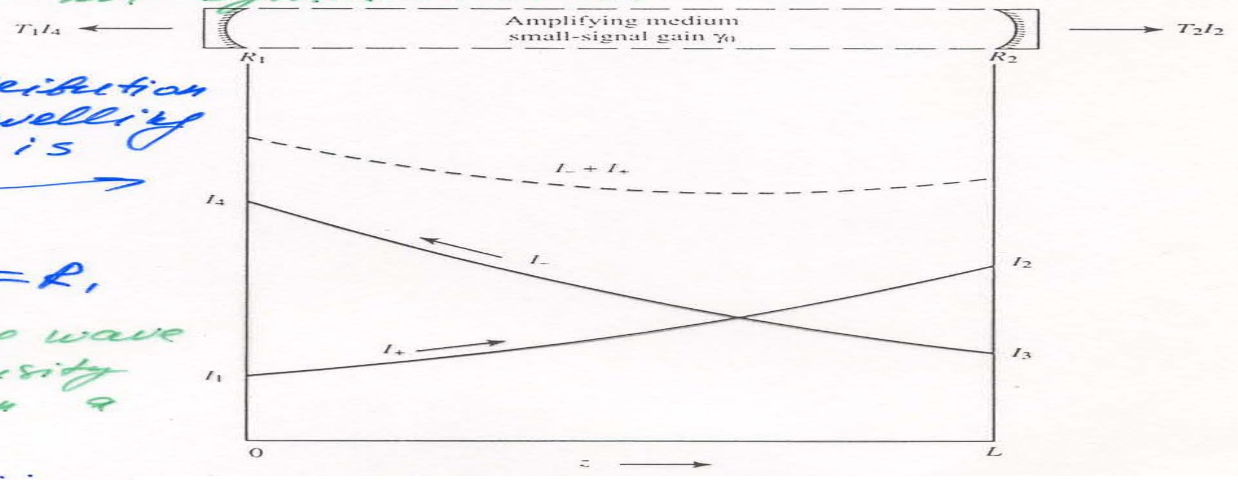
3) If we mix this beat frequency with a 150 MHz oscillator, the observed beat frequency would be

$$150 - 149.1 = 0.9 \text{ MHz}$$

The power output of a laser

- When a laser oscillates, the intracavity field grows in amplitude until saturation reduces the gain to the loss line for each oscillating mode.
- For an asymmetrical resonator, whose mirror reflectances are not equal, the distribution of standing wave energy within the resonator is not symmetrical.

Fig. 5.16. Distribution of wave intensities in an oscillating laser cavity with unequal laser intensities.



- if $R_2 > R_1$ the distribution of intracavity travelling wave intensity is shown

$$\frac{I_3}{I_2} = R_2 \quad ; \quad \frac{I_1}{I_4} = R_1$$

- The left travelling wave I_- grows in intensity from I_3 to I_4 on a single pass
- the right I_+ ... grows from I_1 to I_2
  - The total output intensity $I_{out} = T_2 I_2 + T_1 I_4$
- Consider purely homogeneously broadened system saturated gain $\gamma(z) = \dfrac{\gamma_0}{1 + (I_+ + I_-)/I_s}$
- I_- and I_+ grows according to $\gamma(z)$; $dI = I\gamma(z)dz$

$$\Rightarrow \frac{1}{I_+}\frac{dI_+}{dz} = -\frac{1}{I_-}\frac{dI_-}{dz} = \gamma(z)$$

$$\frac{d[I_+ \cdot I_-]}{dz} = I_+\frac{dI_-}{dz} + I_-\frac{dI_+}{dz} = -\gamma_z I_+ \cdot I_- + \gamma_z I_+ \cdot I_- = 0$$

- consequently $I_+ \cdot I_- = \text{constant} = C$.

$$\Rightarrow I_4 \cdot I_1 = I_2 \cdot I_3 = C.$$

$$\Rightarrow \text{from } \frac{I_3}{I_2} = R_2 \ ; \ (2) \qquad \frac{I_1}{I_4} = R_1 \ (1) \qquad \Rightarrow \frac{I_2}{I_4} = \sqrt{\frac{R_1}{R_2}}$$

$$\frac{(1)}{(2)}: \frac{I_1 \cdot I_2}{I_4 \cdot I_3} = \frac{R_1}{R_2}$$

$$\frac{C}{I_4^2} \cdot \frac{I_2}{\frac{C}{I_2}} = \frac{I_2^2}{I_4^2} = \frac{R_1}{R_2}$$

$$\Rightarrow \frac{I_2}{I_4} = \sqrt{\frac{R_1}{R_2}}$$

- For the right travelling wave

$$\frac{1}{I_+}\frac{dI_+}{dz} = g(z) = \frac{g_0}{1+\left(I_+ + \frac{C}{I_+}\right)/I_s}$$

- integration gives

$$g_0 L = \ln\left(\frac{I_2}{I_1}\right) + \frac{(I_2 - I_1)}{I_s} - \frac{C}{I_s}\left(\frac{1}{I_2} - \frac{1}{I_1}\right)$$

+ for the left travelling wave

$$g_0 L = \ln\left(\frac{I_4}{I_3}\right) + \frac{I_4 - I_3}{I_s} - \frac{C}{I_s}\left(\frac{1}{I_4} - \frac{1}{I_3}\right)$$

---

using $\frac{I_3}{I_2} = R_2$; $\frac{I_1}{I_4} = R_1$

$$I_4 I_1 = I_2 I_3 = C$$

$$\frac{I_2}{I_4} = \sqrt{\frac{R_1}{R_2}}$$

$$I_2 = \frac{I_s\sqrt{R_1}\,(g_0 L + \ln\sqrt{R_1 R_2})}{(\sqrt{R_1}+\sqrt{R_2})(1-\sqrt{R_1 R_2})}$$

- $I_4 = I_2\sqrt{\frac{R_2}{R_1}}$; $T_1 = 1 - R_1 - A_1$; $T_2 = 1 - R_2 - A_2$

$I_{out} = T_2 I_2 + T_1 I_1$; if $A_1 = A_2 = A$

$$\boxed{I_{out} = I_s\,\frac{(1-A-\sqrt{R_1R_2})}{1-\sqrt{R_1R_2}}\,(g_0L + \ln\sqrt{R_1R_2})}$$

for $T_1 = 0$; $R_1 = 1$

$$\boxed{I_{out} = T_2 I_2 = \frac{T_2 I_s\left[g_0 L + \frac{1}{2}\ln(1-A_2-T_2)\right]}{A_2+T_2}}$$

for a symmetrical resonator

$R_1 \cdot R_2 = R^2$

$R = 1 - A - T$

$$\frac{I_{out}}{2} = \frac{I_s}{2}\frac{(1-A-R)}{(1-R)}(g_0L + \ln R)$$

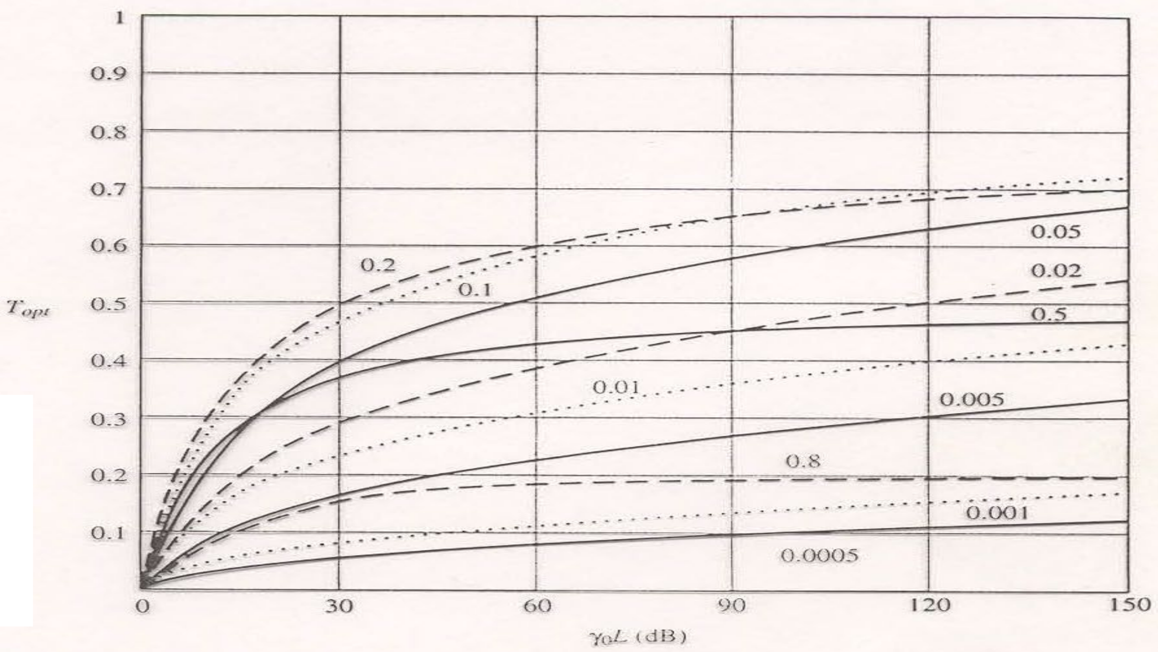
Optimum coupling

- to maximize the output intensity from the symmetr. resonator we must find the value of R such that

$$\frac{\partial I_{out}}{\partial R} = 0$$

$$\frac{T_{opt}}{A} = \left(\frac{1-A-T_{opt}}{A+T_{opt}}\right)\left[\gamma_0 L + \ln(1-A-T_{opt})\right]$$

Fig. 5.17. Calculated optimum coupling for a symmetrical resonator for various values of the loss paramater A and the unsaturated gain.



- for small losses $A + T_{opt} \ll 1$

$$\frac{T_{opt}}{A} = \sqrt{\frac{\gamma_0 L}{A}} - 1$$

Class problem. A laser (λ=2.09 μm, σ=1.15x10^{-20} cm^2, τ=8 ms) measured to have an intensity of 100 W/cm^2 emerging from one end of the laser, which has two identical mirrors each with transmission of 15%. The gain of the laser is also measured to be 0.5.

a) What is the loss parameter "A" in the cavity?

b) What is the optimum output mirror transmission?

$$a)\ \frac{I_{out}}{2} = \frac{I_s}{2}\frac{(1-A-R)}{(1-R)}(\gamma_o L + \ln R)$$

$$I_s = \frac{h\nu}{\sigma\tau} = \frac{6.62\times10^{-34}\cdot1.435\times10^{14}}{1.15\times10^{-20}\cdot8\times10^{-3}} = 1.03\frac{kW}{cm^2}$$

$$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{2.09\times10^{-6}} = 1.435\times10^{14}\,Hz$$

$$R = 0.85;\ I_{out} = 100W/cm^2;\ \gamma_o L = 0.5$$

Everything is given. Let us find A

$$\frac{100}{2} = \frac{1030}{2}\frac{(1-A-0.85)}{(1-0.85)}(0.5+\ln 0.85)$$

$$50 = 515\frac{(0.15-A)}{0.15}\cdot0.337;\quad A = 0.107$$

$b)$ Use $\dfrac{T_{opt}}{A} = \sqrt{\dfrac{\gamma_o L}{A}} - 1$ to find T_{opt}

$$T_{opt} = A\sqrt{\frac{\gamma_o L}{A}} - A = 0.107\sqrt{\frac{0.5}{0.107}} - 0.107 = 0.124 = 12.4\%$$