MA 125, Test 1

1. (4 each) Give the definition of the following terms:

a) the derivative of f at x is L;

b) f is continuous at x.

2. (4 each) Which of these two statements is true? Write "true" beside the true one, and give an example which disproves the false one:

a) If f is continuous at x, then f is differentiable at x;

b) If f is differentiable at x, then f is continuous at x.

3. (4 each) Calculate the derivative of f, any way you can, where $f(x)$ is:

a) $e^{\pi x}$;

b) $\sin(3x) + x\cos x$

c) $\sqrt{\tan x}$

d) $\frac{x^2 \sin x}{e^x}$;

e) e^{e^x}.

f) $x\left(\sqrt[3]{x}+e^{x^2}\right)$

g) $\frac{\sin(3x)}{\cos(2x)}$.

h) $xe^x \sin x$.

3. (4 each) Find $f'(x)$, straight from the definition, not using formulas, where:

a) $f(x) = x^2$;

b) $f(x) = \cos(x)$. (You may use $\lim_{h \to 0} \frac{\sin(h)}{h}$ without deriving.)

(4 each) Calculate the following limits, any way you can:

a) $\lim_{x \to 4^+} \left(\frac{x-2}{\sqrt{x}-2} \right)$;

b) $\lim_{h\to 0}\left(\frac{e^{(x+h)^2}-e^{x^2}}{h}\right)$.

5. (6) If the velocity of an object moving on the x axis at time t is te^t, find the acceleration.

6. (6)Find the equation of the tangent line to $y = e^{x^3}$ at $x = 1$.

7. (6) If the graph of the position of an object moving on the x axis is like this, for what t is the acceleration positive?

8. (6) Given the following two graphs, say which one is velocity and which one is acceleration. Say why.

9. (6) The length, width and height of a rectangular solid of kryptonite, being materialized from a poorly functioning matter transmitter in a lab, are all changing rapidly rapidly with time; their rate of increase is not constant. Recall that the volume of a rectangular solid is length times width times height. At $T = 78$ minutes, the height is 1 foot, the length is 2 feet, and the width is three feet. The rate of increase of height is is $\frac{1}{2}$ foot per second, that of length is 2 feet per second, and that of width is -3 feet per second (in other words, it is getting narrower). Find the rate of increase (or decrease) of volume.

10. (6) Is there an $x > 0$ such that

$$x^2 \sin(x) + x = \pi?$$

If so, prove it. If not, say why not.