MA 125, Test 2

1. (5 each) Find $\frac{dy}{dx}$ where

a) $y = 2^{x^2}$,

b) $y = \ln\left(x^8 + 5\right)$

c) $x^4 + y^2 + 2xy = x^2 y$

d) $y = \tan^{-1}(e^x)$

e) $y = \cos^{-1}\left(e^{x^2}\right)$.

f) $\tan^2 y + xy^2 = x \sin x$

g) $y = \frac{e^{x^2} \cos^2\left(x^3\right)\left(\frac{x^2+1}{x^2}\right)^{10}}{(x^2+5)^7 (x^3+1)^{11} (x^4+3)^{10}}$.

h) $y = e^{\ln\left(\tan^{-1}\left(e^{x^2}\right)\right)}$

2. (10) A circular oil slick spreads in the ocean. The oil slick is 1 inch thick throughout. The volume of the oil spilling into the ocean is 1000 cubic feet per hour. Find the rate at which the radius of the oil slick is increasing when its radius is 5000 feet. (One inch is $\frac{1}{12}$ of a foot.)

3. (10) A person runs around an elliptical track at 6MPH. The equation of the track, units in miles, is

$$4x^2 + 12y^2 = 1$$

A friend is standing at the point $\left(\frac{1}{2}, 0\right)$, which is just at the end of the track. Find the rate at which the distance between the two is changing, when the runner is at the point $\left(\frac{\sqrt{3}}{4}, \frac{1}{\sqrt{48}}\right)$. Hint: the speed of the runner is $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$.

4. (10) Two trains are moving away from the terminal. They started 1 hour apart. The train that started earliest is going straight northeast at 60 miles per hour. The second train is going straight east at 70 miles per hour. What is the rate at which the distance between the trains is changing, three hours after the first one started?

5. (10) Water is being pumped into a conical tank at 5 cubic feet per minute, where t is the time in minutes since the process started. The volume of a cone is

$$V = \frac{1}{3}\pi r^2 h$$

The tank has radius 10 feet and height 20 feet. How fast is the height changing when the water is 3 feet deep?

6. (10) Graph

$$y = \frac{x^3}{3} - 2x^2 + x + 1,$$

showing concavity, critical points, inflection points and intervals on which the function is increasing or decreasing.

7. (10) Graph

$$y = xe^{-5x}$$

labelling all information clearly.