

TEST 2

Duration 105min; Max. Points: 40

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results on this paper. Good luck!

1. (4 pts.) (a) Set up the integral for the length of the graph of $y = x^2$ for $0 \leq x \leq 1$. Do not try to evaluate it!
2. (4 pts.) Sketch the region bounded by the graphs of $y = \sqrt{1 - x^2}$, $2 - \sqrt{x}$, and the lines $x = 2$. Compute the volume of the solid obtained by rotating this region about the x -axis.
3. (8 pts.) Which of the following improper integrals are convergent? Evaluate the ones which are.

(a) $\int_0^\infty \sin(3x) dx$

(b) $\int_0^3 x \ln(x) dx$

(c) $\int_0^\infty \frac{1}{x^2 + 1} dx$

(d) $\int_{-2}^2 \frac{1}{|x|^{3/2}} dx$

4. (8 pts.) Determine whether the given sequence converges. If so, find the limit.

(a) $a_n = \frac{\ln(n^2)}{n}$

(b) $a_n = \frac{n - 1/n + 1}{\sqrt{n} + 5}$

(c) $a_n = \sin(n\pi/2) + \frac{(-1)^n}{\sqrt{n}}$

(d) $a_n = n \tan(2/n)$

5. (6 pts.) Determine whether the give series converges. If so, find its sum.

(a)
$$\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$$

(b)
$$\sum_{n=0}^{\infty} 3^n e^{-n}$$

(c)
$$\sum_{n=0}^{\infty} \arctan(n+1) - \arctan(n)$$

6. (4 pts.) Write the number $1.\overline{23}$ as a ratio of two integers.

7. (6 pts.) Determine whether the give series converges or divergent.

(a)
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$

(b)
$$\sum_{n=1}^{\infty} \frac{\sin(n)}{2^n + 1}$$

(c)
$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$