MA 126, Calculus 2 UAB, Fall 2002.

FINAL EXAM

Duration 140 min, Max. Points: 42 (+ 4 Bonus).

1. Determine whether the given series is convergent or divergent. Be sure to justify the validity of the tests you use. (6 points)

$$(a) \quad 1 - \frac{1}{2^{1/3}} + \frac{1}{3^{1/3}} - \frac{1}{4^{1/3}} + \frac{1}{5^{1/3}} - \dots$$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^3 + 8}$

(c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{5^n}$

2. Find the radius and the interval of convergence. (6 points)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n)!} x^n$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \sqrt{n} (x+3)^n$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2 4^n}$

3. Find the Maclaurin series of the function $f(x)$ and its interval of convergence.

(a) $f(x) = 2^x$

(b) $f(x) = \dfrac{x^2}{1+x}$

(6 points)

4. Find the Taylor series for $f(x) = \sin(x)$ about $x = \pi/4$. (4 points)

5. Find the largest number d for which Taylor's inequality guarantees that the approximation

$$e^{-x} \simeq 1 - x + x^2/2, \qquad \text{for } 0 \leq x \leq d$$

is accurate to within 0.01. (4 points)

6. Make an accurate sketch of the graph of $y = 2 - x^2$, and shade the region bounded by this graph and the line $y = 1$. Then find the volume of the solid obtained by rotating this region about the x-axis. (4 points)

7. Let $\vec{a} = \langle 3, 1, -1 \rangle$ and $\vec{b} = \langle 4, 0, 2 \rangle$. (6 points)

(a) Compute the length of $\vec{a}$ and the length of $\vec{b}$.

(b) Find the dot product of $\vec{a}$ and $\vec{b}$ and the angle between these two vectors.

(c) Compute the vector product $\vec{a} \times \vec{b}$

8. Evaluate two of the following four integrals. (If you do more than two, label those you want credit for.) (6 points)

(*a*) $\displaystyle\int_0^1 \frac{x}{1+x^2}\,dx$

(*b*) $\displaystyle\int \frac{\cos(1/t)}{t^2}\,dt$

(*c*) $\displaystyle\int x^3 \ln(x)\,dx$

(*d*) $\displaystyle\int \frac{x}{3x+2}\,dx$

Bonus. The parametric equations (4 points)

$$\begin{aligned} x(t) &= e^{-t}\sin(t) \\ y(t) &= e^{-t} \end{aligned}$$

with $0 \leq t < \infty$ describe the (infinite) curve below. Set up an improper integral for the length of this curve. Show that this improper integral converges and that the length of this curve is less than $\sqrt{3}$.