MA 126, Calculus 2 UAB, Spring 2002

TEST 1

Duration 105min; Max. Points: 36

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results on this paper. Good luck!

1. (6 pts.) Consider the integral

$$I = \int_0^2 \frac{1-x^2}{2}\,dx$$

(a) Write it as a limit of Riemann sums (with right endpoints) where the number n of subintervals tends to infinity. Define all notation you introduce (e.g. $x_i = ...?$)

(b) Evaluate the Riemann sum of part (a) for $n = 2$ subintervals.

(c) Interpret the value of the integral I in terms of areas. To this end, sketch a graph of the integrand, label the various areas in this graph, and express I in terms of these areas.

2. (4 pts.) Evaluate the following integrals

$$(a) \quad \int_1^2 \frac{x^2+1}{\sqrt{x}}\,dx \qquad\qquad (b) \quad \int_0^\pi (2e^x - 4\sin(x))\,dx$$

3. (4 pts.) Find the derivative of the function f

$$(a) \quad f(x) = \int_1^x \sin(t^2)\,dt \qquad\qquad (b) \quad f(x) = \int_{\sqrt{x}}^{3x} \frac{\cos(t)}{t}\,dt$$

4. (4 pts.) (a) Make a suitable substitution to evaluate

$$\int \frac{1+4x}{\sqrt{1+x+2x^2}}\,dx.$$

(b) Use integration by parts to find

$$\int x\cos(2x)\,dx.$$

5. (4 pts.) Write the following rational function as a sum of partial fractions

$$\frac{2x^4 + 5x^2 - x + 3}{x^3 + x}.$$

6. (10 pts.) Evaluate the following integrals

(a) $\displaystyle\int_{-2}^{2} x^2\sqrt{x^3 + 8}\, dx$

(b) $\displaystyle\int xe^{x/3}\, dx$

(c) $\displaystyle\int \frac{\cos(x)}{\sin^2(x)}\, dx$

(d) $\displaystyle\int x^3 \ln(x)\, dx$

(e) $\displaystyle\int \frac{x^3}{x^2 - 1}\, dx$

7. (4 pts.) The pressure of the air $p(z)$ as a function of the height z above ground satisfies the equation

$$\frac{dp}{dz} = -g\varrho, \qquad g = 9.81\ m/s^2, \quad \varrho = 1.3\ kg/m^3$$

where g is the gravitation constant and ϱ is the density of the air. Use the evaluation theorem (second part of the Fundamental Theorem of Calculus) to find the difference in the air pressure between your nose and your toes assuming that your nose is 1.6m ($\cong$ 5ft. 3in) above your toes.