MA 125, CALCULUS I
December 03, 2007

Name (Print last name first): ....

Student Signature: ....

TEST IV

No calculators are permitted!

PART I - Basic Skills

Part I consists of 6 questions. Clearly write your answer in the space provided after each question.

Each question is worth 6 points.

Question 1

Find the absolute minimum value of the function $f(x) = x^3 - 3x + 1$ on the closed interval $[0, 2]$. (Be sure to give the y-coordinate!)

Answer: ....

Question 2

The function $f(x) = x - \frac{1}{2}x^2$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$. Find the number c that satisfies the conclusion of the Mean Value Theorem.

Answer: ....

Question 3

Find the critical number(s) of the function $f(x) = \frac{1}{3}x^3 + \frac{1}{5}x^5$.

Answer: ....

Question 4

Find the open interval on which the function $g(x) = x - \ln x$ is decreasing. (Clearly indicate the end-points of your interval!)

Answer: ....

Question 5

Find the part of the x-axis on which the function $h(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2$ is concave down.

Answer: ....

Question 6

Find the most general antiderivative of the function $f(x) = e^x - 5x^{2/3} + \frac{1}{1+x^2}$.

Answer: ....

PART II - Problem Solving Skills

Each problem is worth 16 points.

Part II consists of 5 problems. You must show your work to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Suppose that the derivative of a function f is given by

$$f'(x) = (x-2)^7(x^2-4)$$

Answer all the following questions.

(a) Find all the critical numbers of the function f.

(b) On what interval(s) is the function f increasing? (Justify your answer!)

(c) On what interval(s) is the function f decreasing? (Justify your answer!)

Problem 2

This problem has two separate questions. (Answer all the questions.)

(1) Find the dimensions of a rectangle with perimeter 80 ft and whose area is as large as possible.

(2) Find two positive numbers whose product is 49 and whose sum is minimal.

Problem 3

An object moves along a straight line with acceleration

$$a(t) = 4\cos t - \sin t.$$

Use antiderivatives to answer the following questions.

(a) Find the velocity function $v(t)$ of the object if its initial velocity is $v(0) = 3$.

(b) Find the position function $s(t)$ of the object if its initial position is $s(0) = 0$.

Problem 4

Consider the function f given by

$$f(x) = xe^{-x} \quad \left(\text{which can also be written as } f(x) = \frac{x}{e^x}\right).$$

Answer all the following questions.

(a) Find the x and y-intercept(s) of the curve.

(b) Find, if any, the vertical and horizontal asymptote(s) of the curve. [Hint: L'Hospital's Rule might prove useful here!]

(c) Find the (open) interval(s) of increase, and the (open) interval(s) of decrease. [Hint: Factoring out might prove useful in your calculations!]

(d) Find, if any, all local maximum and minimum value(s). [Be sure to give the y-coordinate(s)!]

(e) Find the open interval(s) where the function is concave down, and the open interval(s) where it is concave up. [Hint: Factoring out might prove useful in your calculations!]

(f) Find the inflection point. [Be sure to give the x and the y coordinate!]

(g) Use the information from parts (a)–(f) above to sketch the graph.