Math125 Final Examination
December 14, 2001

Show all your work/reasoning/computations.
You may use results as discussed in class as long as they are quoted correctly
Calculators may be used *only* for numerical computations, that is,
no graphing and no programming functions are allowed

1.(15 pts) Find values of the following limits

a)

$$\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 4x + 3}.$$

b)

$$\lim_{x \to +\infty} \frac{4x^2 - 5x}{2x^2 + 1}.$$

c)

$$\lim_{x \to 0} \frac{\ln(x+1)}{x}$$

Hint:Express the limit as a derivative of a function and thus evaluate it.

2.(15 pts) Differentiate the following functions.

a)

$$f(x) = (2x+1)^5(3x-2)^2(4x+1)^3$$

b)

$$f(x) = (1+\frac{1}{x})^3.$$

c)

$$f(x) = \int_0^x \sin t^2 dt.$$

3.(10 pts) Evaluate the following definite integrals.

a)

$$\int_1^4 \frac{1}{\sqrt{x}} dx$$

b)

$$\int_0^{\pi/2} \sin x dx$$

4.(10 pts) **a)** Show that the function $f(x) = |x|$ is continuous at $x = 0$ *using* the defintion of continuity of a function.

b) Show that $f(x) = |x|$ is NOT differentiable at $x = 0$ *using* the definition of derivative of a function.

5.(10 pts) **a)** Find the linear approximation of the function $f(x) = \sqrt{x+1}$ at $x = 3$

b) Estimate the value of $\sqrt{3.95}$ using the result from the part **a**.

6.(10 pts) Find the absoulute maximum and minimum of the funtion $f(x) = \sin x + \cos x$ over $[0, \pi/3]$.

7.(10 pts) If V is the volume of a cube with edge length x and the cube expands at the constant rate of $300cm^3/sec.$. Find the rate at which the edge length x increases when $x = 10cm$.

9.(10 pts) A box with a square base and open top must have a volume of $32cm^3$. Find the dimension of the box (i.e. the height and the side length of the square) that minimizes the amount/area of the material.

9.(10 pts) The graph of a function f is given as below.

a) List the following quantities A, B, C, D from the smallest to largest. Explain your reasoning.

$$A = \int_0^9 f(x)dx, \quad B = \int_6^9 f(x)dx, \quad C = \int_0^5 f(x)dx, \quad D = \int_0^6 f(x)dx.$$

b) Define $g(x)$ to be $\int_0^x f(t)dt$. Find the value of $g'(6)$.