Mathematics 125 **Midterm 1**
Feb. 7, 2002

1.(15 pts) Find the following limits *if* they exists. The values of the limits may take $+\infty$ or $-\infty$. Briefly justify your answers for each case. (If the limit does not exit, explain why not.)

a)

$$\lim_{x\to 2} \frac{1}{x^2+2x-3}.$$

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b)

$$\lim_{x\to 1} \frac{x-1}{x^2+2x-3}.$$

c)

$$\lim_{x \to 1} \frac{\sqrt{x-1}}{x-1}$$

d)

$$\lim_{x \to 0^+} \frac{|x|}{x}.$$

2.(20 pts) Suppose

$$f(x) = \frac{x^2 + 4}{(x-2)^2}.$$

a) Find the values of $f(0)$, $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2^-} f(x)$, $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

b) Using your answers from **a)**, draw the graph of $y = f(x)$.

3.(20 pts) Match the graph of each function in (a)-(d) with the grapf of its derivative in I-IV.

4.(20 pts) **a)** f is defined as follows;

$$f(x) = \begin{cases} x \cos \frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

Show f is continuous at $x = 0$. (Hint: What was the definition of continuity?)

b) g is defined as follows;

$$g(x) = \begin{cases} x^2 \cos \frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

Show g is differentiable at $x = 0$. (Hint: what was the definition of differentiability?)

5.(20 pts) The table shows the polulation $P(t)$ of Nepal (in millions) as of June 30 of the given year.

t	1975	1980	1985	1990	1995
P(t)	14.0	15.4	17.1	19.4	22.0

a) Estimate the value of $P'(1985)$. Explain your reasoning for the estimate.

b) Using your answer from **a)**, find the equation of tangent line to the graph $y = P(t)$ at $t = 1985$.

c) Estimate the value of $P(1986)$ using linear apporximation.