

Calculus II, Final Exam, Spring 2009

Name: _____

Show all your work and give reasons for your answers. Good luck!

Part I

- (1) (5 points) Find the angle between the vectors $\langle 1, -3, 4 \rangle$ and $\langle -2, 3, -1 \rangle$. (You can express your answer in terms of $\arccos$.)

- (2) (5 points) Find the equation of the line perpendicular to the plane $x + y - z = 4$ which passes through the point $(2, 1, 3)$.

(3) (5 points) Evaluate $\int_0^{1/2} \frac{1}{x^2+1} dx$.

(4) (5 points) Evaluate $\int x^2 \sin(x^3) dx$.

(5) (5 points) Evaluate $\int x \sin(x) dx$.

(6) (5 points) If $F(x) = \int_0^x \sin(t^2) dt$, find $F'(x)$.

- (7) (5 points) Find the area between the graphs of the functions $y = \sqrt{x}$ and $y = x^2$ for $0 \leq x \leq 1$.

- (8) (5 points) Find the sum and the interval of convergence of the series $\sum_{n=0}^{\infty} (\frac{x}{2})^n$.

Part II

(1) (12 points) Evaluate $\int \frac{1}{x^3+x} dx$.

- (2) Given the region bounded by the graphs of $y = x^2$, $y = x^3 + x + 5$, $x = 0$ and $x = 1$, **set up but do not evaluate** integrals for the the following solids of revolution:
- a) (6 points) Set up an integral for the volume of the solid obtained by rotating the above region about the line $y = -1$.

b) (6 points) Set up an integral for the volume of the solid obtained by rotating the above region about the line $x = -1$.

- (3) (12 points) Find the work done in pumping all the water out of an inverted cone of height $h = 10\text{ m}$ and radius $r = 3\text{ m}$. (You may use that the density of water is 1000 kg/m^3 and that $g \approx 10\text{ m/sec}^2$).

- (4) (12 points) Find the Maclaurin series for the function $f(x) = \frac{x^2}{5+x}$ and **state the interval of convergence.**

- (5) (12 points) Use Maclaurin series to evaluate $\int_0^{1/10} e^{(-x^2)} dx$ with an error less than 10^{-5} . (**You do not need to compute and add the terms of the series.**)