CALCULUS I

April 20, 2006

Name (Print last name first): ....

Student ID Number: ....

TEST IV

PART I

Part I consists of 7 questions. Clearly write and circle your answer (only) in the space provided after each question. Do not show your work for this part of the test. No partial credit is awarded on Part I problems.

Each question is worth 4 points.

Question 1

Find the open interval where the function $f(x) = x^3 - 3x$ is decreasing.

Question 2

Find the open interval where the function $g(x) = 3x^4 - 4x^3$ is concave down.

Question 3

Does the curve $y = \frac{9}{10}\, x^{5/3}$ have an inflection point? [Your answer should be either yes or no!]

Question 4

Evaluate the following limit.

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

Question 5

Find the most general antiderivative of the function

$$f(x) = \frac{1}{1 + x^2}$$

Question 6

Find the general indefinite integral $\displaystyle\int \sec^2 x\, dx$

Question 7

Evaluate the definite integral $\displaystyle\int_0^2 x\, dx$

PART II

Each problem is worth 8 points.

Part II consists of 9 problems. You must show your relevant work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant step(s) will not get full credit.

Problem 1

Consider the function

$$f(x) = 2x^3 - 3x^2 - 12x.$$

(a) Find the local maximum value of the function $f(x)$.

(b) Find the local minimum value of the function $f(x)$.

Problem 2

Consider the curve

$$y = \frac{x^4}{12} - \frac{x^3}{6}.$$

(a) Find the open intervals where the curve is concave up. [You should find two intervals in all!]

(b) Find the inflection *points* of the curve.

Problem 3

Evaluate the following limits.

(a) $\displaystyle\lim_{x\to\infty} \frac{x+3}{2x-1}$

(b) $\displaystyle\lim_{x\to 0} \frac{x-\sin x}{x^2}$

Problem 4

A box with a *square base* and *open top* must have a volume of 4 ft^3. Find the *dimensions* of the box that minimize the amount of material used.

[Hint: The volume of a box is given by $V = x^2 h$, where x is the length of each edge of the square base and h is the height of the box. In this case, the area to be minimized is given by $A = x^2 + 4xh$ for an open top box.]

Problem 5

Let $s'(t) = 1 + e^t + \cos t$.

(a) Find $s(t)$. [Here you are required to write the most general expression for $s(t)$.]

(b) Find $s(t)$ if it is known that $s(0) = -4$.

Problem 6

Find the most general antiderivative of the following functions.

(a) $f(x) = 1/x$.

(b) $f(x) = -\sin x$.

(c) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$.

Problem 7

A particle moves along a straight line with (constant) acceleration $a(t) = 4$ ft/s^2.

(a) Find the velocity $v(t)$ of the particle if it is known that its initial velocity $v(0) = 0$.

(b) Find the position $s(t)$ of the particle if it is known that its initial position $s(0) = 0$.

Problem 8

Find the following general indefinite integrals.

(a) $\displaystyle\int \frac{x^2 + x + 1}{x}\,dx$

(b) $\displaystyle\int (\sin x - 2\cos x + e^x)\,dx$

Problem 9

Evaluate the following definite integral:

$$\int_0^1 \left(1 + 4x^3 - 6x + \frac{4}{1+x^2}\right)\, dx.$$

(Hint: Recall that $\tan^{-1} 1 = \dfrac{\pi}{4}$.)