MA 126, Calculus 2 UAB, Spring 2002

TEST 2

Duration 105min; Max. Points: 40

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results on this paper. Good luck!

1. (6 pts.) (a) Use the Simpson rule with two subintervals to find an approximation for

$$\int_0^1 e^{-x^2}\,dx.$$

(b) Estimate the error in part (a). You may use that the 4th derivative of $f(x) = e^{-x^2}$ is given by $f^{(4)}(x) = (12 - 48x^2 + 16x^4)e^{-x^2}$.

(c) How large do you need to choose the number n of subintervals to guarantee an accuracy of 0.001 for the estimate by the Simpson rule? (If you did not solve (b), work with $K = 90$.)

2. (4 pts.) (a) Set up the integral for the length of the curve given by the parametric equations

$$x = \sqrt{2}\cos(t), \quad y = \sin(t), \quad 0 \le t \le \pi.$$

(b) Use the trapezoidal rule with $n = 2$ subintervals to find an approximation for the integral you obtained in part (a).

3. (4 pts.) Sketch the region bounded by the graph of $y = 1/x$ and the lines

$$y = 0, \quad x = 1, \quad x = 2.$$

Compute the volume of the solid obtained by rotating this region about the x-axis.

4. (6 pts.) Which of the following improper integrals are convergent? Evaluate the ones which are.

$$(a) \quad \int_0^\infty \cos(x)\,dx$$

$$(b) \quad \int_e^\infty \frac{\ln(x)}{x}\,dx$$

$$(c) \quad \int_0^3 \frac{1}{\sqrt{x}}\,dx$$

5. (4 pts.) Does the following integral converge? Prove your answer by referring by name or statement to a theorem discussed in class.

$$\int_1^\infty \frac{\sin(x^2)}{1+x^2}dx.$$

6. (8 pts.) Determine whether the given sequence converges. If so, find the limit.

$$(a) \quad a_n = \frac{1+2n+3n^2}{n^{2/5}-1-1/n}$$

$$(b) \quad a_n = \frac{\ln(n)}{\ln(n^2+1)}$$

$$(c) \quad a_n = \frac{(-1)^n \arctan n}{n}$$

$$(d) \quad a_n = \frac{2}{n} + \sin(n\pi/2)$$

7. (8 pts.) Determine whether the give series converges. If so, find its sum.

$$(a) \quad \sum_{n=1}^{\infty} 5^{2-n}2^n$$

$$(b) \quad \sum_{n=1}^{\infty} \frac{n-1/n}{n}$$

$$(c) \quad \sum_{n=1}^{\infty} \cos(1/n) - \cos(1/(n+1))$$

$$(d) \quad \sum_{n=1}^{\infty} \frac{2}{n^{0.5}} - (0.1)^{n-1}$$