MA 125, Test 3

1. (5) Define

$$\int_a^b f(x)\,dx.$$

2. Show that if F and G are two antiderivatives of f, where f is continuous on $[a,b]$, then there is a constant C such that

$$F(x) = G(x) + C.$$

2. (5 each) Find:

a:

$$\lim_{x\to\infty} \frac{x^3}{e^x}.$$

b:

$$\lim_{x\to 1} \frac{x}{\ln x}$$

2. (5 each) Find:

a: $\int_1^2 x^2 e^{x^3}\,dx.$

b: $\int_1^3 \frac{x-x^2}{\sqrt{x}}\,dx$

2. (5 each) Find the most general antiderivative of f :

a: $f(x) = \frac{1}{1+4x^2}$

b: $f(x) = \sec^2(2x).$

3. (5 each) Find F'', when

a: $F(x)$ is the area under $y = \sin^2(s^2)$, between $s = 0$ and $s = x$.

b: $F(x) = \int_0^{x^2} e^{s^2}\,ds.$

4. (10) Find

$$\lim_{N\to\infty} \frac{2}{N} \sum_{i=1}^{N} \sin\left(3 + \frac{2i}{N}\right).$$

5. (10) Using the right end point, write a Riemann sum for

$$\int_0^2 e^{x^2}\,dx$$

dividing the interval up into 6 parts.

6. (10) The acceleration of an object moving on the real line is given by

$$A(t) = t^2.$$

When $t = 1$, the object is at $x = 2$ and has velocity 3. Find the position of the object as a function of time.

7. (10) Find the closest point on the curve $y = x^2$ to the point $(1,2)$. Show your point is closest.

8. (10) A cylindrical can with no top must hold a volume of 100 cubic inches. The cost of the bottom of the can is 2 cents per square inch. The cost of the sides is 1 cent per square inch. Find the dimensions of cheapest can and show it is cheapest.