Name:_____

Calculus II; Spring 2009, Exam I

Part I

Show your work; justify your answers

(1) (5 points) Find the equation of the sphere with center $(1, 2, 3)$ that passes through the point $(-2, 3, 1)$.

(2) (5 points) Find the angle between the vectors $< 1, -1, 3 >$ and $< -1, 3, -2 >$.

(3) (5 points) Find the equation of the line through the point (1,2,3) and parallel to the line

$$\ell = \begin{cases} x = -1 + 2t \\ y = 1 + t \\ z = 4 - t \end{cases}$$

(4) (5 points) Find the equation of the plane containing the points $(0, 0, 0)$, $(1, 2, 3)$ and $(-1, -1, 1)$.

(5) (5 points) Are the vectors $< -1, -1, 3 >$ and $< 3, 0, 1 >$ orthogonal?

(6) (5 points) If $\vec{r}(t) =< t^3, \cos(t), \sin(t) >$ find the tangent vector $\vec{r}\,'(t)$.

Part II

Show all of your work–justify your answers.

(1) (14 points) Find the distance from the point $(1, 2, 3)$ to the plane $3x - y + z = 4$.

(2) (14 points) Find the intersection of the plane $2x + y - z = 4$ and the line

$$\begin{cases} x = 1 + t \\ y = 2 - t \\ z = 3 + 2t \end{cases}$$

(3) (14 points) Are the vectors $< 1, 2, 3 >$, $< 0, 1, 2 >$ and $< -1, 0, 3 >$ coplanar?

(4) (14 points) If $\vec{r}(t) =< t^2, \ln(t), \sin(\pi t) >$ represents the position of a particle at time $t > 0$, find
   (a) The velocity vector at time $t = 1$,
   (b) the speed at time $t = 1$,
   (c) the unit tangent vector at time $t = 1$.

(5) (14 points) Given two lines

$$\ell_1 = \begin{cases} x = 1 + t \\ y = 1 - t \\ z = 2 + 3t \end{cases} \quad \text{and } \ell_2 = \begin{cases} x = 1 - t \\ y = 1 + 2t \\ z = 1 - t \end{cases}$$

determine if they are parallel, skew or intersect.
If they intersect, find the point of intersection; if they are skew find the distance between the two lines.