

Mathematics 125 **Midterm 1**

Spet. 26, 2001

1.(25 pts) Find the following limits *if* they exists. The values of the limits may take $+\infty$ or $-\infty$. Briefly justify your answers for each case.

a)

$$\lim_{x \rightarrow 1} \frac{1}{x^2 - 4}.$$

b)

$$\lim_{x \rightarrow 2^+} \frac{1}{x^2 - 4}.$$

c)

$$\lim_{x \rightarrow 2} \frac{1}{x^2 - 4}.$$

d)

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}.$$

e)

$$\lim_{x \rightarrow +\infty} \frac{x^2 + 4}{x^2 - 4}$$

2.(20 pts) **a)** f is defined as follows;

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

Show f is continuous at $x = 0$.

b) g is defined as follows;

$$g(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

Show g is differentiable at $x = 0$.

3.(15 pts) Given $f(x) = x^3 + 2\sin(\pi x) + 3$, show that there exists a number c such that $f(c) = 15$

4.(20 pts) **a)** Use the definition of a derivative to find $f'(2)$, where $f(x) = x^2 - 2x$.

b) Find an equation of the tangent line to the curve $y = x^2 - 2x$ at the point $(2, 0)$.

c) Find a linear approximation of f at $x = 2$ to estimate the values $f(1.9)$ and $f(2.05)$.

d) Are the answers in c) underestimated or overestimated? Explain.

Hint: First draw the graphs of f and its linear approximation at $x = 2$.

5.(20 pts) Water is flowing into three containers of different shapes; A) spherical, B) cylindrical and C) conical (point-down). Let $H_A(t)$ represent the height of the water level of the container A at time t , and $H_B(t)$ $H_C(t)$ defined accordingly.

a) For each function, select a graph which best represents its behavior from the six graphs below.

b) For the derivative of each function (that is, $H'_A(t)$, $H'_B(t)$ and $H'_C(t)$) select a graph which best represent its behavior from the six graphs below.

Remark: You may want to compare the answers from a) and b) to see they are indeed consistent.