MA 126, Calculus 2 UAB, Fall 2002

TEST 1

Duration 105min;

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results on this paper. Good luck!

1. Consider the integral

$$I = \int_0^2 \left(\frac{x^3}{4} - 1\right) dx$$

(a) Write it as a limit of Riemann sums (with right endpoints) where the number n of subintervals tends to infinity. Define all notation you introduce (e.g. $x_i = ...?$)

(b) Evaluate the Riemann sum of part (a) for $n = 2$ subintervals.

2. Evaluate the following derivatives

$$(a) \quad \frac{d}{dx}\int_1^x \frac{1}{\sin(t)}\, dt \qquad\qquad (b) \quad \frac{d}{dx}\int_{-1}^{1/x} e^{\tan(t)}\, dt$$

3. (a) Make a suitable substitution to evaluate

$$\int \frac{5 - 14x}{\sqrt{1 + 5x - 7x^2}}\, dx.$$

(b) Use integration by parts to find

$$\int_0^1 xe^x\, dx.$$

4. Write out the form of the partial fraction expansion of the function. Do not determine the numerical values of the coefficients.

$$2\frac{2x^2 + 5 - 4x}{(x-1)(x^2+1)}.$$

5. Evaluate the following integrals

(a) $\displaystyle\int_1^2 \frac{1+u}{u^2}\,du$

(b) $\displaystyle\int \frac{1}{4x-7}\,dx$

(c) $\displaystyle\int x^{1/2}\sin(x^{3/2})\,dx$

(d) $\displaystyle\int x^2\cos(x)\,dx$

(e) $\displaystyle\int_{\pi/2}^{\pi}(\sin x)^3\,dx$

(f) $\displaystyle\int \frac{\ln x}{x^3}\,dx$

(g) $\displaystyle\int \frac{x^3}{x^2-4}\,dx$

6. (a) Approximate

$$4\int_0^1 \frac{1}{1+x^2}\,dx$$

using Simpson's rule with $n = 2$ subintervals.

(b) How large do you need to take n in the Trapezoidal Rule to guarantee that the approximation of

$$\int_0^1 e^{-x^2/2}\,dx$$

is accurate to within 0.001? (Error bound: $K(b-a)^3/(12n^2)$).