1 Shorter Problems (3 points each part)

1. Give the definition of $f'(x)$.

2. Give the definition of $\int_a^b f(x)\,dx$.

3. Give the definition of what it means for f to be continuous at x.

4. State the intermediate value theorem.

5. State the mean value theorem.

6. Find $f'(x)$, any way you like.
a) $f(x) = e^{x^3}$;

b) $f(x) = \sin(x)\cos\left(x^2\right)$

c) $f(x) = \frac{\tan x^2}{\tan x}$.

d) $f(x) = \frac{x \tan x \sin x}{e^x \cos x}$

e) $f(x) = y$, where

$$x^2 + 2xy^2 + e^y = 1.$$

f) $f(x) = \int_0^{x^2} \sin\left(x^2\right) dx$.

g) $f^{-1}(x)$, where

$$f(x) = \int_0^x e^{s^2} ds$$

7. Find $\int_2^3 f(x)\,dx$ any way you like.
a) $f(x) = \frac{x+x^2}{\sqrt[3]{x}}$.

b) $f(x) = \sec^2 x$.

c) $f(x) = x^2 \cos\left(x^3\right)$.

d) $f(x) = \frac{1}{2x}$.

2 Longer problems: 5 points each, except no. 10

1. Find, directly from the definition, the derivative of $f(x) = x^{\frac{1}{2}}$.

2. Write a Riemann sum for $\int_0^1 e^{x^2} dx$, using right endpoints, dividing the interval up into 4 parts, and CALCULATING THE ANSWER.

3. A spherical balloon is being inflated at 2 cubic feet per second (it is a big balloon). Find the rate at which the radius is changing when the radius is 2.

4. Find

$$\lim_{N\to\infty} \frac{5}{N} \sum_{i=1}^{N} x_i \cos\left(x_i^2\right).$$

5. Find

$$\lim_{h \to 0} \frac{e^{(2+h)^2} - e^4}{h}.$$

6. The acceleration of an object moving on the x axis is given by

$$a(t) = t^3.$$

When $t = 0$, the object is at $x = 1$. When $t = 1$, the velocity is 2. Find the position as a function of time.

7. Graph the function

$$y = \frac{x^2 + 1}{x}$$

on the interval $(0, \infty)$. Give all the information you can give.

8. Find the dimensions of the rectangle of largest area which has its base on the x axis and its other two vertices above the x axis and lying on the parabola $y = 8 - x^2$.

9. A paper cup has the shape of a cone with height 10 cm and radius 3cm at the top. If water is poured into the cup at a rate of 2 cubic centimeters per second, and water is running out a hole in the bottom at the rate of 1 centimeter per second, how fast is the water rising when the water is 5 centimeters deep?

10. (10 points) A metal storage tank with volume V, a constant, is to be constructed in the shape of a right circular cylinder with a hemisphere on top. No bottom of the tank is needed, as the bottom will be made of wood. What dimensions will require the least amount of metal? The area of a sphere is $4\pi r^2$. A hemisphere is a half-sphere.