Calculus I

TEST 1A

September 22nd, 2004

Name: _____

- Calculators are allowed *only* for numerical calculations, that is you may not graph functions on your calculator.
- Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.

1. (20pts) Evaluate the following limits:

(a) (4pts) $\lim_{x\to 3} \frac{x^2-9}{x-3}$

(b) (4pts) $\lim_{x\to \infty} \frac{9-2x}{2+x^2}$

(c) (4pts) $\lim_{x\to \infty} \sqrt{x} - \sqrt{x-2}$

(d) (4pts) $\lim_{x\to \pi} \frac{3}{\sin x}$

(e) (4pts) $\lim_{x\to \infty} \frac{\sqrt{3x^2-1}}{x}$

2. (14pts) Sketch the graph of a function f that satisfies all of the following conditions:
$\lim_{x\to 3^-} f(x) = \infty$, $\lim_{x\to 3^+} f(x) = -\infty$, $\lim_{x\to \infty} f(x) = 5$, $\lim_{x\to -\infty} f(x) = -1$,
$\lim_{x\to 1^+} f(x) = -3$, $\lim_{x\to 1^-} f(x) = 2$, $f(1) = -1$.

3. (10pts) Let the function $f(x)$ be given by

$$f(x) = \begin{cases} \frac{x}{x^2+2x} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

Find the value for c that makes f continuous at $x = 0$.

4. (16pts) Consider the function $f(x) = \frac{1}{x+2}$.
(a) (8pts) Find $f'(x)$ by using the definition of the derivative.

(b) (8pts) Find an equation for the tangent line to the graph of $y = f(x) = \frac{1}{x+2}$ at the point (-3,-1). (If you are unable to obtain the answer to part (a) you may use that $f'(-3) = -1$).

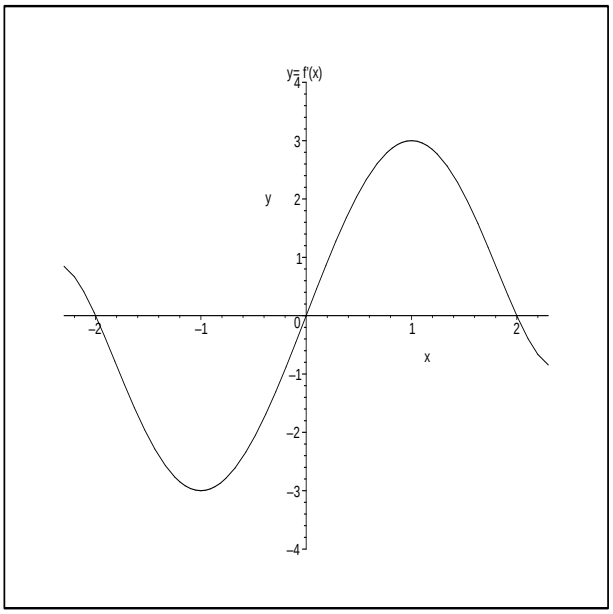
5. (10pts) Prove that there exists a number c such that $c^3 = 31$. State the name of the Theorem(s) you are using.

6. (12pts) If a ball is thrown up vertical with an initial velocity of 20 meters per second, then after t seconds its height in meters above the ground is given by $s(t) = 20t - 5t^2$.
(a) (8pts) Using the definition of the derivative, find the velocity of the ball at time t.

(b) (4pts) At what time does the ball have a velocity of 15 m/s?

(c) (Bonus, 5pts) At what time does the ball reach its maximal height? What is the maximal height?

7. (18pts) The graph of $y = f'(x)$ is given below. Note that this is **not** the graph of $y = f(x)$.



(a) (3pts) On what intervals is f decreasing or increasing?

(b) (3pts) At what values of x does f have a local maximum or minimum?

(c) (3pts) Where is the graph of f concave upward or downward?

(d) (3pts) Sketch a graph of $f''(x)$.

(e) (3pts) State the $x-$coordinate(s) of the point(s) of inflection of f.

(f) (3pts) If $f(0) = 0$, sketch a possible graph of f