Name: ________________

Calculus I; Fall 2007

Part I

Part I consists of 6 questions, each worth 5 points. Clearly show your work for each of the problems listed.

Evaluate the following limits:

(1)

$$\lim_{x\to\infty} \frac{2x^5 - 3x^2 + 7}{3x^5 + 3x^3 - 100}$$

(2)

$$\lim_{x\to 5} \frac{\sin(x+2)}{x^2+7}$$

(3)

$$\lim_{x\to 1} \frac{x^2 - 2x + 1}{x^2 - 1}$$

(4) Given that $\frac{-1}{x} \leq \frac{\sin(x)}{x^2+x} \leq \frac{1}{x}$ for $x > 0$, find $\lim_{x\to\infty} \frac{\sin(x)}{x^2+x}$

(5) If $f(3) = 5$ and $f'(3) = -1$, find the equation of the tangent line to the graph of $y = f(x)$ at $x = 3$.

(6) Given the graph of the function $y = f(x)$ below, list all places where this function is **not** continuous.

Part II

Part II consists of 5 problems each worth 14 points. If a problem has two parts, the first is worth 10 points and the second 4 points. Displaying only answers (even if correct) will not get you any points. You must show the relevant steps and justify your answer to earn credit.

(1) (a) If $f(x) = \frac{1}{x}$, find $f'(3)$ **using the limit definition**.

(b) Find the equation of the tangent line to $f(x) = \frac{1}{x}$ at $x = 3$.

(2) Evaluate the limit: $\lim_{x\to\infty} \sqrt{x} - \sqrt{x+100}$

(3) If $S(t) = -t^2 + 2t$ gives the position of a ball (in meters above the ground) at time t seconds $(0 \le t \le 2)$.

(a) Find the instantaneous velocity (using the limit definition) at time $t = 1$.

(b) What can you say about the position of the particle at time $t = 1$?

(4) Given the function

$$f(x) = \begin{cases} x^2 + 2 & \text{when } x \geq 2, \\ -x + 8 & \text{when } x < 2. \end{cases}$$

(a) Is the function $y = f(x)$ continuous at $x = 2$?

(b) Is the function $y = f(x)$ differentiable at $x = 2$? As always you must explain your answers!

(5) Given the graph of the function below, draw a reasonable graph of its derivative. [I suggest that the graph is a horizontal and vertical shift of $y = \frac{1}{x}$]