Math125 Final Examination
April 25, 2002

Show all your work/reasoning/computations.
You may use results as discussed in class as long as they are quoted correctly
Calculators may be used *only* for numerical computations, that is,
no graphing and no programming functions are allowed

1.(20 pts) Find values of the following limits

a)

$$\lim_{x \to 2} \frac{x^2 - 4}{x^2 - x - 2}.$$

b)

$$\lim_{x \to +\infty} \frac{4\sqrt{x} - 5x}{2\sqrt{x} + 1}.$$

c)

$$\lim_{x \to 0^-} \frac{|x|}{x}$$

d)

$$\lim_{x \to 0} \frac{\tan x}{x}$$

Hint: Consider the limit

$$\lim_{x \to 0} \frac{\tan x - 0}{x - 0}$$

and recognize it as a derivative.

2.(20 pts) Differentiate the following functions.

a)

$$f(x) = (x^2 + 1)^5(3x - 2)^2$$

b)

$$f(x) = (1 + \frac{1}{x})^3.$$

c)

$$f(x) = \ln\{\tan(x^2 + 1)\}$$

d)

$$f(x) = \frac{\cos x}{\sin x}$$

3.(10 pts) Evaluate the following definite integrals.

a)

$$\int_1^4 (\sqrt{x})^3 dx$$

b)

$$\int_0^{2\pi} \cos x \, dx$$

4.(10 pts) Find the equation of the tangent line to the curve $x^3 + y^3 = 6xy$ at the point $(3, 3)$. (First show that $(3, 3)$ lies on the curve.)

6.(10 pts) A farmer had 1200 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

7.(15 pts) The graph of a function f is given as below.

a) List the following quantities A, B, C, D from the smallest to largest. Explain your reasoning.

$$A = \int_0^9 f(x)dx, \quad B = \int_6^9 f(x)dx, \quad C = \int_0^5 f(x)dx, \quad D = \int_0^6 f(x)dx.$$

b) Define $g(x)$ to be $\int_0^x f(t)dt$. Find the value of $g'(6)$.

c) Find x in $[0, 6]$ which maximizes $g(x)$.